AN ROINN OIDEACHAIS AGUS EOLAÍOCHTA | DEPARTMENT OF EDUCATION AND SCIENCE

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Matamaitic *Gnáthleibhéal*

Marking Scheme *Leaving Certificate Examination, 2001*

Mathematics *Ordinary Level*

An Roinn Oideachais agus Eolaíochta

Leaving Certificate Examination 2001

Marking Scheme

MATHEMATICS - Ordinary Level

Paper 1

General Instructions

Penalties are applies as follows:

Numerical slips, misreadings	(-1)
Blunders, major omissions	(-3)

Note 1: The list of slips, blunders and attempts given in the Marking Scheme is not exhaustive.

Note 2: A serious blunder, omission or misreading which oversimplifies merits the attempt mark at most.

Note 3: The same error in the same section of a question is penalised once only.

Note 4: The attempt mark is awarded only after relevant deductions are made.
A mark between 0 and the attempt mark may not be awarded.
Particular cases or verifications usually qualify for the attempt mark.

Note 5: Mark all of the candidate's work, including excess answers and repeated answers whether cancelled or not, and allow the highest scoring answers.

Note 6: The phrase "and stops" used in the marking scheme means that no more work is shown by the candidate.

Note 7: Special notes relating to the marking of a particular part of a question are indicated by *.

Note 8: Solution methods are labelled I, II, etc.

QUESTION 1

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 marks** **Att 3**

> **(a)** A cookery book gives the following instruction for calculating the amount of time for which a turkey should be cooked:
>
> "Allow 15 minutes per 450 grammes plus an extra 15 minutes."
>
> For how many hours and minutes should a turkey weighing 9 kilogrammes be cooked?

(a) Time **10 marks** **Att 3**

I $\frac{9000}{450} = 20$	**II** $\frac{450}{9000} = \frac{1}{20}$ or 0.05	**III** $1g = \frac{15}{450}$ or 0.033	**IV** $\frac{450}{15}$ or 30g	step 1 ...3m
(20x15) = 300 m	$15 \div \frac{1}{20} = 300m$	$9000 \times \frac{15}{450} = 300m$	$\frac{9000}{30} = 300m$	step 2 ...7m
300 + 15 = 315m = 5 h 15m	300 + 15 = 315m = 5 h 15m	300 + 15 = 315m = 5h 15m	300 + 15 = 315m =5h 15m	step 3 ..10m

* 5h 15m, without work: 10 m. 315m, no work: 9m. 300 without work and stops: 7m
Step 1 without work and stops (i.e. 20, 1/20, .05, 15/450, .03, 450/15, or 30): 3m.

* Candidate may reverse steps 1 and 2, or combine them into one calculation.

Blunders (-3)

B1 9kg $\neq$ 9000g, and continues, e.g. 9 / 450 in **I** above, and not rectified. Or, 1000/450.

B2 Incorrect operation, e.g. multiplies instead of divides, or vice versa. Apply once.

B3 Calculation error handling fractions (e.g. multiplying, cancelling or dividing). Once.

B4 Error handling extra 15m, e.g. 20 x (15+15) = 600 = 10hr, or failure to add 15m on.

B5 Each incomplete calculation after step 1, e.g. (9000 x 15)/450 and stops: 2xB5 => 4m.

Attempts (3 marks)

A1 Mention of 9000.

A2 Part of a step correct and stops, e.g. 9000/450 and stops (see method **I**).

A3 Any correct relevant statement and stops, e.g. 15m = ¼ h, or 450g = .45kg.

A4 9 ÷ 4= 2 ¼ (hours) and stops, or 9 x 15 = 135 (minutes) and stops.

A5 Final 15 minutes added to cooking time.

Slips (-1)

S1 Incorrect or no conversion from mins to hours and mins,
e.g. 315m and stops, or 315m = 3h.15m, or 5h 25m, or 6h 15m.

Worthless (0)

W1 15 x 450 = 6750 and stops.(May earn marks if continued).

Part (b) **20 (10, 10) marks** **Att 6(3, 3)**

> **(b) (i)** The answer to 3.58 + 2.47 was given as 6.50.
> What was the percentage error correct to one decimal place?
>
> **(ii)** Calculate the value of $\frac{3.1 \times 10^5 - 1.5 \times 10^4}{5.9 \times 10^6}$
> and write your answer as a decimal number.

(b) (i) % error **10 marks** **Att 3**

I:	**II:**
3.58 + 2.47 = 6.05 …(3m)	3.58 + 2.47 = 6.05 ...(3m)
6.5 – 6.05 = 0.45 …(7m)	$\frac{6.50}{6.05}$ x 100 = 107.43 ...(7m)
$\frac{0.45}{6.05}$ x 100 = 7.43	107.43 - 100 = 7.43
= 7.4 % …(10m)	= 7.4 % ...(10m)

* Answers without work: 7.4 merits 10m; 7.43 merits 9m; 0.45 or 107.43 merit 7m; and 6.05 merits 3m. Other incorrect answers without work and stops: 0m

Blunders (-3)

B1 Incorrect ratio, e.g. 0.45 / 6.50 in method **I**, or 6.05/6.50 in method **II**.

B2 0.45 = 45% and stops.

Misreading (-1)

M1 Multiplies, subtracts or divides the given numbers, and continues.

Slips (-1)

S1 Incorrect or no rounding off.

Attempts (3 marks)

A1 A fraction multiplied by 100, e.g. (6.5/3.58) x 100 = 181%

A2 An effort to add the given numbers and stops, e.g. 3.58 + 2.47 = 5.05 and stops.

(b)(ii) Exp. **10 marks** **Att 3.**

$\frac{10^4(3.1 \times 10 - 1.5)}{5.9 \times 10^6}$ …(3m)	$\frac{3.1 \times 10^5 - 0.15 \times 10^5}{5.9 \times 10^6}$ …(3m)	$\frac{3.1 \times 100000 - 1.5 \times 10000}{5.9 \times 1000000}$ …(3m)
$= \frac{10^4(29.5)}{5.9 \times 10^6}$ …(7m)	$= \frac{2.95 \times 10^5}{5.9 \times 10^6}$ …(7m)	$= \frac{295000}{5.9 \times 10^6}$ …(7m)
= 0.05 …(10m)	= 0.05 …(10m)	= 0.05 …(10m)
IV: Calculator: Correct answer without work: 10m.		

* For 0.5 x 10^{-1}, *or* 0.5 ÷ 10, or equivalent: award 7m.

* Numerator calculated correctly (with or without work) and stops: award 7m e.g. 295000, or 29.5x10^4, or equivalent, merits 7m.

* (310000 –15000)/ 5900000 and stops, *or* 310000 - 15000 and stops: award 4m.

* No penalty if candidate writes 3.1 x 10^5 as 3.1^5 unless it is *used* as such.

Blunders (-3)

B1 Mathematical error in indices, subtraction, multiplication, cancellation, etc., and continues. Apply *each time*. e.g. cancelling powers 4 and 6; adding or subtracting powers 5 and 4. And 1.6 x 10^n where n = 1, 9 or 20 etc, is *two* blunders on numerator.

For $\frac{3100\not{0}\not{0} - 15\not{0}\not{0}\not{0}}{59\not{0}\not{0}\not{0}\not{0}\not{0}} = \frac{3100 - 15}{59}$ apply B1 + B1.

B2 Error in decimals or the number of zeros, e.g. 3.1 x 10^5 = 3100000. Once if consistent.

B3 Answer not expressed as a decimal.

Attempt (3 marks)

A1 One correct operation, 3.1 x 10^5 = 310000; 1.5 x 10^4 = 15000; or 5.9x10^6=5900000.

A2 A division done correctly, e.g. 1.6 ÷ 5.9 – 0.27.

A3 $\frac{3.1 \times 50 - 1.5 \times 40}{5.9 \times 60}$ and then continues correctly.

Part (c) C.I. **20 (5, 5; 10)** **Att 7 (2, 2; 3)**

(c)	IR£5000 was invested for 3 years at compound interest. The rate for the first year was 4%. The rate for the second year was 4½%. **(i)** Find the amount of the investment at the end of the second year. At the beginning of the third year a further IR£4000 was invested. The rate for the third year was *r*%. The total investment at the end of the third year was IR£9811.36. **(ii)** Calculate the value of *r*.

(c) (i) year 1 **5 marks** **Att 2**

P(1) = 5000 I(1) = <u>200</u> P(2) = 5200 …(5m)	A = 5000 x 1.04 = 5200 …(5m)	$A = P(1 + r/100)^n$ $A = 5000(1 + 4/100)^1$ A = 5200 …(5m)	$I = \frac{P.T.R}{100}$ $I = \frac{5000x1x4}{100}$ I=200 =>A = 5200 …(5m)

* For marking scheme see after next section. Mark (i) out of 5m + 5m, not out of 10m.

and year 2 **5 marks** **Att 2**

I (2) = 234 A = 5434 …(5m)	A = 5200 x 1.045 A = 5434 …(5m)	$A = 5200(1 + 4.5/100)^1$ A = 5434 …(5m)	$I = \frac{5200x1x4.5}{100}$ …(2m) I=234=> A = 5434 …(5m)

* Correct answer(s), without work: full marks (5m + 5m).

* Simple interest for 2 years with *r* = 4, 4½ , 4¼ or 8½ %, finished correctly: att 2+ att 2

* Simple interest for 2 years with some other rate: award att 2 *once.*

* $A = 5000(1.04)^2$ and stops: att 2 + att 2, but if finished correctly: 5m + att 2.
$A = 5000(1.045)^2$ and stops: att 2 + att 2, but if finished correctly: 5m + att 2.

* $A = 5000(1.04)^3$ and stops: att 2 + att 2 – whether finished or not.
$A = 5000(1.045)^3$ and stops: att 2 + att 2 – whether finished or not.

Blunders (-3)
B1 Error getting interest (e.g. 4% = ¼th or other incorrect method). Each time.
B2 Incorrect rate used, unless M1 applies. (See below). Each time.
B3 Does not add interest. Each time.
B4 $A = P(1 – r/100)^n$ used. Apply once.

Slips (-1)
S1 Each error in addition or multiplication.

Misreading (-1)
M1 Uses 4.5% and then 4%. (Reverses the order). Apply once.

Attempt (2 marks)
A1 $A = P(1 + r/100)^n$, or $A = 5000(1 + 4/100)^1$, or $A = 5000(1 - 4/100)^1$, and stops.
A2 Gets I(1) = 200 and stops.

Worthless (0)
W1 $A = P(1 - r/100)^n$ and stops.

(c) (ii) rate **10 marks** **Att 3**

I: P(3) = 9434 => I(3) = 377.36 …(3m) $r = \frac{377.36}{9434} \times 100$ …(7m) = 4% …(10m)	**II:** P(3) = 9434 => I(3) = 377.36 …(3m) $r = \frac{100 \times I}{P \times T} = \frac{100 \times 377.36}{9434 \times 1}$ …(7m) = 4 % …(10m)
III: $\frac{9811.36}{9434}$ …(3m) = 1.04 …(7m) => r = 4% …(10m)	

* Correct answer without work: 10m, if it is clear that candidate is doing (c)(ii).

* Trial and error: If successful (4%), award full marks; if not, and *with work*: att 3m.

Blunders (-3)

B1 Doesn't add the 4000, or subtracts it.

B2 Doesn't subtract to find I(3), or adds.

B3 Incorrect relevant fraction used. For example: 377.36 / 9811.36 in **I**,
or incorrect P used in **II**, e.g. P = 5000 or 5200 or P = 9811.36,
or 9434 / 9811.36 in **III**. This latter gives 0.9615 => 96.15% => 3.8%: one blunder.

B4 Some/all of last step (calc.) missing in method **III**, e.g. stops at 1.04 or 104%.

B5 $A = P(1 - r/100)^n$ used.

B6 Decimal error, e.g. 40%.

Attempts (3 marks)

A1 9811.36 – 9434 and stops. Or, subtraction of candidate's figures from previous work.

A2 Adds on £4000 to P or A found in (c)(i).

A3 $R = \frac{100 \times I}{P \times T}$ and stops.

A4 $5000(1.04)^3$ or $5000(1.045)^3$ in this part, even if continued correctly.

QUESTION 2

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 6**

Part (a) **10 marks** **Att 3**

> **(a)** Find the solution set of
>
> $$11 - 2n > 3, \quad n \in \mathbf{N}.$$

(a) Inequality **10 marks** **Att 3**

I $-2n > 3 - 11$ …(3m)	**II** $11 - 3 > 2n$ …(3m)	$11 - 2(0) > 3$, yes …any one or two values
$-2n > -8$ …(7m)	$8 > 2n$ …(7m)	$11 - 2(1) > 3$, yes correct (3m)
$2n < 8$	$4 > n$	$11 - 2(2) > 3$, yes …any three correct…(7m)
$n < 4$		$11 - 2(3) > 3$,yes
=> $\{0,1,2,3\}$ …(10m)	=> $\{0,1,2,3\}$ …(10m)	=> $\{0,1,2,3\}$ …all four correct …(10m)

Blunders (-3)

B1 Transposition or sign error. Each time, e.g. $2n > 3 - 11$ is B1. If this is followed by $2n > 8$, then apply B1 twice. (Note: in other circumstances, $2n > 8$ is B2 once).

B2 Inequality sign error, e.g. $-2n > -8 \Rightarrow n > 4$.

B3 $n \notin \mathbf{N}$.

B4 $n < 4$ and stops in method **I**, or (in method **II**).

B5 Finite set given as answer when candidate's work should have given an infinite set of naturals, i.e. incorrect set using candidate's data.

Slip (-1)

S1 After candidate's final inequality in **I** or **II**, each incorrect element of set to a max of 3.
e.g. $n < 4 \Rightarrow n \in \{1,2,3\}$; or $n < 4 \Rightarrow n \in \{0,1,2,3,4\}$, but 

is B3.

S2 Uses ≥ instead of >, i.e. includes the equality sign from the first line. Once only.

Attempts (3 marks)

A1 $11 - 2n = 3$ solved correctly, $n = 4$, and then stops.

A2 Finds one correct value of set in method **III**.

A3 Any correct transposition, whether helpful or not, e.g. $11 - 2n - 3 > 0$.

Worthless (0)

W1 $11 - 2n = 3$ and stops, or solved incorrectly.

Part (b) **20 marks** **Att 7**

> Solve for x and y
>
> $$x + 2y = 3$$
> $$x^2 - y^2 = 24$$

(b) Solve eqns. 20 marks Att 7

I		II	
$x = -2y + 3$	...(7m)	$y = (3 - x)/2$	...(7m)
$(-2y+3)^2 - y^2 = 24$	...(8m)	$x^2 - \{(3-x)/2\}^2 = 24$	...(8m)
$4y^2 - 12y + 9 - y^2 = 24$		$x^2 - (9 - 6x + x^2)/4 = 24$	
		$4x^2 - 9 + 6x - x^2 = 96$	
$3y^2 - 12y - 15 = 0$	...(11m)	$x^2 + 6x - 105 = 0$	...(11m)
$y^2 - 4y - 5 = 0$		$x^2 + 2x - 35 = 0$	
$(y-5).(y+1) = 0$	...(14m)	$(x+7)(x-5) = 0$	...(14m)
$y = 5$ and $y = -1$	...(17m)	$x = -7$ and $x = 5$	...(17m)
=> $x = -7$ and $x = 5$.	...(20m)	=> $y = 5$ and $y = -1$	...(20m)

* Generates a linear equation from $x^2 - y^2 = 24$ and solves simultaneous equations for *both* x and y values: att 7m.
If no effort made to solve for 2nd variable: 0m instead.
e.g. $x+2y = 3$ and $x - y = 24$ solved to $x = 17$, $y = -7$, merits 7m.
$x+2y = 3$ and $x - y = \sqrt{24}$ solved to $x = 1 + (2\sqrt{24})/3$, $y = 1 - (\sqrt{24})/3$, merits 7m.
$x+2y = 3$ and $x + y = \sqrt{24}$ solved to $x = (2\sqrt{24}) - 3$, $y = 3 - \sqrt{24}$, also merits 7m.

* Candidate finds first variables, substitutes into 2nd degree equation, finds correct and incorrect values and presents them all as solutions: no penalty (ignore excess answers). However, if only the incorrect ones are found as solutions, apply B(-3).

* Correct point/points found without work or by some 'trial and error':

2 points verified in 2 eqns:	allow 20m.	1 point verified in 2 eqns.:	Att 7.
2 points verified in 1 eqn.:	Att 7.	1 point verified in 1 eqn.:	0 m.
2 points, no verification:	Att 7.	1 point, no verification:	0 m.

Blunders (-3)

B1 Each missing or incorrect step.
B2 Error multiplying out brackets. Apply once.
B3 Algebraic error when totting/simplifying, etc. Example: $4y^2 - 12y - 15 = 0$.
Candidate may solve the quadratic generated without further penalty.
B4 Quadratic formula error (in formula, substitution or simplification). Each time to a maximum of 2 blunders (equivalent to two steps).
B5 Incorrect factors. Apply once.
B6 Incorrect root(s) from candidate's factor(s). Apply once.
B7 One value for x when two available, or one value for y when two available.
B8 Two x values and stops, or two y values and stops. (B7 and B8 could both apply).

Attempts (7 marks)

A1
$$\begin{aligned} x^2 + 4y^2 &= 9 \\ \underline{x^2 - y^2} &\underline{= 24} \end{aligned}$$
$5y^2 = -15 \Rightarrow y^2 = -3 \Rightarrow y = i\sqrt{3} \Rightarrow x^2 = 21 \Rightarrow x = \sqrt{21}$ (2nd variable needed).
As in note 1, if no effort made to solve for 2nd variable: award 0 marks instead.
A2 Correct quadratic formula and stops.
A3 An effort to find the second variable, having found the first variable.

Worthless (0)

W1 Incorrect values without work.
W2 Invented values substituted, and continues, e.g. let $x = 1 \Rightarrow y = 1$. See Note 3 above.

Part (c) 20 (10, 10) marks Att 6 (3, 3)

(c) Solve each of the following equations for p

(i) $9^p = \frac{1}{\sqrt{3}}$

(ii) $2^{3p-7} = 2^6 - 2^5$.

(c) (i) Index eqn. **10 marks** **Att 3**

I $(3^2)^p = \frac{1}{3^{1/2}}$ ...(3m) $3^{2p} = 3^{-1/2}$ ...(7m) $2p = -1/2$ $\Rightarrow p = -1/4$ ...(10m)	$\sqrt{3}.9^p = 1$ ...(3m) $3^{1/2}.(3^2)^p = 1 \Rightarrow 3^{1/2+2p} = 3^0$ ...(7m) $\Rightarrow 2p + 1/2 = 0$ $\Rightarrow p = -1/4$ ...(10m)

* In (c)(i) and (c)(ii), if candidate employs trial and error using a calculator, then: correct answer, without work: 10m. Incorrect answer, without work: 0 marks.

Blunders (-3)

B1 Each index error, e.g. $(3^2)^p \neq 3^{2p}$, or $\sqrt{} \neq$ power of ½, or $1/3^{1/2} \neq 3^{-1/2}$, etc.

B2 Incorrect index-equation from candidate's previous work in (i).

B3 Transposition or cross-multiplication error. Each time.

B4 Finds $2p = -1/2$ and stops, or $2p + ½ = 0$ and stops.

Attempts (3 marks)

A1 Any correct relevant index statement, and stops, e.g. $9 = 3^2$, or $(3^2)^p = 3^{2p}$, or $\sqrt{3}=3^{1/2}$.

A2 Any correct use of indices in simplifying the left or right hand sides.

A3 $9^p = 3^{-1/2} \Rightarrow p = -1/2$.

A4 $3^a = 3^b$ with powers a and b incorrect $\Rightarrow a = b$.

A5 Squares both sides, e.g. $81^p = 1/3$ and stops.

Worthless (0)

W1 $\sqrt{3} = 1.732$ and stops, or $1/\sqrt{3} = .5774$ and stops.(Continuing may lead to correct ans.)

(c) (ii) 2nd eqn. **10 marks** **Att 3**

I $2^{3p-7} = 64 - 32 = 32$ ...(3m) $2^{3p-7} = 2^5$ ...(7m) $3p - 7 = 5$ $p = 4$ ...(10m)	**II** $2^{3p-7} = 2^5(2-1)$ ...(3m) $2^{3p-7} = 2^5$ ...(7m) $3p - 7 = 5$ $p = 4$ ...(10m)

Blunders (-3)

B1 Each index error, e.g. $2^6 \neq 64$, or $2^{3p-7} = 2^1$, or $3p - 7 = 6 - 5$, and continues.

B2 Forms incorrect index-equation.

B3 Transposition or cross-multiplication error. Each time.

B4 Finds $3p - 7 = 5$ and stops.

Attempts (3 marks)

A1 Any correct relevant index statement, and stops, e.g. $2^6 = 64$.

A2 Any correct use of indices in simplifying the right hand side.

A3 $2^{3(-1/4)-7} = 64 - 32$ and stops.

QUESTION 3

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 6**

Part (a) **10 marks** **Att 3**

(a) Given that $u^2 + 2as = v^2$,
calculate the value of a when $u = 10$, $s = 30$ and $v = 20$.

(a) manipulation **10 marks** **Att 3**

I	II
$10^2 + 2a(30) = 20^2$ ...(3m) $100 + 60a = 400$ ...(7m) $60a = 300$ $a = 5$ ...(10m)	$2as = v^2 - u^2$ ...(3m) $a = \frac{v^2 - u^2}{2s}$ ...(4m) $a = \frac{20^2 - 10^2}{2(30)}$ ...(7m) $a = \frac{400 - 100}{60} = 5$ ...(10m)

* Correct answer without work: 10 m. Incorrect answer without work: 0 m.
* 3 steps: substitute, isolate and simplify – or – isolate, substitute and simplify.

Blunders (-3)
B1 Transposition error, e.g. $2as = v^2 + u^2$, or $u^2 - v^2$, or $a = (2s)/(u^2 - v^2)$. Apply once.
B2 Incorrect squaring, once if consistent, e.g. $10^2 = 20$ and $20^2 = 40$.
B3 Incorrect cancellation.
B4 Last step omitted.
B5 Incorrect substitution.

Misreadings (-1)
M1 Misreading, each time, of values of u, s or v, e.g. $s = 3$.
M2 Isolating incorrect letter – may result in later errors and penalties.

Attempts (3 marks)
A1 Step one and stops, or part of step one and stops, e.g. any relevant substitution.
A2 Any correct transposition (whether helpful or not) and stops, e.g. $u^2 + 2as - v^2 = 0$.

Part (b) **20 (10, 10) marks** **Att (3, 3)**

(b) (i) Simplify $(x + \sqrt{x})(x - \sqrt{x})$ when $x > 0$.

(ii) Hence ,or otherwise, find the value of x for which
$(x + \sqrt{x})(x - \sqrt{x}) = 6$.

(b)(i) mult. **10 marks** **Att 3**

$x^2 - \sqrt{x}.(x) + \sqrt{x}.(x) - (\sqrt{x})^2$ …(3m) or: $x^2 - x^{1/2}(x) + x^{1/2}(x) - (x^{1/2})^2$ …(3m) $= x^2 - \sqrt{x}.(x) + \sqrt{x}.(x) - \boldsymbol{x}$ …(7m) $= x^2 \quad - x$ ..(10m)	$x + \sqrt{x}$ $\underline{x - \sqrt{x}}$ $x^2 + x\sqrt{x}$ …(3m) $\underline{- x\sqrt{x} - \boldsymbol{x}}$ …(7m) $x^2 \quad - x$ ..(10m)	$x^2 - (\sqrt{x})^2$ …(7m) $x^2 - \boldsymbol{x}$ ..(10m)

* Correct answer without work: 10m.
* Only one term incorrect and without work, e.g. $x^2 + x$: 7m. (But x^2 alone: 0m).
  Other incorrect answers: 0m.

Blunders (-3)
B1 Each error in calculation, multiplication or sign, e.g. $-(\sqrt{x})^2 = x^2$, or $2x$ instead of x^2.
B2 Error dealing with central terms. Apply once.
B3 Index error, e.g. $(x^{1/2})^2 = x^{1/4}$. Each time.

Misreadings (-1)
M1 Any misreading that does not oversimplify the question, e.g. $(x + \sqrt{x})(x + \sqrt{x})$

Attempts (3 marks)
A1 A correct relevant multiplication, e.g. $(\sqrt{x})^2 = x$.
A2 Mentions $a^2 - b^2 = (a - b)(a + b)$, or $x^2 - y^2 = (x - y)(x + y)$.
A3 $\sqrt{x} = x^{1/2}$ and stops.

Worthless (0)
W1 Numerical substitution
W2 $x + \sqrt{x} = 2\sqrt{x}$, or similar, or $(x + \sqrt{x})(x - \sqrt{x}) = (x + \sqrt{x}) \pm (x - \sqrt{x})$.

(b)(ii) equation **10 marks** **Att 3**

$x^2 - x = 6$ ...(3m)
$x^2 - x - 6 = 0$ ...(4m)
$(x - 3)(x + 2) = 0$ ...(7m)
$x = 3,\ x = -2$ …(10m) (No penalty for leaving $x = -2$ in the answer)

* Candidate's *quadratic* expression = 6, correctly solved: award 10m.
* If (b)(i) had a linear result, max available for (b)(ii) is att 3m.
* (b)(ii) can include (b)(i) if (b)(i) was not done – in such a case (b)(ii) could merit 20 marks if done correctly.
* $x = 3$ without work: att 3m. But if $x = 3$ is *verified* allow full marks (10m).

Blunders (-3)
B1 Solves $x^2 - x = 0$, or candidate's quadratic expression = 0. There may be other errors.
B2 Incorrect factors. Apply once.
B3 Factorises and stops.
B4 Incorrect roots from factors. Apply once.
B5 Quadratic formula error in formula, substitution or simplification. Max of 2 blunders.

Misreadings (-1)
M1 $x^2 + x = 6$, if it is not from (b)(i).

Attempts (3 marks)
A1 Candidate's (b)(i) answer = 6, and stops.
A2 Any effort to multiply out, if done again in (b)(ii).
A3 Quadratic formula correct, and stops.

Worthless (0)
W1 Incorrect answer without work.

Part (c) **20 (10,10) marks** **Att 6 (3, 3)**

(c)	Let $f(x) = x^3 + ax^2 + bx - 6$ where a and b are real numbers. Given that $x - 1$ and $x - 2$ are factors of $f(x)$ **(i)** find the value of a and the value of b **(ii)** hence, find the values of x for which $f(x) = 0$.

(c) (i) find a, b **10 marks** **Att 3**

$x - 1$ a factor => $x = 1$ a root

$f(1) = (1)^3 + a(1)^2 + b(1) - 6 = 0$

$f(1) = 1 \quad + a \quad + b \quad - 6 = 0$

$a + b = 5$ ... (3m)

$x - 2$ a factor => $x = 2$ a root

$f(2) = (2)^3 + a(2)^2 + b(2) - 6 = 0$

$f(2) = 8 \quad + 4a \quad + 2b \quad - 6 = 0$

$4a + 2b = -2$ ... (7m)

$\underline{2a + 2b = 10}$

$2a = -12$

$a = -6$

$b = 11$ ...(10m)

$(x - 1)(x - 2) = x^2 - 3x + 2$

$$\begin{array}{r l}
 & x + (a+3) \\
x^2 - 3x + 2 \,\big)\!\! & \overline{x^3 + ax^2 + bx - 6} \\
 & \underline{x^3 - 3x^2 + 2x} \\
 & (a+3)x^2 + (b-2)x - 6 \\
 & \underline{(a+3)x^2 - 3(a+3)x + 2(a+3)}
\end{array}$$

...(3m)

rem. = 0 => $b - 2 = -3(a + 3)$ and $-6 = 2(a + 3)$ ...(7m)

$b - 2 = -3a - 9$ and $-6 = 2a + 6$

$3a + b = -7$ and $-12 = 2a$

$-6 = a$

$-18 + b = -7$

$b = 11$ ...(10m)

III: $(x - 1)(x - 2) = x^2 - 3x + 2$ ...(3m)

$(x^2 - 3x + 2)(x - 3)$ ...(7m)

$= x^3 - 6x^2 + 11x - 6$

which is $x^3 + ax^2 + bx - 6$

=> $a = -6$ and $b = 11$ ...(10m)

* In method **I**, steps 1 and 2 are reversible.

* For $f(1) = 1 + a + b - 6 = 0$ and $f(2) = 2^3 + a(2)^2 + b(2) - 6 = 0$ and stops: award 4m.

Blunders (-3)

B1 Substitutes (–1) for 1 and/or (–2) for 2, i.e. incorrect root. Apply once only.

B2 Mathematical error in indices or brackets, e.g. $(2)^3 = 6$. Apply once, if consistent.

B3 Missing or incomplete step, e.g. only one solution found.

B4 Algebraic error(s) in multiplying factors. Apply once per multiplication of brackets.

Attempts (3 marks)

A1 $x = 1$ is a root and/or $x = 2$ is a root, and stops.

A2 Substitution (even partial) of candidate's <u>root(s)</u>into $f(x)$ and stops, e.g. subs. $x = -1$.

A3 $x^2 - 3x + 2$ formed and stops.

A4 Sets up division by $x - 1$, or $x - 2$, or $x^2 - 3x + 2$.

Worthless (0)

W1 Gets $f(x - 1)$ or $f(x - 2)$ even if continued.

(c) (ii) find x **10 marks** **Att 3**

I $(x-1)(x-2) = x^2 - 3x + 2$ $\qquad\qquad x - 3$ $x^2 - 3x + 2 \overline{)\, x^3 - 6x^2 + 11x - 6}$ ...(3m) $x^3 - 3x^2 + 2x$ $-3x^2 + 9x - 6$ $-3x^2 + 9x - 6$ ...(7m) $\Rightarrow x - 3 = 0$ $\Rightarrow x = 3 \Rightarrow$ Ans: 1, 2, 3 ...(10m)	**II** Remainder Theorem *used* on $f(x) = x^3 - 6x^2 + 11x - 6$ ...(3m) $f(3) = 3^3 - 6(3)^2 + 11(3) - 6$ $= 0$ ...(7m) $\Rightarrow x = 3 \Rightarrow$ Ans 1,2,3 ...(10m)
III: $\qquad x^2 - 5x + 6$ …(3m) $x - 1 \overline{)\, x^3 - 6x^2 + 11x - 6}$ $x^3 - x^2$ $-5x^2 + 11x$ $-5x^2 + 5x$ $6x - 6$ $6x - 6$ $\qquad x - 3$ …(7m) $x - 2 \overline{)\, x^2 - 5x + 6}$ $x^2 - 2x$ $-3x + 6$ $-3x + 6$ $\Rightarrow f(x) = (x-1)(x-2)(x-3)$ $\Rightarrow x = 1, 2, 3$ …(10m)	**IV**: $f(x) = 0$ ...(3m) $(x-1)(x-2)(\ x - 3\ ) = x^3 - 6x^2 + 11x - 6$ ...(7m) $\Rightarrow x = 1, 2, 3$ ...(10m)

* Answers to <u>(c)(ii)</u> without work: $x = 1, 2$: award 3m;
$x = 3$: award 7m;
$x = 1,2,3$: award 10m.

Blunders (-3)

B1 Division errors, to a max of 2 blunders, i.e. errors in multiplication, addition, subtraction, signs, cancellations, etc.

B2 Incorrect factors in method **IV**, to a max of 2 blunders.

B3 Missing step.

B4 Incorrect roots from factors. Once.

B5 Quadratic formula error (in formula, substitution or simplification).Max of 2 blunders.

B6 In method **III**, divides by $x + 1$ and $x + 2$. (See M1 below).

Misreadings (-1)

M1 Divides by $x + 1$ and $x - 2$; *or,* divides by $x - 1$ and $x + 2$. (See B6 above).

Attempts (3 marks)

A1 Writes down the cubic substituting the values found in (c)(i) for a and b, and stops.

A2 Sets up division in method **I** or **III**, or sets up multiplication in method **IV**.

A3 Remainder theorem used – incomplete and/or unsuccessful.

A4 Quadratic formula correct, and stops.

Worthless (0)

W1 $f(x - 1)$ or $f(x - 2)$ even if continued.

QUESTION 4

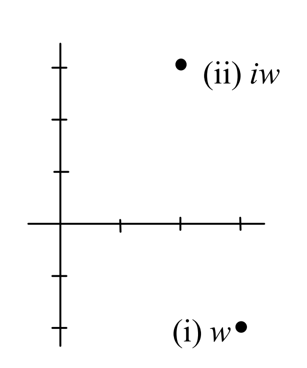
Part (a)	**15 marks**	**Att 5**
Part (b)	**15 marks**	**Att 5**
Part (c)	**20 marks**	**Att 6**

Part (a) **15 (10, 5) marks** **Att 5 (3, 2)**

(a) Let w = 3 – 2i where $i^2 = -1$.
Plot (i) w, (ii) iw on an Argand diagram.

(a) (i) plot w **10 marks** **Att 3**

w plotted correctly ... (10m)



- * If axes reversed they must be identified, or B1 applies.
- * If axes unlabelled, assume horizontal one is real axis. e.g. point (-2, 3) plotted on unlabelled axes is B1.
- * Points (-2,3) and (3,2) plotted on unlabelled axes: award 7m for (i) and 5m for (ii), i.e. penalise reversal of axes *once* in (a).
- * Axes drawn once or twice, then one or two points plotted incorrectly: award 7 marks for (i) and 2 marks for (ii).
- * One unnamed point plotted: assume it is w.

Blunders (-3)
B1 Incorrect plotting, e.g. plots (-2,3) on unlabelled axes.

Attempts (3 marks)
A1 A correct set of scaled axes (ticks sufficient).

(a) (ii) plot *iw* **5 marks** **Att 2**

$i.w = i.(3 - 2i) = 3i - 2i^2$
$= 2 + 3i$ ...(2m)
and correctly plotted ...(5m)

- * If axes reversed, penalise in (a)(i) – do not penalise again here.
- * Scaled axes do not attract both attempts unless drawn each time.

Blunders (-3)
B1 Mixing of real and imaginary parts when calculating $i.w$
B2 Incorrect plotting of candidate's $i.w$
B3 Plots i. Oversimplified (calculation avoided).
B4 Index error, e.g. $i.(-2i) \neq -2i^2$
B5 $i^2 \neq -1$.

Misreadings (-1)
M1 $i + w$ written down (instead of iw) and continued.

Attempts (2 marks)
A1 $iw = i.(3 - 2i)$ and stops.
A2 iw incorrectly calculated but result plotted correctly. See Note 4 in (a)(i)

Part (b) **15 marks** **Att 5**

> Solve
> $(x + 2yi)(1 - i) = 7 + 5i$
> for real x and y.

(b) find x, y **15 marks** **Att 5**

$x + 2yi = \dfrac{7+5i}{1-i}$ ...(5m)	$x - xi + 2yi - 2yi^2 = 7 + 5i$ ...(5m)
$= \dfrac{(7+5i).(1+i)}{(1-i).(1+i)}$ ...(9m)	$x - xi + 2yi + 2y = 7 + 5i$ ...(9m)
$= \dfrac{7 + 7i + 5i + 5i^2}{1 - i^2}$	or:
$= \dfrac{2 + 12i}{2}$ or $1 + 6i$ ...(12m)	$(x + 2y) + (-x + 2y)i = 7 + 5i$ ...(9m)
$\Rightarrow$ $x = 1$ and $y = 3$ ...(15m)	$x + 2y = 7$, $-x + 2y = 5$ (subtract) ...(12m)
	$4y = 12$
	$\Rightarrow y = 3$ and $x = 1$ ...(15m)

Blunders (-3)

B1 Incorrect conjugate, or different numerator than denominator in the conjugate.

B2 Error multiplying out brackets. Once per multiplication of brackets.

B3 $i^2 \neq -1$. Apply once in part (b).

B4 Forgets to multiply denominator by conjugate, or denom. not real after multiplication.

B5 Multiplies out numerators and denominators, and stops.

B6 Real and imaginary parts mixed up,
e.g. real ≠ real, imaginary ≠ imaginary when equating or adding.

B7 Finds both equations and stops. (Note: One correct equation and stops, 9m max.).

B8 Finds one value and stops.

Attempts (5 marks)

A1 Correct or partially correct multiplying out of any brackets.

A2 Correct transposition and stops.

A3 Correct conjugate and stops.

A4 Transposition error, e.g. $x + 2yi = (7 + 5i) - (1 - i)$, and continues.

A5 $x + 1 = 7$ and/or $2y - 1 = 5$, and stops or continues.

Part (c) **20 (10, 10) marks** **Att 6 (3, 3)**

> **(c)** Let $z_1 = 3 + 4i$ and $z_2 = 12 - 5i$.
> $\bar{z}_1$ and $\bar{z}_2$ are the complex conjugates of z_1 and z_2, respectively.
> **(i)** Show that $z_1\bar{z}_2 + \bar{z}_1 z_2$ is a real number.
> **(ii)** Investigate if $|z_1| + |z_2| = |z_1 + z_2|$.

(c)(i) $z_1\overline{z_2} + \overline{z_1}z_2$ real **10 marks** **Att 3**

$z_1\overline{z_2} = (3+4i)(12+5i)$ ...(3m) $= 36 + 15i + 48i + 20i^2$ $= 16 + 63i$ ...(7m) $\overline{z_1}z_2 = (3-4i)(12-5i)$ $= 36 - 15i - 48i + 20i^2$ $= 16 - 63i$ $\Rightarrow z_1\overline{z_2} + \overline{z_1}z_2 = 32$ ...(10m)	$z_1\overline{z_2} + \overline{z_1}z_2 = (3+4i)(12+5i) + (3-4i)(12-5i)$ ...(3m) $= 36 + 15i + 48i + 20i^2 + 36 - 15i - 48i + 20i^2$ $= 36 + 63i - 20$ $= 16 + 63i$ ...(7m) $+ 36 - 63i - 20$ $+ 16 - 63i$ $= 32$ ...(10m)

* In method **II**, separate into components and mark as in **I**.

* If conjugate is confused with modulus: max award is Att 3 for substitution (see A1).

Blunders (-3)

B1 Error multiplying out brackets. Apply once per multiplication of brackets.

B2 $i^2 \neq -1$. Apply once if consistent.

B3 Mixing real and imaginary parts.

B4 Incorrect cancelling.

B5 Incorrect $\overline{z_2}$ and/or incorrect $\overline{z_1}$: apply one blunder only.

Attempts (3 marks)

A1 Substitutes and stops.

A2 One correct conjugate and stops, e.g. $\overline{z_1}$ correct, or $\overline{z_2}$ correct.

A3 Some relevant multiplication.

A4 A correct conjugate formula and stops, e.g. $\overline{a+bi} = a - bi$, and stops.

(c) (ii) mods **10 marks** **Att 3**

$\lvert z_1\rvert = \sqrt{3^2+4^2} = \sqrt{25} = 5$	$\lvert z_1{}^2\rvert$ or $\lvert z_1\rvert^2 = z_1.\overline{z_1} = 25 \Rightarrow \lvert z_1\rvert = 5$	...(3m)
$\lvert z_2\rvert = \sqrt{12^2+(-5)^2} = \sqrt{169} = 13$	$\lvert z_2\rvert^2 = z_2.\overline{z_2} = 169 \Rightarrow \lvert z_2\rvert = 13$	...(7m)
$\lvert z_1+z_2\rvert = \lvert 15-i\rvert = \sqrt{15^2+(-1)^2} = \sqrt{226}$	$\lvert z_1+z_2\rvert^2 = (z_1+z_2)(\overline{z_1+z_2}) = 226 \Rightarrow \lvert z_1+z_2\rvert = \sqrt{226}$	...(10m)

* The three steps are interchangeable (for 3m, 7m and 10m in the candidate's order).

* Award 3m for $\sqrt{(a^2 - b^2)}$, or $a^2 + b^2$, each with a and b substituted but not finished.
Award 3m for incorrect coord. geom. distance formula substituted but not finished.

* If not using $\lvert z\rvert^2 = z\overline{z}$, modulus confused with conjugate: att 3 *maximum*. (See A1).

Blunders (-3)

B1 $\lvert z_1\rvert = a^2 + b^2 = 25$, or $\sqrt{a^2 - b^2}$ substituted, incl. $\sqrt{3^2 - 16i^2} = \sqrt{9+16} = \sqrt{25} = 5$,
or incorrect substitution into correct formula, e.g. retains the i [leading to $\sqrt{-7}$ or $\sqrt{25}$],
or incorrect squaring (using the candidate's terms) including incorrect sign.
Apply B1 once overall to finding $\lvert z_1\rvert$, $\lvert z_2\rvert$ and $\lvert z_1 + z_2\rvert$.

B2 Square root error, e.g. $\sqrt{9+16} = 3 + 4$, or stops at $\sqrt{9+16}$. Apply once.

Attempts (3 marks)

A1 Substitutes (again) in (c)(ii), and stops.

A2 $\sqrt{a^2 + b^2}$ and stops.

A3 Coordinate Geometry distance formula correct and stops, or $\lvert z\rvert^2 = z.\overline{z}$ (or equivalent).

A4 Plots $\lvert z_1\rvert$, $\lvert z_2\rvert$ or $\lvert z_1 + z_2\rvert$ correctly and stops.

Worthless (0)

W1 $\sqrt{a^2 - b^2}$ without substitution, or $a^2 + b^2$ without substitution. (See B1).

W2 Other incorrect formula with or without substitution.

QUESTION 5

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 6**

Part (a) **10 marks** **Att 3**

(a) 5, 13, 21, 29, ... is an arithmetic sequence.
Which term of the sequence is 813?

(a) T*n* = 813, *n* =? **10 marks** **Att 3**

I	II / III
$a + (n-1)d$ $a = 5,\ d = 8$ ... (3m) for either or both lines. $\Rightarrow\ 5 + (n-1)(8) = 813$ ...(7m) $\Rightarrow\ 5 + 8n - 8 = 813$ $\Rightarrow\ 8n = 816$ $\Rightarrow\ n = 102$ ..(10m)	**II**: Lists out all the terms... 5, 13, 21, 29, 37, 5th term correct ...(3m) 45,...,813. all correct...(7m) and counts them correctly: 102 terms ...(10m) **III**: 813 - 5 ...(3m) 808 ÷ 8 ...(7m) = 101 + 1 = 102 ...(10m)

* Here, and elsewhere in Q5, no penalty for notation errors if the formula is correct, e.g. no penalty for $Sn = a + (n-1)d = 813$.

* Correct answer without work: 10m. Incorrect answer without work: 0m.

* In method **II**, if 2 terms are omitted or incorrect using candidate's values for *a* and *d* then max available is att 3m.

Blunders (-3)
- B1 Incorrect substitution into correct formula, e.g. mixes up *a* and *d*: one blunder.
- B2 $a + nd$ substituted correctly, or $a(n-1)d$ substituted correctly. Any other incorrect formula is worthless (0 marks), but att marks may be given for *a* and *d* values. See A1.
- B3 Error solving equation, e.g. transposition, or multiplying out bracket. Apply once.
- B4 Candidate's $Tn \neq 813$ and continues, or finds T_{813} and continues. (= 6501).
- B5 One incorrect term of list in method **II**, or incorrect total when counted.
- B6 Forgets to add the 1 in method **III**.
- B7 Incorrect *d* without work. (With work, S1 may apply.)

Slips (-1)
- S1 Numerical slip calculating *d* (with work shown), e.g. $d = 13 - 5 = 7$.

Attempts (3 marks)
- A1 Correct *a* or *d* values stated, or substituted.
- A2 Correct T*n* formula and stops.
- A3 States T*n* = 813 and stops.

Worthless (0)
- W1 Incorrect formula and stops; e.g. GP formula -- but may earn marks for the correct *a*.
- W2 Incorrect answer without work.

Part (b) **20 (5, 5, 10) marks** **Att 7 (2, 2, 3)**

(b) The nth term of a geometric series is given by $T_n = 3^n$ **(i)** What is the value of a, the first term? **(ii)** What is the value of r, the common ratio? **(iii)** Show that S_{10}, the sum of the first 10 terms, is $\frac{3}{2}\left(3^{10}-1\right)$.

(b)(i) find *a* **5 marks** **Att 2**

$a = T_1$ ...(2m)	$T_1 = 3^1$ or 3 ...(5m)

* Correct answer without work: full marks (5m). Incorrect answer without work: 0 m.

Blunders (-3)

B1 $n \neq 1$.

B2 ar^n, $(ar)^n$, $(ar)^{n-1}$ or $(ar)^{n+1}$ *used*. Other incorrect formulae are worthless.

Attempts (2 marks)

A1 $T_1 = a$ and stops, or $T_n = ar^{n-1}$ and stops.

Worthless (0)

W1 Incorrect answer without work.

W2 Incorrect formula (e.g. AP formula) and stops.

(b)(ii) find r **5 marks** **Att 2**

$T_2 = 3^2 = 9$	...(2m)
$r = \frac{T_2}{T_1} = \frac{9}{3}$ *or* $\frac{9}{3^1}$ *or* $\frac{3^2}{3^1}$ *or* 3.	…(5m)

* Correct answer without work: full marks (5m). Incorrect answer without work: 0 m.

Blunders (-3)

B1 $T_2 \neq 3^2$, or $3^2 \neq 9$.

B2 $r \neq T_2 \div T_1$, e.g. $r = 3/9$.

B3 ar^n, $(ar)^n$, $(ar)^{n-1}$ or $(ar)^{n+1}$ *used*. Other incorrect formulae are worthless.

B4 $T_1 \neq 3$ or 3^1 in (b)(ii).

Attempts (2 marks)

A1 $T_2 = 9$ and stops, or $ar = 9$ and stops, or $3r = 9$ and stops.

A2 Correct formula for T_n, or r, and stops.

A3 Value of a substituted into correct T_n formula or into a formula listed in B3 above.

Worthless (0)

W1 Incorrect r without work.

W2 Incorrect formulae and not substituted.

(b)(iii) S_{10} **10marks** **Att 3**

I	**II**	**III**
$Sn = \frac{a(r^n - 1)}{r - 1}$ ...(3m) $S_{10} = \frac{3(3^{10}-1)}{3-1}$ *or* $\frac{3(3^{10}-1)}{2}$ ...(10m)	**II** $Sn = \frac{a(1 - r^n)}{1 - r}$ ...(3m) $= \frac{3(1 - 3^{10})}{1 - 3}$ ...(7m) $= \frac{3(1 - 3^{10})}{-2}$ still 7m $= \frac{3(3^{10}-1)}{2}$ ..(10m)	$S_{10} = 3+9+27+81+243+729$ $+2187+6561+19683$ $+59049$. ...(3m) $S_{10} = 88572$ ...(7m) And $(3/2)(3^{10}-1)$ $= 1.5(59049-1)$ $= 88572$ also ...(10m)

* Penalise candidate's incorrect values for a and r from previous parts of (b) if not already penalised.

* In method **III**, step 3 may be done first followed by steps 1 and 2: apply 3m,7m, 10m.

* Using candidate's data, 2 terms of list in **III** omitted or incorrect: *max att 3 available.*

Blunders (-3)

B1 Incorrect relevant formula *used* for Sn of GP (see examples):

Examples: $Sn = \frac{a(r^n + 1)}{r + 1}$; or $Sn = \frac{a(r - 1)^n}{r - 1}$ or $Sn = \frac{a(r - 1)^n}{1 - r}$

or Sn of GP formula *used* with *only one error*.

B2 Incorrect a or r (different to previous values) substituted into Sn. Max of 2 blunders.

B3 Indices error . Each time.

B4 Error in sign when simplifying in method **II**.

B5 One term omitted or incorrect in list in method **III**. See Note 3.

Attempts (3 marks)

A1 $a = 3$ or $r = 3$ mentioned (again) or used in a formula.

A2 Correct Sn of GP formula and stops.

A3 Incorrect relevant formula for Sn of GP (as in B1 above), *each with some substitution.*

A4 Lists at least *one* term correctly in method **III** and stops.

A5 Says $S_{10} = a + ar + ar^2 + ar^3 + \ldots + ar^9$ and stops, or $S_{10} = T_1 + T_2 + \ldots + T_{10}$ and stops.

Worthless (0)

W1 Incorrect answers without work.

W2 Incorrect formula and not substituted.

Part (c) **20 (10, 10) marks** **Att 6 (3, 3)**

(c)	The sum of the first n terms of an arithmetic series is given by $Sn = 4n^2 - 8n$.
(i)	Use S_1 and S_2 to find the first term and the common difference.
(ii)	Starting with the first term, how many terms of the series must be added to give a sum of 252?

(c)(i) *a* and *d* **10 marks** **Att 3**

$S_1 = 4(1)^2 - 8(1) = 4 - 8 = -4$ …(3m) Order of calculation

$S_2 = 4(2)^2 - 8(2) = 16 - 16 = 0$ …(7m) of S1 and S2 is reversible

$\Rightarrow T_2 = S_2 - S_1 = 0 - (-4) = 4$

$\Rightarrow d = T_2 - T_1 = 4 - (-4) = 8$ ..(10m)

Blunders (-3)

B1 a not found, or $a \neq S_1$ or $S_1 \neq T_1$. Example: $a = S_2$.

B2 T_2 not found, or $T_2 \neq S_2 - S_1$. Example: $T_2 = S_1 - S_2$.

or d not found, or $d \neq T_2 - T_1$. Example: $d = T_1 - T_2$, or $T_2 = T_2/T_1$.

B3 Index error, e.g. $4(1)^2 = 4(2)$ or $4(2)^2 = 8^2$. Apply once if consistent.

Attempts (3 marks) for Q5(c)(i)

A1 Has $S_1 = 4(1)^2 - 8(1)$ and stops, <u>or</u> $S_1 = 4 - 8$ and stops,
<u>or</u> $S_2 = 4(2)^2 - 8(2)$ and stops, <u>or</u> $S_2 = 16-16$ and stops.

A2 Correct S*n* of AP formula and stops.

Worthless (0)

W1 Incorrect formula and not substituted.

(c)(ii) Sn = 252 **10 marks** **Att 3**

I:	$Sn = (n/2)\{2a + (n-1)d\}$ …(3m)	**III**:
	$(n/2)\{-8 + (n-1)8\} = 252$	Terms calculated and added:
	$(n/2)\{-8 + 8n-8\} = 252$	
$Sn = 252$	$(n/2)\{8n - 16\} = 252$	
$4n^2 - 8n = 252$ …(3m)	$4n^2 - 8n = 252$ still (3m)	$-4+4+12$ ...the next term (3m)
$4n^2 - 8n - 252 = 0$ …(4m)	$4n^2 - 8n - 252 = 0$ …(4m)	
$n^2 - 2n - 63 = 0$ still 4m	$n^2 - 2n - 63 = 0$ still 4m	$+20+28+36+44+52+60$
$(n-9)(n+7) = 0$ …(7m)	$(n-9)(n+7) = 0$ …(7m)	$= 252$ … (7m)
$n = 9, \ -7$ …(10m)	$n = 9, \ -7$ …(10m)	=> 9 terms …(10m)

* Correct answer without work: full marks (10m).

* Trial/Error used with $4n^2 - 8n$ or correct S*n* of AP formula:10m if $n = 9$ found. See A4.

* Accept the candidate's values for *a* and *d* from (c)(i), e.g. if $d = 4$ in (c)(i) then in (c)(ii) - 4,0,4,8,12,....,40 or 44 => $n = 12.8$. Allow 10m for $n = 12$, 12.8 or13.
For reference: $S_{12} = 216$ and $S_{13} = 260$, in - 4,0,4,8,12,....

* Using candidate's data, 2 terms of list in **III** omitted or incorrect: *max att 3 available.*

Blunders (-3)

B1 Equation not equal (or implied equal) to 0 before factorizing, or $n^2 - 2n - 252 = 0$.

B2 Incorrect factors. Apply once.

B3 Incorrect or no roots from factors. Apply once.

B4 *Incorrect relevant* S*n* of AP formula <u>used</u>. Note: *Incorrect relevant* = one error only.
If the formula has more than one error, then att 3m is max possible: see A3, W3.

B5 Incorrect substitution of *a* or *d* into S*n* of AP formula. Apply once.
Note: incorrect substitution into *incorrect relevant* S*n* of AP formula is B4 + B5.

B6 In **III**, one term omitted or incorrect term using candidate's data, or terms not added.
B6 applies twice if a term is omitted and list is not added. See also note 4 above.

B7 Quadratic formula error in formula, substitution or simplification. Max of 2 blunders.

Slips (-1)

S1 Arithmetic errors.

Attempts (3 marks)

A1 Correct S*n* of AP formula and stops, or says '$Sn = 252$' and stops.

A2 Evaluates S_{252}, or substitutes into S_{252} formula (even if not finished correctly).

A3 *a* or *d* correctly substituted into a formula containing more than 1 error, and stops or continues, e.g. subst. into S*n* of AP formula with 2 errors, T*n* of AP, or GP formulae

A4 Unsuccessful trial and error of S*n*, e.g. S_3, S_{20}, etc found, but not S_9.

A5 Quadratic formula correct and stops.

Worthless (0)

W1 $n = 252$ and stops.

W2 Incorrect answer without work.

W3 Incorrect formula and not substituted.

QUESTION 6

Part (a)	**10 marks**	**Att 4**
Part (b)	**40 marks**	**Att 13**

Part (a) **10 (5, 5) marks** **Att 4 (2, 2)**

> **(a)** Let $g(x) = \dfrac{1}{x^2 + 1}$ for $x \in \mathbf{R}$.
> Evaluate:
> **(i)** $g(2)$
> **(ii)** $g(3)$ and write your answers as decimals

(a)(i) $g(2)$ **5 marks** **Att 2**

$$g(2) = \frac{1}{2^2 + 1} \quad ...(2m)$$
$$= 0.2 \quad ...(5m)$$

Blunders (-3)

B1 Taking $g(x) = x^2 + 1$. Each time.

B2 Index error, e.g. $3^2 = 6$ in (ii). Each time.

B3 Substitutes and stops (at line 1). Each time.

B4 Decimal error, e.g. 1/5 = .5 in (i). Each time.

B5 Division error, e.g. 1/(4 + 1) = 1 ¼ or 1/(9 + 1) = 1 1/9 . Each time.

B6 Failure to decimalise, i.e. stops at 1/5 in (i) or 1/10 in (ii). Each time.

Misreadings (-1)

M1 $g'(x)$ used instead of $g(x)$. Each time.

Attempts (2 marks)

A1 Any correct substitution, and stops.

(a)(ii) $g(3)$ **5 marks** **Att 2**

$$g(3) = \frac{1}{3^2 + 1} \quad ...(2m)$$
$$= 0.1 \quad ...(5m)$$

* Same scheme as in (a)(ii).

* Mark (a) out of 5m + 5m, not out of 10m.

Part (b) **40 (10,10, 5, 10, 5) marks** **Att 13 (3,3,2,3,2)**

(b)	Let $f(x) = 2 - 9x + 6x^2 - x^3$ for $x \in$ **R**.
(i)	Find $f(-1)$, $f(2)$ and $f(5)$.
(ii)	Find $f'(x)$, the derivative of $f(x)$.
(iii)	Find the coordinates of the local maximum and the local minimum of $f(x)$.
(iv)	Draw the graph of $f(x)$ in the domain $-1 \le x \le 5$.
(v)	Use your graph to find the range of real values of k for which $f(x) = k$ has more than one solution.

(b) (i) *f(x)* values **10 marks** **Att 3**

$f(x) = 2 - 9x + 6x^2 - x^3$ ...given

$f(-1) = 2 + 9(-1) + 6(-1)^2 - (-1)^3$

$= 2 + 9 \quad + 6 \quad + 1 \quad = 18$ ...(3m)

$f(2) = 2 - 9(2) + 6(2)^2 - (2)^3$

$= 2 - 18 \quad + 24 \quad - 8 \quad = 0$ ...(7m)

$f(5) = 2 - 9(5) + 6(5)^2 - (5)^3$

$= 2 - 45 \quad + 150 \quad - 125 \quad = -18$...(10m)

II

x	-1	2	5
2	2	2	2
$-9x$	9	-18	-45
$6x^2$	6	24	150
$-x^3$	1	-8	-125
$f(x)$	18	0	-18

10m

III

x	-1	2	5
$f(x)$	18	0	-18

10m

* Values may be worked out in any order, but marking scheme keeps to 3,7,10m order.

* In all methods, 1 $f(x)$ value correct, with or without work: 3m See A1 and B1.
2 $f(x)$ values correct, with or without work: 7m.
3 $f(x)$ values correct, with or without work: 10m.

* $f(x) = x^3 - 6x^2 + 9x - 2$ in (b): apply B(-3) in *each* of (i), (ii), (iii) and (iv) if used.

Misreadings (-1)

M1 $f(x) = -9x + 6x^2 - x^3$ or $f(x) = 2 + 9x + 6x^2 - x^3$ and continues. See B1.

Blunders (-3)

B1 Consistent error across a line of table or term, if not M1. Apply B1 twice at most.

Attempt (3 marks)

A1 Any correct substitution for x in the cubic expression, e.g. $-9(-1)$,
or any term of the cubic (other than the '2') correctly evaluated, e.g. $-9x = 9$.

Worthless (0)

W1 Three incorrect values, each without work.

(b) (ii) *f '(x)* **10 marks** **Att 3**

(b)(ii) $f'(x) = -9 + 12x - 3x^2$

Blunders (-3)

B1 Calculus error. Once per term. (4 terms to check):

# terms differentiated correctly:	4	3	2	1	0
marks earned:	10m	7m	4m	att 3m	0m

Attempts (3 marks)

A1 Any term differentiated correctly, e.g. $+9 - 12x + 3x^2$ merits att 3 (for the initial '0').

Worthless (0)

W1 No term differentiated correctly.

(b) (iii) max.,min **5 marks** **Att 2**

(b)(iii)	$f'(x)$ <u>or</u> $-9+12x-3x^2 = 0$ ...(2m)
	$\Rightarrow\quad x^2-4x+3 = 0$
	$\Rightarrow\quad (x-1)(x-3) = 0$
	$\Rightarrow\quad x = 1,\ x = 3$
	$\Rightarrow\quad y = -2,\ y = 2$ …(5m)

* No need to distinguish between max and min points – not asked in question.

* Ignore errors in candidate's work relating to 2nd derivative.

* If (iii) not calculated, but values read from the candidate's graph in (iv):
  5m for a max and a min both correct;
  2m for a max or a min correct;
  0m for neither correct.
'Correct' means both coordinates. If not both coordinates apply blunder (-3).

* In line 1, an implied use '= 0' is acceptable when factors found etc.

* If (b)(ii) is <u>not</u> attempted and $f'(x)$ is found in (b)(iii) then mark (b)(iii) out of 10m (for derivative) + 5m (for max and min).

Blunders (-3)

B1 $f'(x)$ not equated to zero, or $f'(0)$ found.

B2 Incorrect factors. Apply once.

B3 Incorrect roots from factors. Apply once.

B4 Quadratic formula error in formula, substitution or simplification. Max of 2 blunders.

B5 Calculates only one turning point and stops, e.g. $x = 1$, $y = -2$ and stops

B6 Error(s) in calculating y (apart from a totting error which is a slip; see S1).

B7 y coordinates not found. Apply once.

Slips (-1)

S1 Numerical error, e.g. totting y values.

Misreadings (-1)

M1 $f(x) = -9x + 6x^2 - x^3$ and continues, or $f(x) = 2 + 9x + 6x^2 - x^3$ and continues.

Attempts (2 marks)

A1 Statement that derivative = 0, e.g. 'max and min at $f'(x) = 0$'.

A2 Candidate's $f'(x)$ equated to zero and stops.

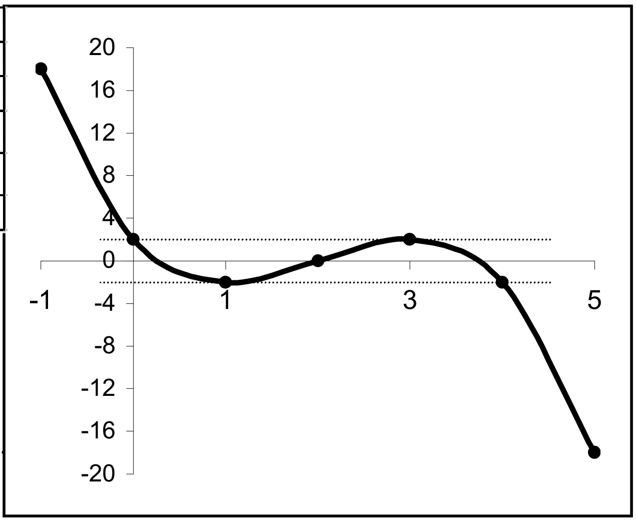
A3 Quadratic formula correct and stops.

Worthless (0)

W1 Effort to solve $f(x) = 0$, or finds $f(0)$.

(b) (iv) graph **10 marks** **Att 3**

x	-1	0	1	2	3	4	5
2	2	2	2	2	2	2	2
$-9x$	9	0	-9	-18	-27	-36	-45
$6x^2$	6	0	6	24	54	96	150
$-x^3$	1	0	-1	-8	-27	-64	-125
$f(x)$	18	2	-2	0	2	-2	-18



* If $f(x) = x^3 - 6x^2 + 9x - 2$ is tabulated and graphed, apply B(-3) once, and mark as per scheme.

* Candidate may use points from (i) along with the max and min points to sketch graph without table: 10m. No penalty if $f(0)$ and $f(4)$ omitted.

* If $f(-1)$, $f(2)$ and $f(5)$ are re-calculated in this part, mark the candidate's new data.

Blunders (-3)

B1 Incorrect $f(x)$ value, *unless* note 1, M1 or A4 applies. Apply B1 twice at most.
B2 Consistent error across a line of table or term, if not M1. Apply B2 twice at most.
B3 Incorrect plotting of candidate's points. Apply twice at most.
e.g. table compiled but no points plotted => 2 x B3.
B4 Omission of candidate's turning point. Each time.
B5 Part of the domain omitted on left or on right. Each time.
B6 Serious errors in scaling of axis or axes. Apply once.

Misreadings (-1)

M1 $f(x) = -9x + 6x^2 - x^3$ or $f(x) = 2 + 9x + 6x^2 - x^3$ and continues. See B2.

Slips (-1)

S1 Each point plotted but not joined, subject to a max of 3.
S2 Incorrect total from incorrect internal values if not related to sign.

Attempts (3 marks)

A1 One correct $f(x)$ value calculated, or one term of $f(x)$ correct (apart from the '2')
A2 One point (of candidate's data) correctly plotted.
A3 Two suitable axes drawn (ticks sufficient).
A4 Graphs $f'(x)$. A serious misreading (oversimplified to a quadratic graph).

(b) (v) *k* **5 marks** **Att 2**

I **(v)** $-2 \le k \le 2$ ...(5m)	**II** [-2, 2] ...(5m)

* Allow without penalty: $-2 \le y \le 2$; and (- 2, 2).

Blunders (-3)

B1 $0 \le k \le 4$, or $0 \le x \le 4$

Slips (-1)

S1 $-2 < k < 2$.
S2 $-2 \le x \le 2$.

Attempts (2 marks)

A1 Any mention of 2 and/or –2 in candidate's (incorrect) answer.

QUESTION 7

Part (a)	**10 marks**	**Att 4**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 8**

Part (a) **10 (5, 5) marks** **Att 4 (2, 2)**

(a) Differentiate with respect to x
(i) $6x^5 + x^2$
(ii) $(x - 3)(x + 3)$.

(a)(i) Differentiate **5 marks** **Att 2**

$30x^4 + 2x$. …(5m)

* Correct answer without work or notation: full marks, 5m.

* If done from first principles, ignore errors in procedure – just mark the answer.

Blunders (-3)
B1 Calculus error. Once per term. (Two terms to check. See A1 below).

Attempts (2 marks)
A1 Only one term differentiated correctly, e.g. $6x^4 + \underline{2x}$.
A2 Unsuccessful effort at first principles, e.g. $y + \Delta y$ on L.H.S., or x replaced by $x + \Delta x$ on R.H.S., etc.

Worthless (0)
W1 No term differentiated correctly.

(a)(ii) Differentiate **5 marks** **Att 2**

$y = (x - 3)(x + 3)$ ...given.. (0) $y = x^2 - 9$ …(2m) $\frac{dy}{dx} = 2x$ …(5m)	$\frac{dy}{dx} = u.\frac{dv}{dx} + v.\frac{du}{dx}$ …Tables…(0) $= (x - 3)(1) + (x + 3)(1)$ …(5m) or $x - 3 + x + 3$ (Two terms) or $2x$

Blunders (-3)
B1 Incorrect terms after multiplying out brackets. Apply once.
B2 Calculus error. Apply once per term. (Two terms to check).
B3 Error in $u.v$ formula, e.g. central sign.

Attempts (2 marks)
A1 Only one term differentiated correctly, e.g. $2x - 9$ in method **I**.
A2 Unsuccessful effort at first principles, e.g. $y + \Delta y$ on L.H.S., or x replaced by $x + \Delta x$ on R.H.S., etc.
A3 Multiplies out correctly and stops, or some term correctly multiplied out and stops
A4 $u = x - 3$, $v = x + 3$ (or vice versa) and stops.
A5 One or both brackets differentiated correctly and stops, e.g. (1)(1).

Worthless (0)
W1 No term differentiated correctly, *unless* A2, A3 or A4 applies.

Part (b) **20 (10, 10) marks** **Att 6 (3, 3)**

(b)	**(i)**	Find $\frac{dy}{dx}$ when $y = \frac{x^2}{x-4}$, $x \neq 4$.
	(ii)	Find the value of $\frac{dy}{dx}$ at $x = 0$ when $y = (x^2 - 7x + 1)^5$.

(b) (i) *u/v* **10 marks** **Att 3**

I: $\frac{dy}{dx} = \frac{(x-4).2x - x^2.1}{(x-4)^2}$ ...(10m)	**II** $y = x^2.(x-4)^{-1}$ …(3m) $dy/dx = x^2.(-1)(x-4)^{-2} + (x-4)^{-1}.2x$ …(10m)

* No marks for writing down *u/v* or *u.v* formula from Tables, and stopping.

* If identified as *u/v* but *u.v* used, apply B2 + B3 (and others if required).

Blunders (-3)

B1 Calculus errors. Once per term. (Both methods have two terms.)

B2 Central sign incorrect in formula.

B3 No ÷ by v^2 in method **I**.

B4 Vice versa substitution for du/dx and dv/dx in *u/v* formula. Apply once.

B5 $y \neq x^2.(x-4)^{-1}$ in method **II**.

Attempts (3 marks)

A1 $u = x^2$ and $v = x - 4$ and stops.

A2 Any correct derivative, du/dx or dv/dx, and stops.

A3 $dy/dx = 2x/1$, or $2x$.

(b) (ii) chain rule **10 marks** **Att 3**

$5(x^2 - 7x + 1)^4.(2x - 7)$ ...(7m) $x = 0 \Rightarrow 5(1)^4.(-7)$ $= -35.$ …(10m)	$u = x^2 - 7x + 1$ and $du/dx = 2x - 7$. $y = u^5 \Rightarrow dy/du = 5u^4$ $\frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$ $= 5u^4 . (2x - 7) = 5(x^2 - 7x + 1)^4.(2x - 7)$ ...(7m) $x = 0 \Rightarrow 5(1)^4.(-7)$ $= -35.$ …(10m)

* Treat $5(x^2 - 7x + 1)^4$ and $(2x - 7)$ as two separate terms. See B1 and B2 below.

* No penalty for omission of brackets, as long as multiplication is implied.

* Answer $5(x^2 - 7x + 1)^4$: treat as B2+B3.

* Answer $5(2x - 7)^4$: treat as B1+B2+B3 and award attempt marks; see A2 below.

* If candidate tries to multiply out first, mark using slips and blunders:

Blunders (-3)

B1 Calculus error(s) in $5(x^2 - 7x + 1)^4$ part of derivative. Apply once.

B2 Calculus error(s) in $(2x - 7)$ part of derivative. Apply once.

B3 Does not substitute $x = 0$ in dy/dx, or puts $dy/dx = 0$.

B4 The two parts not multiplied, but *implied multiplication* is acceptable <u>unless</u> the substitution made contradicts the implied multiplication, e.g. $5(x^2 - 7x + 1)^4 .2x - 7$ and stops, merits 7m. (Only B3 applies).

B5 Calculation error in last step, e.g. error in sign, or $5(1)^4(-7) = 5^4(-7)$ or $6(-7)$. (See S1)

Attempts (3 marks)
A1 $u = x^2 - 7x + 1$ and stops, or $du/dx = 2x - 7$and stops.
A2 Some correct element of chain rule, e.g. coefficient 5, or power 4.
A3 $y = 5x^2 - 35x + 5 \Rightarrow dy/dx = 10x - 35$. Oversimplified.
A4 Effort to multiply out expression, e.g. $x^2 - 7x + 1$ multiplied by x^2 or $-7x$.

Slips (-1)
S1 Numerical slip in calculation of –35, e.g. $5(1)^4(-7) = -36$.

Worthless (0)
W1 $x = 0$ substituted in $y = (x^2 - 7x + 1)^5$ and stops.

Part (c) 20 (5; 5 , 5; 5) marks Att 8 (2; 2, 2; 2)

(c)	Two fireworks were fired straight up in the air at $t = 0$ seconds. The height, h metres, which each firework reached above the ground t seconds after it was fired is given by: $h = 80t - 5t^2$. The first firework exploded 5 seconds after it was fired. **(i)** At what height was the first firework when it exploded? **(ii)** At what speed was the first firework travelling when it exploded? The second firework failed to explode and it fell back to ground. **(iii)** After how many seconds did the second firework reach its maximum height?

(i) height 5 marks Att 2

$h = 80(5) - 5(5)^2$	…(2m)
$= 400 - 125 = 275$	…(5m)

Blunders (-3)
B1 Incorrect t value substituted into $80t - 5t^2$, i.e. $t \neq 5$, e.g. $t = 0 \Rightarrow h = 0$ merits 2m.
B2 Incorrect equation substituted, e.g. subst. into dh/dt. [B1 and B2 may both occur].
B3 Mathematical error in calculations, e.g. $80(5) - 5(5)^2 = 400 - 5(10) = 350$.

Attempts (2 marks)
A1 $80(5) - 5(5)^2$ and stops.

Slips (-1)
S1 Arithmetical slip in calculation, e.g. 400 – 125 = 375.

Worthless (0)
W1 $80t - 5t^2$ equated to 5, or 0, or some other number, whether solved or not.

(ii) speed 10 (5, 5) marks Att 4 (2, 2)

I: $dh/dt = 80 - 10t$ …(5m...att 2)	**II :**
$80 - 10(5)$ …(2m) $= 30$ …(5m)	30 without work …award 5m + 5m for (ii).

* No differentiation (to find dh/dt): award 0m from first five, *unless* method **II** applies. If $t = 5$ substituted into h equation: award 0 (from first five) + att 2 (from second 5m)
* Differentiates twice and then subst. $t = 5$ into the *first* derivative: award 5m + 5m.
* Differentiates twice, then tries to subst. $t = 5$ into *second* derivative: award 5m + 2m.

Blunders (-3)
B1 Calculus error, once per term. (Two terms to check).
B2 $t \neq 5$, e.g. $t = 0$ substituted into speed (dh/dt) equation.

B3 Mathematical errors in calculation, e.g. $80 - 10(5) = 70(5)$. Apply once

Attempts (2 marks) for 7(c)(ii):

A1 One term differentiated correctly and stops.

A1 $t = 5$ substituted into any incorrect equation, i.e. the second att 2m.

A3 dh/dt mentioned, e.g. speed = dh/dt, or mentions ds/dt, dy/dx or $f'(x)$.

Worthless (0)

W1 Both terms incorrectly differentiated, but dh/dt mentioned merits att marks.(See A3)

W2 Substitutes $t = 0$ into h, i.e. 0m for differentiation + 0m for incorrect substitution.

W3 Incorrect answer to second part without work.

W4 $h = 275$ and Speed = Distance/Time => Speed = $275/5 = 55$.

W5 $80 - 10t = 275$ solved: 0 marks from the 2nd five.

(iii) t **5 marks** **Att 2**

I:	$dh/dt = 0$ or $80 - 10t = 0$ ...(2m) $t = 8$ ...(5m) (See method **II** below)
II:	$80t - 5t^2 = 0$ … (2m) $t(80 - 5t) = 0$ $t = 0, t = 16$ => max. at $t = 8$. …(5m)
III:	Tabulates, graphs or adopts trial/error approach to $h = 80t - 5t^2$ and then estimates that max occurs at $t = 8$ …(5m)

* For reference:

t	0	1	2	3	4	5	6	7	8	9	10 …
$80t$	0	80	160	240	320	400	480	560	640	720	800
$-5t^2$	0	-5	-20	- 45	- 80	-125	-180	-245	-320	- 405	-500
h	0	75	140	195	240	275	300	315	320	315	300 …

* Correct answer without work: 5m.

* Accept candidate's dh/dt from (c)(ii).

Blunders (-3)

B1 Solves dh/dt = an incorrect number, i.e. not 0, e.g. $80 - 10t = 5$.

B2 Transposition error.

Attempts (2 marks)

A1 $dh/dt = 0$ and stops, or $80 - 10t = 0$ and stops, or answer (c)(ii) = 0 and stops.

A2 Effort to tabulate, graph or use trial/error, e.g. 2 points correctly calculated or plotted, or 2 correct values found using a trial/error approach.

A3 Substitutes t = 0 into dh/dt (to get 80 seconds).

Worthless (0)

W1 Solves h = some number other than 0. (See method **II**)

QUESTION 8

Part (a)	**10 marks**	**Att 4**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 (5, 5) marks** **Att 4 (2, 2)**

(a) Let $g(x) = x^4 - 32x$ for $x \in \mathbf{R}$.
(i) Write down $g'(x)$, the derivative of $g(x)$.
(ii) For what value of x is $g'(x) = 0$?

(a) (i) $g'(x)$ **5 marks** **Att 2**

$4x^3 - 32$ ...(5m)

* Correct answer without work or notation: full marks, 5m.
* If done from first principles, ignore errors in procedure – just mark the answer.

Blunders (-3)
B1 Calculus error. Once per term. (Two terms to check. See A1 below).

Attempts (2 marks)
A1 Only one term differentiated correctly, e.g. $4x^3 - 32x$.
A2 Unsuccessful effort at first principles, e.g. $y + \Delta y$ on L.H.S., or x replaced by $x + \Delta x$ on R.H.S., etc.

Worthless (0)
W1 No term differentiated correctly, unless A2 applies.

(a) (ii) $g'(x) = 0$ **5 marks** **Att 2**

I $4x^3 - 32 = 0$ ...(2m)	**II** $4x^3 - 32 = 0$ ...(2m)
$x^3 - 8 = 0$	$x^3 - 8 = 0$
$x^3 = 8$	$(x-2)(x^2+2x+4) = 0$
$x = \sqrt[3]{8}$	
$x = 2$ ...(5m)	$x = 2$ ...(5m)

* If method **II** used, no penalty for errors solving $x^2 + 2x + 4 = 0$.
* $g'(x) = 4(2)^3 - 32 = 0$: award full marks (5m).

Blunders (-3)
B1 Transposition or sign error. Apply once.
B2 Incorrect cube root, e.g. 8/3.
B3 Incorrect factors in method **II**. Apply once.
B4 Incorrect root, instead of $x = 2$, in method **II**.

Misreadings (-1)
M1 Solves $g(x) = 0$, i.e. solves $x^4 - 32x = 0$. (Ans: $x = 0$, $\sqrt[3]{32}$).

Part (b) **20 marks** **Att 7**

(b) Differentiate $3x^2 - x$ from first principles with respect to x.

(b) first principles **20 marks** **Att 7**

$y + \Delta y = 3(x + \Delta x)^2 - (x+\Delta x)$	...(7m)...	$f(x+h) = 3(x + h)^2 - (x + h)$
$y + \Delta y = 3[x^2 + 2x\Delta x + (\Delta x)^2] - x - \Delta x$		$f(x+h) = 3[x^2 + 2hx + h^2] - x - h$
$y + \Delta y = 3x^2 + 6x\Delta x + 3(\Delta x)^2 - x - \Delta x$ $y = 3x^2 - x$	… 11 m	$f(x+h) = 3x^2 + 6hx + 3h^2 - x - h$ $f(x+h)-f(x) = 3x^2 + 6hx + 3h^2 - x - h - 3x^2 + x$
$\Delta y = 6x\Delta x + 3(\Delta x)^2 - \Delta x$	… 14 m	$f(x+h)-f(x) = 6hx + 3h^2 - h$
$\frac{\Delta y}{\Delta x} = 6x + 3\Delta x - 1$	… 17 m	$\frac{f(x+h) - f(x)}{h} = 6x + 3h - 1$
$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = 6x - 1$	… 20 m	$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 6x - 1$

- * In method **II**, candidate may start with $\frac{f(x+h) - f(x)}{h}$.
- * Overlook $\Delta x = 0$ or $h = 0$ in limit, and the use of dy/dx instead of lim $\Delta y/\Delta x$.
- * If first mention of LHS is in last line, then B4 and B5 apply; i.e. 14m for RHS correct.
- * Correct RHS but _no_ LHS => B4 + B5 + B6 apply, i.e. 11m.
- * After substit. and further work, B(-3) for each major step omitted; see steps … above.

Blunders (-3)

B1 Error multiplying out $(x + \Delta x)^2$ or $(x + h)^2$. Apply once.

B2 Brackets error, e.g in $3[x^2 + 2x\Delta x + (\Delta x)^2]$ or $-(x + h)$. Apply once per brackets.

B3 $(\Delta x)^2 = \Delta^2 . x^2$ (if it affects the solution); or $6x\Delta x = 6\Delta x^2$, _but_ allow Δx^2 for $(\Delta x)^2$.

B4 Omits $y + \Delta y$, or Δy, or $f(x + h)$, or $f(x + h) - f(x)$ on LHS. Apply once.

B5 Omits $\Delta y/\Delta x$ or $\{f(x + h) - f(x)\}/h$ on L.H.S.

B6 Omits limiting idea (word "lim" unnec.) or has other than $\Delta x \to 0$ or $h \to 0$ on L.H.S., i.e. should have "lim", or "$\Delta x \to 0$" (allow "$\Delta x = 0$" instead), or "dy/dx".

B7 Evaluates limit where Δx or h will not divide, e.g. no Δx on R.H.S. at that stage.

B8 Limit error, e.g. $\Delta y/\Delta x = 6x + 3\Delta x - 1$ but lim $\Delta y/\Delta x \neq 6x - 1$.

B9 Differentiates from first principles $3x^2 + x$, $3x^2$, $x^2 - x$ or $2x^2$.

Misreadings (-1)

M1 Differentiates from first principles $3x^2 - 3x$, $2x^3 - x$, $2x^3 - 3x$.

Slips (-1)

S1 Correct term such as $6x\Delta x$ subsequently "becomes" $6\Delta x$. (Misreads own work.)

Attempts (7 marks)

A1 $y+\Delta y$ or $f(x+h)$ on LHS; or $x+\Delta x$ or $x+h$ substituted somewhere on RHS.

A2 Linear function differentiated from 1st principles: if correct, award Att 7; if not, 0m.

A3 Writes $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ and stops, or $\lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$ and stops.

Worthless (0)

W1 Answer $6x - 1$ without work (i.e. not from first principles).

Part (c)	**20 (10, 5, 5) marks**	**Att 7 (3, 2, 2)**

> **(c)** Let $f(x) = \dfrac{1}{x+1}$ for $x \in \mathbf{R}$ and $x > -1$.
>
> **(i)** Find $f'(x)$.
>
> **(ii)** Find the co-ordinates of the point on the curve of $f(x)$ at which the tangent has slope of $-\dfrac{1}{4}$.
>
> **(iii)** Find the equation of the tangent to the curve which has slope $-\dfrac{1}{4}$.

(c) (i) $f'(x)$	**10 marks**	**Att 3**

I	**II**
$\dfrac{(x+1).0 - (1).(1)}{(x+1)^2}$ ...(10m)	$f(x) = (x+1)^{-1}$ ...(3m) $f'(x) = -1(x+1)^{-2}(1)$ or $-(x+1)^{-2}$ ...(10m)

* No marks for writing down u/v or $u.v$ formula from Tables, and stopping.

* Do not penalise subsequent errors in methods **I** and **II** of (c)(i), but *penalties may apply later in (c)(ii) and/or (c)(iii)*.

* If identified as u/v but $u.v$ used, apply B2 + B3 (and others if required).

Blunders (-3)

B1 Calculus errors. Once per term. Both methods have 2 terms to check.

B2 Central sign incorrect in u/v formula.

B3 No division by v^2 in method **I**.

B4 Vice versa substitution for du/dx and dv/dx in method **I**.

B5 $y \neq (x+1)^{-1}$ in method **II**.

Attempts (3 marks)

A1 Identifies $u = 1$ and $v = x + 1$ and stops.

A2 Any correct derivative, du/dx or dv/dx, and stops.

A3 $dy/dx = 0/1$, or $dy/dx = 0$.

A4 $y = (x+1)^{-1}$ and stops, in method **II**.

Worthless (0)

W1 No term differentiated correctly correctly, unless A1 or A4 applies.

(c) (ii) point **5 marks** **Att 2**

<table>
<tr><td>

I: $\dfrac{-1}{(x+1)^2} = \dfrac{-1}{4}$ ...(2m)

$(x+1)^2 = 4$ cross mult. or equate denom.

$x^2 + 2x + 1 = 4$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3,\ x = 1$

$y = 0.5$ ...(5m)

</td><td>

II : $\dfrac{-1}{(x+1)^2} = \dfrac{-1}{4}$ ...(2m)

$x = 1$ without work (by inspection) ...(2m) still

$y = 0.5$ ...(5m)

III:

$x = 1, y = 0.5$ without work ...(5m)

</td></tr>
</table>

* If $f'(x)$ from (i) is incorrectly simplified, slips and blunders apply here in (ii).

* If $f'(x)$ is linear or cannot produce a quadratic, att mark at most in (c)(ii).

* Ignore extra point supplied in answer ($x = -3, y = -1/2$) and any work related to it.

Blunders (-3)

B1 $f'(x)$ equated to a number other than $-1/4$.

B2 Error multiplying out brackets. Apply once.

B4 Quadratic formula error in formula, substitution or simplification. Max. of 2 blunders.

B5 Incorrect factors. Apply once.

B6 Incorrect root(s) from candidate's factor(s). Apply once.

B7 Finds x and stops. No y value found.

Attempts (2 marks)

A1 Substitutes $x = -1/4$ into dy/dx.

A2 $f'(x)$ from (i) simplified to $x/(x+1)^2$, and continues. [i.e. $f'(x) = (x+1).0 \ldots = x+1\ldots$].

A3 Quadratic formula correct and stops.

Worthless (0)

W1 Incorrect point without work.

(c) (iii) tangent **5 marks** **Att 2**

$$y - y_1 = m(x - x_1) \qquad \text{...(2m)}$$

$$y - \frac{1}{2} = -\frac{1}{4}(x - 1) \qquad \text{...(5m)}$$

* Allow candidate's (x_1,y_1) point found in (c)(ii).

* If $f'(x)$ calculated again, remark it but subject to a max of (-3)m if incorrect. See B3.

Blunders (-3)

B1 Incorrect relevant formula, substituted with candidate's coordinates or slope.

B2 Incorrect coordinates used for (x_1,y_1), unless M1 applies.
Examples: coordinates in reverse order, or uses a point not found in (c)(ii).

B3 $m \neq -\frac{1}{4}$, or no slope substituted.

Misreadings (-1)

M1 Uses (-3, - ½) as (x_1,y_1).

Attempts (2 marks)

A1 $y - y_1 = m(x - x_1)$ and stops. (No marks for $y = mx + c$ in this question).

Worthless (0)

W1 Incorrect and unsubstituted formula.

An Roinn Oideachais agus Eolaíochta

Leaving Certificate Examination 2001

Marking Scheme

MATHEMATICS - Ordinary Level

Paper 2

General Instructions

Penalties are applies as follows:

	Numerical slips, misreadings	(-1)
	Blunders, major omissions	(-3)

Note 1: The list of slips, blunders and attempts given in the Marking Scheme is not exhaustive.

Note 2: A serious blunder, omission or misreading which oversimplifies merits the attempt mark at most.

Note 3: The same error in the same section of a question is penalised once only.

Note 4: The attempt mark is awarded only after relevant deductions are made.
A mark between 0 and the attempt mark may not be awarded.
Particular cases or verifications usually qualify for the attempt mark.

Note 5: Mark all of the candidate's work, including excess answers and repeated answers whether cancelled or not, and allow the highest scoring answers.

Note 6: The phrase "and stops" used in the marking scheme means that no more work is shown by the candidate.

Note 7: Special notes relating to the marking of a particular part of a question are indicated by *.

Note 8: Solution methods are labelled α, β, etc.

Question 1

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 marks** **Att 3**

(a) A running track is made up of two straight parts and two semicircular parts as shown in the diagram.

The length of each of the straight parts is 90 metres.

The diameter of each of the semicircular parts is 70 metres.

Calculate the length of the track correct to the nearest metre.

(a) **10 marks** **Att 3**

(a) Semicircular ends $= 2\pi r = 2 \times {}^{22}/_{7} \times 35$ or 220
Straights $=$ 90 + 90 or 180
Total length $=$ 220 + 180 or 400

* Accept π = 3 or more accurate.

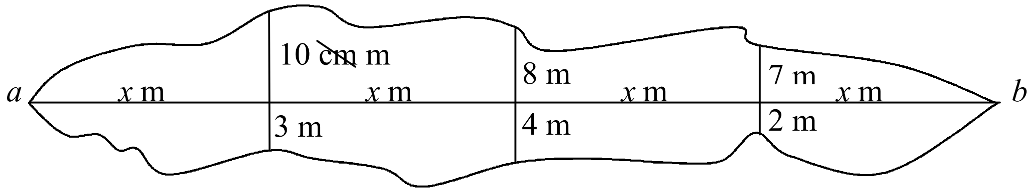
Blunders (-3)
B1 Incorrect relevant formula e.g. area or L×B or ½ πr (once if same error, e.g. area of both)
B2 Incorrect substitution or omission, e.g. $r \neq 35$, no subs for π in formula (once each step).
B3 Omits last step.
B4 Subtracts instead of adds.
B5 Mathematical errors.
B6 Correctly fills in formulae and stops (3m for calculations).
B7 Two ends + perimeter of rectangle, e.g. 220 + 180 + 70 +70 or equivalent.

Slips (-1)
S1 Each arithmetic slip to a maximum of 3.
S2 Failure to round off.

Attempts (3 marks)
A1 Correct relevant formula not in tables and stops.
A2 One correct substitution and stops.
A3 Substitutes into one formula and stops.
A4 Any manipulation of 70, 35, 90 and stops.
A5 Deals only with straight sides, e.g. 90 + 90 or 90 + 90 + 70 + 70.
A6 400 or 399·something without work.
A7 One step only, i.e. 220 only **or** 180 only. [Note: 220, 180 merits 7 marks.]
A8 $r = 35$ or 35 only and stops.

Part (b) **20 (10, 5, 5) marks** **Att 7 (3, 2, 2)**

(b) The sketch shows a flood caused by a leaking underground pipe that runs from a to b.



At equal intervals of x m along [ab] perpendicular measurements are made to the edges of the flood. The measurements to the top edge are 10 m, 8 m and 7 m. The measurements to the bottom edge are 3 m, 4 m and 2 m. At a and b the measurements are 0 m.

Using Simpson's Rule the area of the flood is estimated to be 672 m^2.

Find x and hence, write down the length of the pipe.

(b) Use formula	**10 marks**	**Att 3**
Calculations x	**5 marks**	**Att 2**
Length of pipe	**5 marks**	**Att 2**

(b) α: Area $= {}^h/_3\{ F + L + 2(\text{odds}) + 4(\text{evens})\}$

${}^h/_3\{0 + 0 + 2(8) + 4(10 + 7)\} + {}^h/_3\{0 + 0 + 2(4) + 4(3 + 2)\}$

$= {}^h/_3\{ 0 + 0 + 2(8 + 4) + 4(10 + 3 + 7 + 2)\}$

$= {}^h/_3\{ 24 + 88\}$

$= {}^h/_3\{ 112\}$

$= {}^{112h}/_3$

${}^{112h}/_3 = 672$

$h = 672 \times {}^3/_{112}$

$h = 18$

Length = 18 × 4 = 72

β: ${}^h/_3\{F + L + TOFE\}$

F/L	O	E
0	8	10 + 3
	4	7 + 2
${}^h/_3\{$	× 2	× 4$\}$

* A candidate may obtain Att 3 marks for 1st stage and 5 marks for a later stage.

* Accept 10 cm instead of 10 m→${}^x/_3\{44·4\}+{}^x/_3\{28\}$or ${}^x/_3\{72·4\}$→x = 27·845→pipe =111·38.

* Allow ${}^h/_3 = \{ F + L + 2(\text{odds}) + 4(\text{evens})\}$. Penalise in calculations, if used.

Blunders (-3)

B1 Incorrect ${}^h/_3$ (once).
B2 Incorrect F and/or L or extra terms with F and L (once).
B3 Incorrect TOFE (once).
B4 E or O omitted (once).
B5 Distribution error. Penalise once.
B6 Transposing error.
B7 Incorrect use of 672m^2, e.g. $({}^{112}/_3)\times 672$.
B8 Length = n × 18 where $n \neq 4$, e.g. (writes) pipe = 18 merits 2 marks in last stage.
B9 Deals with top (x = 24) **or** bottom (x = 72) but not both. [17 marks are still possible.]

Slips (-1)

S1 Numerical slip to a maximum of 3.

Attempts (3m for use of formula and 2m for each calculation).

A1 Identifies F and/or L or odds or evens and stops. (3m)
A2 Statement of Simpson's Rule, i.e. ${}^h/_3\{F + L + TOFE\}$. (3m)

A3 E **and** O omitted. (3m) (at most for 1st stage)
A4 Some correct relevant calculation only. (2m)
A5 Completes one rectangle or area of one rectangle. (2m)
A6 Completes all rectangles, but no calculations. (3m)
A7 Completes all rectangles and adds areas. (3m+2m)
A8 $^{x}/_{3}(112)$ or $^{x}/_{3}(72{\cdot}4)$ without work. (3m)
A9 $x = 18$ or 27·8… without work. (3m+2m)
A10 Pipe = 72 or 111·3… without work. (3m + 2m + 2m)
A11 Pipe = $4x$. (2m) last stage.

Worthless (0 marks)
W1 Formula taken from Tables and stops.
W2 Incorrect answer without work.

Part (c) **20 (5, 5, 10) marks** **Att 7 (2, 2, 3)**

(c) Sweets, made from a chocolate mixture, are in the shape of solid spherical balls. The diameter of each sweet is 3 cm.

36 sweets fit exactly in a rectangular box which has internal height 3 cm.

(i) The base of the box is a square. How many sweets are there in each row?

(ii) What is the internal volume of the box?

(iii) The 36 sweets weigh 675 grammes. What is the weight of 1 cm^3 of the chocolate mixture? Give your answer correct to one decimal place.

(c) (i) **5 marks** **Att 2**

Number of sweets per row = $\sqrt{36}$ or 6

* Accept correct answer without work.

Blunders (-3)
B1 Base not a square.

Attempts (2 marks)
A1 Some manipulation of 36 or 3 or 1·5, e.g. 36^2 or ½(360) or 5 × 7 +1 or some factor of 36.
A2 Knowledge of a square, e.g. x^2 or $4x$ or L × B.

(c) (ii) **5 marks** **Att 2**

Volume = L × B × H = 18 × 18 × 3 = 972

* Accept shape of box consistent with part (i), particularly when applying B2.

Blunders (-3)
B1 Incorrect relevant formula, e.g. L × B.
B2 Incorrect substitution.
B3 Correctly filled in formula and stops.

Slips (-1)
S1 Arithmetic slips to a maximum of 3.

Attempts (2 marks)

A1 Correct formula not from Tables and stops, e.g. L × B × H.
A2 Correct substitution in relevant formula, e.g. 3 × 3 × 3 and stops.
A3 Any correct dimension, including r = 1·5.
A4 Three arbitrary numbers (shown) multiplied, e.g. 8 × 4 × 5 presupposes correct formula.
A5 Correct or consistent answer without work
A6 Volume of 1 or 36 sweets **with work**, i.e. 14·142857 or 509·14286 cm^3.

(c) (iii) **10 marks** **Att 3**

Volume 1 Sweet = $^4/_3\pi r^3 = {}^4/_3 \times {}^{22}/_7 \times 1{\cdot}5^3$ or 14·142857 cm^3	
α: Volume 36 sweets = 36×14·142 or 509·142 cm^3 1 cm^3 weighs $^{675}/_{(36 \times 14\cdot142)}$ or $^{675}/_{509\cdot14286}$ = 1.325g ≅ 1.3g	β: 1 sweet weighs $^{675}/_{36}$ or 18·75 1 cm^3 weighs $^{675}/_{(36 \times 14\cdot142)}$ or $^{18\cdot75}/_{14\cdot142}$ = 1.325g ≅ 1.3g

* Accept π = 3 or more accurate.
* Units other than grammes must be identified.
* Allow " Volume of sphere = $^4/_8\ \pi r^3$ ". This may be due to fault in Tables.

Blunders (-3)

B1 Incorrect relevant sphere formula, e.g. $^2/_3\ \pi r^3$, $4\pi r^2$.
B2 Incorrect substitution, e.g. $r \neq 1{\cdot}5$.
B3 Mathematical error, e.g. mishandles fractions or misplaced decimal point, e.g. $^{14\cdot142}/_{18\cdot75}$.
B4 Correctly fills in formula and stops.
B5 Number of sweets ≠ 36, e.g. 1cm^3 of one sweet $^{675}/_{14\cdot142}$ and continues.
B6 Incorrect transposing.

Slips (-1)

S1 Arithmetic slips to a maximum of 3.
S2 Fails to round off or rounds off incorrectly or rounds off too soon, if it affects the answer.

Attempts (3 marks)

A1 r = 1·5 and stops.
A2 Some correct substitution into relevant formula and stops but not π only, e.g. into πr^2.
A3 1·3… or 1·3 without work.
A4 One sweet = $^{675}/_{36}$ g or 18·75 g or 18·7 g or 18·8 g without work.
A5 972 cm^3 = 675 g ==> 1 cm^3 = $^{675}/_{972}$, even if completed.
A6 Correct calculation with relevant numbers but incorrect operation, e.g. 36 × 675 = 24300.

Worthless (0 marks)

W1 Substitutes only for π, i.e. $^4/_3 \times {}^{22}/_7 \times r^3$ or similar.

Question 2

Part (a)	**10 marks**	**Att 3**
Part (b)	**40 marks**	**Att 14**

Part (a) **10 marks** **Att 3**

(a)	The point $(t, 2t)$ lies on the line $3x + 2y + 7 = 0$. Find the value of t.

(a) **10 marks** **Att 3**

α: Subs. $3(t) + 2(2t) + 7 = 0$	==> $7t + 7 = 0$	==> $t = -\,^{7}/_{7}$ or -1
β: Eqn. Slope = $-\,^{3}/_{2}$	==> Eqn is $y - 2t = -\,^{3}/_{2}(x - t)$	==> $t = -\,^{7}/_{7}$ or -1

* Accept t = - 1 correctly verified.
* Incorrect answer without work merits 0 marks.

Blunders (- 3)
B1 Mixes up x and y entries (penalise once only), i.e. t for y and $2t$ for x.
B2 Error in **more than one** sign when substituting into $x_2 - x_1$ or $y_2 - y_1$ or similar.
B3 Incorrect relevant formula, e.g. $x - x_1 = m(y - y_1)$ or $y = mx - c$.
B4 Incorrect slope (without work) and continues with answer.
B5 Incorrect substitution, if not an obvious misreading (once).
B6 Mathematical error, e.g. $3(-2) = 6$ or $-(-3) = -3$ or $3t + 4t = 7t^2$.
B7 Transposing error, e.g. $7t + 7 = 0 \Rightarrow 7t = 7$ or $7t = -7 \Rightarrow t = {}^{-7}/_{-7} = 1$ (once).

Slips (- 1)
S1 Each numerical slip to a maximum of 3, e.g. $3 \times 0 = 3$.
S2 Error in **one** sign when substituting into $x_2 - x_1$ or $y_2 - y_1$ or similar.

Misreadings (- 1)
M1 Obvious misreading which does not oversimplify task.

Attempts (3 marks)
A1 Substitutes one value and stops.
A2 Correct relevant formula and stops, e.g. equation of line or slope formula.
A3 1st step in either method and stops.
A4 Effort at substituting and stops, i.e. finding a point on given line or $x = t$ or $y = 2t$ or $3(0) + 2y + 7 = 0$.
A5 Correct answer without work.
A6 Formula with $x_2 - x_1$ or $y_2 - y_1$ or similar with some correct substitution.
A7 Plots a correct point of the given line.

Worthless (0 marks)
W1 Draws axes and/or arbitrary line and stops.

Part (b) **40 (10, 10, 5, 5, 10)** **Att 14 (3, 3, 2, 2, 4)**

(b) $a(4, 2)$, $b(-2, 0)$ and $c(0, 4)$ are three points.

(i) Prove that $ac \perp bc$.

(ii) Prove that $|ac| = |bc|$.

(iii) Calculate the area of the triangle bac.

(iv) The diagonals of the square $bahg$ intersect at c.
Find the co-ordinates of h and the co-ordinates of g.

(v) Find the equation of the line bc and show that h lies on this line.

(b) (i) **10 marks** **Att 3**

(b) (i) α	β	γ	δ
Slope = $\frac{y_2 - y_1}{x_2 - x_1}$	$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	$\vec{ac} = \vec{c} - \vec{a}$
Slope $ac = {}^{(4-2)}/_{(0-4)} = {}^{2}/_{-4}$	Eqn ac: $x + 2y = 8$	$\|ac\| = \sqrt{20}$	$\vec{ac} = -4\vec{i} + 2\vec{j}$
Slope $bc = {}^{(4-0)}/_{(0+2)} = {}^{4}/_{2}$	Eqn bc: $2x - y = -4$	$\|bc\| = \sqrt{20}$, $\|ab\| = \sqrt{40}$	$\vec{bc} = 2\vec{i} + 4\vec{j}$
${}^{2}/_{-4} \times {}^{4}/_{2} = -1$	$-{}^{1}/_{2} \times 2 = -1$	$\|ab\|^2 = \|ac\|^2 + \|bc\|^2$	$\vec{ac} \cdot \vec{bc} = 0$

* Accept equivalent expressions, e.g. $(y_1 - y_2)/(x_1 - x_2)$ for slope.
* Accept slope $ac = -\frac{1}{2}$, slope $bc = 2$, $-\frac{1}{2} \times 2 = -1$ for 10 marks.
* Correct use of - 1 or equivalent is required for final 1 mark in methods α and β.

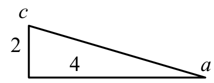
Blunders (- 3)
B1 Incorrectly treats couples as (x_1, x_2) and (y_1, y_2), (once).
B2 Incorrect relevant formula, e.g. or $\frac{y_2 - y_1}{x_1 - x_2}$ or $\frac{x_2 - x_1}{y_2 - y_1}$ or error in **both** signs (once).
B3 x and y switched in substitution (once).
B4 Error in more than one sign when substituting, e.g. ${}^{(4+2)}/_{(0+4)}$.
B5 Mathematical error, e.g. 4 - 2 = - 2.

Slips (- 1)
S1 Each numerical slip to a maximum of 3.
S2 **One** incorrect sign in $(x_2 - x_1)$ **or** $(y_2 - y_1)$.
S3 **One** incorrect substitution, if formula is written, e.g. ${}^{(4+2)}/_{(0-4)}$.

Misreadings (- 1)
M1 Obviously, writes down a coordinate incorrectly from the question paper.
M2 Finds the slope of ab, instead of one of the other lines, correctly.

Attempts (3 marks)
A1 1st step and stops.
A2 Slope = ${}^{\text{vertical}}/_{\text{horizontal}}$ or ${}^{\text{opposite}}/_{\text{adjacent}}$ or tanθ or similar and stops.
A3 Point a and/or b and/or c plotted reasonably well.
A4 Formula with $x_2 - x_1$ and/or $y_2 - y_1$ with some correct substitution.
A5 $m_1 \times m_2 = -1$ or equivalent.

(b) (ii) **10 marks** **Att 3**

(b) (ii) α	β
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	
$\|ac\| = \sqrt{(0-4)^2+(4-2)^2} = \sqrt{(-4)^2+(2)^2} = \sqrt{16+4}$ or $\sqrt{20}$	$\|ac\|^2 = 4^2+2^2 \Rightarrow \|ac\| = \sqrt{16+4}$ or $\sqrt{20}$
$\|bc\| = \sqrt{(0--2)^2+(4-0)^2} = \sqrt{(2)^2+(4)^2} = \sqrt{4+16}$ or $\sqrt{20}$	$\|bc\|^2 = 2^2+4^2 \Rightarrow \|bc\| = \sqrt{4+16}$ or $\sqrt{20}$

* B2 and B3 may both apply, e.g. $|ac| = (x_2 - x_1)^2 - (y_2 - y_1)^2$ and continues.
* Accept proving $|ac|^2 = |bc|^2$.

Blunders (- 3)
B1 Incorrectly treats couples as (x_1, x_2) and (y_1, y_2) or x and y switched when substituting (once).
B2 Incorrect relevant formula, e.g. $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$ or $\sqrt{(x_2 + x_1)^2 + (y_2 + y_1)^2}$.
B3 Inconsistent or incorrect use of $\sqrt{}$, e.g. $|ac| = 20$.
B4 Mathematical errors, e.g. $4^2 = 8$ or $-(-2) = -2$ or $(-4)^2 = -16$.
B5 Omits last step.
B6 Two or more incorrect substitutions.

Note: No formula written anywhere (sign error is blunder at least), e.g.
$|ac| = \sqrt{(0-4)^2 - (4-2)^2} = \sqrt{12}$ One blunder
$|ac| = \sqrt{(0+4)^2 + (4-2)^2} = \sqrt{20}$ or $\sqrt{(0+4)^2+(4+2)^2} = \sqrt{52}$ One blunder
With or without formula written: $|ac| = \sqrt{(0+4)^2 - (4+2)^2} = \sqrt{-20}$ Two blunders.

Slips (- 1)
S1 Each numerical slip to a maximum of 3.
S2 **One** incorrect sign in $(x_2 - x_1)$ or $(y_2 - y_1)$.
S3 **One** incorrect substitution, if formula is written.
S4 Calculates $|ab|$ instead of $|ac|$ or $|bc|$.

Attempts (3 marks)
A1 Correct relevant formula and stops.
A2 Oversimplifies, e.g. $\sqrt{(x_2 - x_1)+(y_2 - y_1)}$ with some correct substitution, even if completed.
A3 Point a and/or b and/or c plotted reasonably well **for this part**.
A4 Formula with $x_2 - x_1$ and/or $y_2 - y_1$ with some correct substitution and stops.
A5 States Pythagoras' Theorem.
A6 $\sqrt{a^2 + b^2}$ applied, e.g. $\sqrt{5^2 + 3^2} = \sqrt{34}$.
A7 $|ac| = \sqrt{20}$ and/or $|bc| = \sqrt{20}$ without work.
A8 Uses translation only.

Worthless (0 marks)
W1 Irrelevant formula, even if completed, e.g. midpoint or $\sqrt{x_2y_1 - y_2x_1}$ or similar.

(b) (iii) **5 marks** **Att 2**

(b) (iii) $a(4, 2)$, $b(-2, 0)$, $c(0, 4)$ $\overrightarrow{ao} \Rightarrow b_1(-6, -2), c_1(-4, 2)$ $\overrightarrow{bo} \Rightarrow a_1(6, 2), c_1(2, 4)$

α	β	γ	δ
$\frac{1}{2}\|ac\|.\|bc\|$	Translation $(0, 4) \rightarrow (0, 0)$	Area =	$\frac{1}{2}\begin{vmatrix} 4 & 2 \\ -2 & 0 \\ 0 & 4 \\ 4 & 2 \end{vmatrix}$
$\frac{1}{2}\sqrt{20}.\sqrt{20}$	$c_1(0, 0); a_1(4, -2); b_1(-2, -4)$	$\frac{1}{2}\|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\|$	
	$\frac{1}{2}\|x_1y_2 - x_2y_1\|$	$\frac{1}{2}\|4(0-4) - 2(4-2) + 0(2-0)\|$	
	$\frac{1}{2}\|4(-4) - (-2)(-2)\|$	$\frac{1}{2}\|4(-4) - 2(2) + 0(2)\|$	
$\frac{1}{2} \times 20$ or 10	$\frac{1}{2}\|-20\|$ or 10	$\frac{1}{2}\|-20\|$ or 10	$\frac{1}{2}\|20\|$ or 10

* Area = - 10, no penalty.

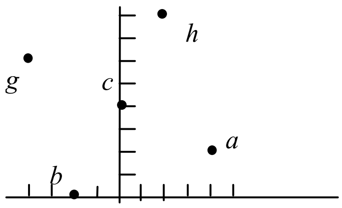
Blunders (- 3)
B1 Incorrect relevant formula and continues, e.g. $\frac{1}{2}|x_1y_2+x_2y_1|$, $\frac{1}{2}|x_1y_2 \times x_2y_1|$ or omits the ½.
B2 Two or more incorrect substitutions.
B3 Multiplies non-perpendicular sides in α method.
B4 Calculates area of Δaoc or Δaob or Δboc.
B5 Mathematical errors, e.g. (- 2)(- 2) = - 4.

Slips (- 1)
S1 Each numerical slip to a maximum of 3.
S2 **One** sign error in formula of method *γ*.
S3 **One** sign error in substitution, if the formula has been written.

Attempts (2 marks)
A1 Correct answer without work.
A2 Correct relevant formula and stops.
A3 Point *a* and/or *b* and/or *c* plotted reasonably well **for this part**.
A4 $\frac{1}{2}|x_1y_2 + x_2y_1|$ or similar with one correct substitution and stops.

(b) (iv) **5 marks** **Att 2**

(b) (iv) α: $a(4, 2) \rightarrow c(0, 4) \rightarrow g(-4, 6)$, i.e. ($x$ down 4, y up 2)
$b(-2, 0) \rightarrow c(0, 4) \rightarrow h(2, 8)$, i.e. ($x$ up 2, y up 4)



β: Midpoint $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

c is midpoint of $[ag] \Rightarrow {}^{(4+x)}/_2 = 0 \Rightarrow x = -4$ and ${}^{(2+y)}/_2 = 4 \Rightarrow y = 6 \Rightarrow g(-4, 6)$
c is midpoint of $[bh] \Rightarrow {}^{(-2+x)}/_2 = 0 \Rightarrow x = 2$ and ${}^{(0+y)}/_2 = 4 \Rightarrow y = 8 \Rightarrow h(2, 8)$

* Accept $\{h\} = bc \cap ah$, using equations of lines and simultaneous equations.
* Accept correct answers without work for 4 + 1 marks.
* Slips and blunders are subject to S2.

Blunders (- 3)
B1 Incorrect direction of the translation, e.g. $\overrightarrow{ca}$ (once) or incorrect centre of symmetry.
B2 Error in translation of *x* or *y* ordinate in *h* or *g*, other than an obvious slip.
B3 Mathematical error.
B4 Incorrectly treats couples as (x_1, x_2) and (y_1, y_2).
B5 Incorrect relevant formula, e.g. $\left(\frac{x_1 - x_2}{2}, \frac{y_1 - y_2}{2}\right)$ or $\left(\frac{x_1 + y_1}{2}, \frac{x_2 + y_2}{2}\right)$.
B6 Two or more signs incorrect in substitution.
B7 Transposing error, e.g. $2 + y = 8 \Rightarrow y = 8 + 2 = 10$.

Slips (- 1)
S1 Each numerical slip to a maximum of 3, e.g. 8 – 2 = 7.
S2 Finds coordinates of *h* **or** *g* correctly (not both).
S3 $h(-4, 6)$, $g(2, 8)$.

Attempts (2 marks)
A1 Correct relevant formula and stops, e.g. $y - y_1 = m(x - x_1)$.

A2 States a relevant theorem, e.g. "diagonals of a parallelogram bisect each other".
A3 Makes some reference to midpoint, e.g. implied by diagram or $g(2, 3)$ = midpoint of [ac].
A4 Point a and/or b and/or c plotted reasonably well **for this part**.
A5 Correct h and/or g indicated but no coordinates written down.
A6 One correct ordinate of h or g.
A7 Calculates the slope or length of one of given line segments correctly in this part.

(b) (v) Equation of bc **5 marks** **Att 2**

(b) (v) Equation of bc	α	β	γ
	$y - y_1 = m(x - x_1)$	$y = mx + c$	$2x - y + c = 0$
slope = 2		$(0) = 2(-2) + c$	$2(-2) - 0 + c = 0$
point (- 2, 0) or (0, 4)	$y - 0 = 2(x + 2)$ or $y - 4 = 2(x - 0)$	$c = 4$	$c = 4$

* Accept slope of bc from previous part of question 2.
* Accept correct or consistent line equation without work.
* Penalise errors in simplifying equation in next part.
* Allow candidate to use slope 2 and point h to find equation of line for 1st 5 marks and to show that b or c is an element of that line for the other 5 marks.

Blunders (- 3)
B1 Last step omitted.
B2 Two or more sign errors in substitution.
B3 Incorrect relevant formula, e.g. $y + y_1 = m(x + x_1)$ or $x - x_1 = m(y - y_1)$.
B4 Mixes up x's and y's in substitution, e.g. $y + 2 = 2(x - 0)$, if not an obvious slip.
B5 Transposing error, e.g. $-4 + 0 + c = 0 \Rightarrow c = -4$.
B6 Sign error that is not an obvious slip, e.g. $y = mx - c$ in method β or $2x + y + c = 0$ in γ.

Slips (- 1)
S1 Each numerical slip to a maximum of 3.
S2 **One** sign incorrect in formula of α method.
S3 One sign incorrect in substitution in any method.

Attempts (2 marks)
A1 Correct relevant formula and stops (including slope formula).
A2 Correct or consistent slope of bc written for this part of question and stops.
A3 $y - 0 = m(x + 2)$ with $m \neq 2$ or $2x - y + c = 0$ with $c \neq 4$ or $x + 2y + c = 0$ without work.

(b) (v) $h \in bc$ **5 marks** **Att 2**

(b) (v) $h \in bc$ α	β	γ
Substitute for $h(2, 8)$ in $2x - y + 4 = 0$	Slope of $bc = 2$	Eqn of line bh
$2(2) - (8) + 4$	Slope of ch or $bh = \frac{y_2 - y_1}{x_2 - x_1}$	$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$
$= 0$	$= {}^{(8-4)}/_{(2-0)} = {}^4/_2$ or 2	${}^{(y-0)}/_{(8-0)} = {}^{(x+2)}/_{(2+2)}$
		$2x - y + 4 = 0$ or equivalent

* Accept area of $\Delta bch = 0$ or $\perp$ distance from h to $bc = 0$ with relevant work.
* Accept correct substitution of coordinates of h from previous work for 5 marks.
* Accept equation of bc from previous part.
* Accept work with no conclusion drawn.

Blunders (- 3)
B1 Last step omitted or incorrect conclusion, e.g. 2(3) - (8) + 4 = 0.
B2 Two or more sign errors in substitution.
B3 Incorrect relevant formula, e.g. two or more signs incorrect in methods other than α.
B4 Mixes up *x*'s and *y*'s in substitution, e.g. 2(8) – 2 + 4, if not an obvious slip.
B5 Mathematical error, e.g. 8 – 4 = - 4 in method β or transposing error in γ.

Slips (- 1)
S1 Investigates a relevant point (some point from the question) but not *h*.

Attempts (2 marks)
A1 Any relevant step and stops.
A2 Point *b* and/or *c* and/or *h* plotted reasonably well **for this part**.
A3 Substitutes the coordinate(s) of an arbitrary (not used previously) point correctly in *bc*.
A4 β method: slope of *bc* written down in this part and stops.

QUESTION 3

Part (a)	**15 marks**	**Att 6**
Part (b)	**15 marks**	**Att 5**
Part (c)	**20 marks**	**Att 7**
Part (a)	**15 (10, 5) marks**	**Att 6 (2, 2, 2)**

(a) The circle S has equation $(x-3)^2 + (y-4)^2 = 25$.

(i) Write down the centre and the radius of S.

(ii) The point $(k, 0)$ lies on S. Find the two real values of k.

(a) (i) Centre	**5 marks**	**Att 2**
Radius	**5 marks**	**Att 2**

(a) (i) α	β
By inspection:	$x^2 - 6x + 9 + y^2 - 8y + 16 = 25$
	$x^2 + y^2 - 6x - 8y + 0 = 0$
Centre = (3, 4)	Centre = (3, 4)
Radius = 5 or $\sqrt{25}$ or $r^2 = 5^2$	$r = \sqrt{g^2 + f^2 - c} = \sqrt{9 + 16 - 0} = \sqrt{25}$ or 5

* Accept correct centre without work or (3, 4) plotted **correctly** without coordinates written.
* Accept correct circle drawn with correct centre and radius without centre or radius written.
* Accept $r = 5$ or $\sqrt{25}$ or $r^2 = 5^2$ without work.
* Award 5 marks for $r = -5$ with correct work. Award 0 marks for $r = -5$ without work.

Blunders (- 3)
B1 Centre = (- 3, - 4), i.e. both signs incorrect.
B2 Centre = (¾, 1) or (9, 16) without work.
B3 Centre = (4, 3), i.e. confuses x and y values.
B4 $r^2 = 25 \Rightarrow r = 12\frac{1}{2}$..
B5 $r^2 = 25$ and stops.
B6 Error expanding $(x-3)^2 + (y-4)^2 = 25$, e.g. $(x-3)^2 = x^2 + 9$ (once).
B7 Incorrect relevant formula and continues, e.g. $r = g^2 + f^2 - c$.

Slips (- 1)
S1 **One** sign of centre incorrect.
S2 Each numerical slip to a maximum of 3.

Attempts (2 marks each)
A1 Correct relevant formula, e.g. $x^2 + y^2 = r^2$ (1 × Att 2), $r = \sqrt{g^2 + f^2 - c}$ (another 1 × Att 2)
A2 Some correct work in expansion of $(x-3)^2 + (y-4)^2 = 25$ (1 × Att 2).
A3 $x^2 + y^2 - 6x - 8y = 0$ and stops, award 4 marks, i.e. Att 2 + Att 2.
A4 Effort at finding a point on S, e.g. substituting some number for x or y in S (1 × Att 2).
A5 Draws axes and circle with correct centre (coords not written) and $r \neq 5$. Award 7m(5+2)
A6 Draws axes and circle without correct centre or radius. Award 4 marks (Att 2 + Att 2).
A7 $r = 25$ with or without work (1 × Att 2).

Worthless (0 marks)
W1 Draws axes without a circle.

(a) (ii) **5 marks** **Att 2**

(a) (ii) α	β
$(k-3)^2 + (0-4)^2 = 25$	Verifies both values of k
$k^2 - 6k + 9 + 16 = 25$	$k = 0 \implies (0-3)^2 + (0-4)^2 = 25$
$k^2 - 6k = 0$	$9 + 16 = 25$
$k(k-6) = 0$ ⋮ $k = \dfrac{--6 \pm \sqrt{36-0}}{2}$	$k = 6 \implies (6-3)^2 + (0-4)^2 = 25$
$k = 0$ or $k - 6 = 0$ ⋮ $k = {}^{6\pm6}/_2$	$9 + 16 = 25$
$k = 0$ or $k = 6$ ⋮ $k = 0$ or $k = 6$	$k = 0$ or $k = 6$

* Accept candidate's centre and radius from (i), if used.

Blunders (- 3)
B1 Incorrect squaring of $(k-3)$, e.g. $(k-3)^2 = k^2 + 9$.
B2 Incorrect factors or incorrect quadratic formula.
B3 Error in the use of the quadratic formula or one root or $k^2 - 6k = 0 \Rightarrow k - 6 = 0 \Rightarrow k = 6$.
B4 Transposing error.
B5 Incorrect relevant formula, e.g. $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$ with some correct substitution.

Slips (- 1)
S1 Each numerical slip to a maximum of 3.
S2 $\sqrt{(x_2 + x_1)^2 + (y_2 - y_1)^2}$, i.e. **one** non-central sign incorrect in formula or in substitution.

Attempts (2 marks)
A1 Substitutes $(k, 0)$ into equation of S and stops.
A2 Correct relevant formula, e.g. $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ with or without substitution and stops.
A3 Tests an arbitrary value for k in S or tests $k = 0$ **or** $k = 6$ but not both.
A4 Correct answer without work.
A5 Some correct work in expansion.

Part (b) **15 marks** **Att 5**

> **(b)** Prove that the line $x - 3y = 10$ is a tangent to the circle with equation $x^2 + y^2 = 10$ and find the co-ordinates of the point of contact.

(b) **15 marks** **Att 5**

(b) α	β	γ : L ⊥ x-3y =10
$x - 3y = 10 \Rightarrow x = 3y + 10$	or $y = {}^{(x-10)}/_3$	$(0, 0) \in$ L or slope of L is - 3
$x^2 + y^2 = 10 \Rightarrow (3y+10)^2 + y^2 = 10$	or $x^2 + [{}^{(x-10)}/_3]^2 = 10$	L is $3x + y = 0$
$9y^2 + 60y + 100 + y^2 = 10$	$x^2 + \dfrac{x^2 - 20x + 100}{9} = 10$	$3x + y = 0 \cap x - 3y = 10$ is $(1, -3)$
$10y^2 + 60y + 90 = 0$	$9x^2 + x^2 - 20x + 100 = 90$	Test if $(1, -3) \in x^2 + y^2 = 10$
$y^2 + 6y + 9 = 0$	$x^2 - 2x + 1 = 0$	
$(y+3)^2 = 0$ ⋮ $y = {}^{(-6 \pm \sqrt{36-36})}/_2$	$(x-1)^2 = 0$ ⋮ $x = {}^{(+2 \pm \sqrt{4-4})}/_2$	
$y = -3$ ⋮ $y = -3$	$x = 1$ ⋮ $x = 1$	$(1)^2 + (-3)^2 = 10$
$x - 3y = 10 \Rightarrow x - 3(-3) = 10 \Rightarrow x = 1$	$(1) - 3y = 10 \Rightarrow y = -3$	$1 + 9 = 10$

* Case: Proves distance from (0, 0) to $x - 3y = 10$ is $\sqrt{10}$ and writes $r = \sqrt{10}$ and stops, 9marks,
* Case: α method, last step: $y = -3 \implies x^2 + (-3)^2 = 10 \implies x^2 = 1 \implies x = 1$. Apply B(- 3).

δ
$xx_1 + yy_1 = r^2$
(1, -3)
[Tests if $(1, -3) \in$ circle]
$(1)^2 + (-3)^2 = 10$
$1 + 9 = 10$

Blunders (- 3)
B1 Transposing error (once for each type of error).
B2 Error in squaring $(3y + 10)$ or $^{(x-10)}/_3$.
B3 Incorrect relevant formula and continues, e.g. $\dfrac{-b \pm \sqrt{b^2 + 4ac}}{2a}$.
B4 Error in use of $^{(-b \pm \sqrt{b^2 - 4ac})}/_{2a}$.
B5 Error in factorising.
B6 Error in finding equation of line through origin and $\perp$ to $x - 3y = 10$ and continues.
B7 Omits last step or substitutes incorrectly in equation of circle in α or β method.
B8 Mathematical blunder.

Slips (- 1)
S1 Each numerical slip to a maximum of 3.

Attempts (5 marks)
A1 Writes $^{|ax_1 + by_1 + c|}/_{\sqrt{a^2+b^2}}$ with or without substitution and stops.
A2 Correct substitution for any value of x or y into $x^2 + y^2 = 10$ or $x - 3y = 10$ and stops.
A3 Correct step in finding equation of line $\perp$ to $x - 3y = 10$ and stops.
A4 Mentions or indicates that a tangent touches a circle at one point.
A5 Accurate graphical solution.
A6 $(1, -3)$ without work, even if substituted into line and/or circle.
A7 $r = \sqrt{10}$ and/or centre $= (0, 0)$ and stops.

Worthless (0 marks)
W1 $\left.\begin{array}{l} x - 3y = 10 \\ x^2 + y^2 = 10 \end{array}\right\}$ ==> $\left.\begin{array}{l} x^2 + 9y^2 = 100 \\ x^2 + y^2 = 10 \end{array}\right\}$ ==> $8y^2 = 90$, even if completed.

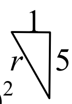
Part (c)	**20 (10, 10) marks**	**Att 7 (4, 3)**

(c) C is a circle with centre (0, 0). It passes through the point (1, -5).

(i) Write down the equation of C.

(ii) The point (p, p) lies inside C where $p \in \mathbf{Z}$.
Find all the possible values of p.

(c) (i) Radius	**5 marks**	**Att 2**

(c) (i) Radius: α	β	γ
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	$x^2 + y^2 = r^2$	
$\sqrt{(1-0)^2 + (-5-0)^2}$	$(1)^2 + (-5)^2 = r^2$	$r^2 = (1)^2 + (5)^2$
$r = \sqrt{26}$ or $r^2 = 26$	$\sqrt{26} = r$ or $r^2 = 26$	$r = \sqrt{26}$ or $r^2 = 26$

* Accept $r = \sqrt{26}$ or $r^2 = (\sqrt{26})^2$ or $r^2 = 26$ without work.

Blunders (- 3)
B1 Incorrect relevant formula, i.e. central sign incorrect in method α or β
or **both** signs in $(x_2 - x_1)$ and $(y_2 - y_1)$.
B2 Two or more sign errors in substitution.
B3 Incorrectly treats couples as (x_1, x_2) and (y_1, y_2), i.e. $\sqrt{(1 - -5)^2 + (0-0)^2}$.
B4 Mathematical error, e.g. $5^2 = 10$. [Note: $r^2 = 26$ ==> $r = 13$. Penalise in next part if used]
B5 $r = 26$, with or without work.

Slips (- 1)
S1 **One** sign incorrect in $(x_2 - x_1)$ or $(y_2 - y_1)$.
S2 **One** sign incorrect in substitution.
S3 Each numerical slip to a maximum of 3.

Attempts (2 marks)
A1 Plots (1, -5) and/or (0, 0) and stops.
A2 Correct relevant formula and stops, e.g. $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ or $x^2 + y^2 = r^2$.
A3 Any formula involving $(x_2 - x_1)$ or $(y_2 - y_1)$ or $(x - h)$ with some correct substitution.
A4 States Pythagoras' Theorem.

(c) (i) Equation of *C* **5 marks** **Att 2**

(c) (i) Equation of *C*: α	β
$x^2 + y^2 = r^2$	$(x - h)^2 + (y - k)^2 = r^2$
$x^2 + y^2 = (\sqrt{26})^2$	$(x - 0)^2 + (y - 0)^2 = (\sqrt{26})^2$
or $x^2 + y^2 = 26$	or $x^2 + y^2 = 26$

* Accept candidate's radius from previous part but for $r = 13$, apply B4 from previous part.
* Accept correct answer without work.
* Accept $(0 - h)^2 + (0 - k)^2 = 26$.
* N.B. $x^2 + y^2 = 26$ without work merits 5 + 5 marks (c) (i).
* $x^2 + y^2 = r^2$, $r^2 = 26$ merits full 10 marks for (c) (i).

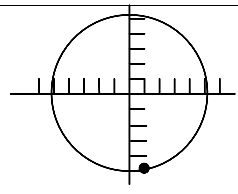
Blunders (- 3)
B1 Centre $\neq (0, 0)$.
B2 Uses radius which is neither correct nor consistent.

Attempts (2 marks)
A1 Correct relevant formula and stops.
A2 Incorrect relevant formula with some correct substitution, e.g. $x^2 - y^2 = r^2$ or $x^2 + y^2 = r$.

Worthless (0 marks)
W1 Linear equation used for equation of a circle.

(c) (ii) **10 marks** **Att 3**

(c) (ii) α	β
Points inside $\Rightarrow x^2 + y^2 < 26$	**Without work**
points $(p, p) \Rightarrow \quad y = x$	1 to 3 correct values or points
$\Longrightarrow x^2 + x^2 < 26$ or $p^2 + p^2 < 26$	
or $x^2 < 13$	
or $-\sqrt{13} < x < \sqrt{13}$	4 to 6 correct values or points [7 - S2, 4 to 7 m]
$\Longrightarrow p \in \{-3, -2, -1, 0, 1, 2, 3\}$	All values or points correct [10 - S2, 7 to 10 m]



* Accept correct answer without work.
* Accept work consistent with candidate's equation of *C*, if it does not simplify the task.
* $x^2 + x^2 \leq 26$ only, award 7 marks.
* $x^2 + x^2 = 26$ only, award 4 marks.
* If the candidate tests only points with $y = x$ and implies $x^2 + x^2 < 26$, treat it as α method. Award 7 marks for 1 to 6 correct points so tested. Award 10 marks for the 7 correct points. If the candidate tests points with $y \neq x$ as well as points with $y = x$, treat it as β method.

Blunders (- 3)
B1 Mathematical error, e.g. $x^2 < 13 \Longrightarrow x < \sqrt{13}$.
B2 Transposing error, e.g. $2x^2 < 26 \Longrightarrow x^2 < 24$.
B3 Takes $p \in R$, i.e. $-\sqrt{13} < p < \sqrt{13}$ with or without work. [7 marks.]

Slips (- 1)
S1 Each numerical slip to a maximum of 3.
S2 Without work: each incorrect value or point, i.e. each value or point outside circle or each point with $y \neq x$ to a maximum of 3.

Attempts (3 marks)
A1 Draws circle and plots at least 1 correct point inside the circle. [Merits at least Att 3.]
A2 1st step and stops.
A3 Writes or indicates that the distance from the centre to the point is less than the radius.
A4 $-5 \leq p \leq 5$ without work.
A5 Names any point or value inside circle and stops.
A6 Names any point with $y = x$ and stops.
A7 Substitutes p for x and/or p for y in equation of circle and stops.

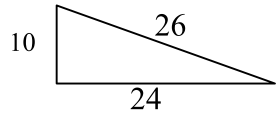
Worthless (0 marks)
W1 Names only values or points which are neither inside circle nor have $y = x$.

Question 4

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 8**

Part (a)	**10 marks**	**Att 3**

(a) Prove that the triangle with sides of lengths 10 units, 24 units and 26 units is right-angled.



(a)	**10 marks**	**Att 3**

$10^2 = 100$ or $24^2 = 576$ or $26^2 = 676$

$10^2 + 24^2 = 100 + 576 = 676 = 26^2$	$26^2 - 24^2 = 676 - 576 = 100 = 10^2$	$26^2 - 10^2 = 676 - 100 = 576 = 24^2$

* Last step is not demanded, if it is written earlier.
* Accept $5^2 + 12^2 = 25 + 144 = 169 = 13^2$, 5 : 12 : 13 = 10 : 24 : 26.

Blunders (- 3)
B1 Mathematical error, e.g. $10^2 = 20$ (once, and if not an obvious slip).
B2 Finds 3 squares and stops, i.e. fails to apply Pythagoras' theorem or applies it incorrectly.
B3 $10^2 + 24^2 = 26^2$ and stops.

Slips (- 1)
S1 Each obvious arithmetical slip to a maximum of 3.

Attempts (3 marks)
A1 States Pythagoras' theorem or gives another practical example, e.g. $3^2 + 4^2 = 5^2$.
A2 5 : 12 : 13 = 10 : 24 : 26 and stops or 5, 12, 13 and stops.
A3 Shows some relevant knowledge, e.g. identifies the right angle.

Part (b)	**20 marks**	**Att 7**

(b) Prove that a line which is parallel to one side-line of a triangle, and cuts a second side, will cut the third side in the same proportion as the second.

(b)	**20 marks**	**Att 7**

(b) α

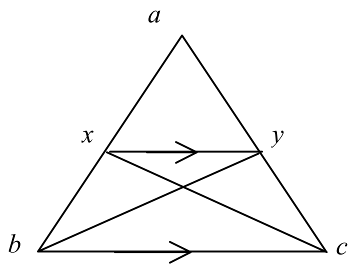
Construction: Join x to c and y to b — 5m

Proof: $\text{Area } \Delta axy / \Delta bxy = [\frac{1}{2}|ax|.h1 / \frac{1}{2}|xb|.h1] = |ax|/|xb|$

and $\Delta axy / \Delta cxy = |ay|/|yc|$ — 5, 2

Area Δbxy = Area Δcxy — 5, 2

$|ax|/|xb| = |ay|/|yc|$ — 5, 2

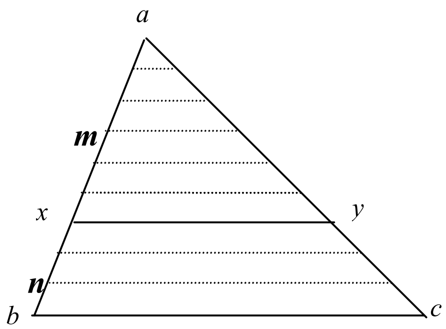


β

Construction:
Divide [ax] into $\boldsymbol{m}$ and [xb] into $\boldsymbol{n}$ equal parts. — 5m
Through each point of division draw a line parallel to bc. — 5, 2

Proof: The parallel lines make intercepts of equal length along [ac]. — 5, 2

$|ax| : |xb| = (\boldsymbol{m} : \boldsymbol{n}) = |ay| : |yc|$ — 5, 2



* If a candidate's work is not worthless, he/she must be awarded at least 7 marks.
* Award 0 marks for a step that is omitted and not implied by subsequent work.
* Allow (***m*** : ***n***) to be omitted in last step.

Blunders (- 3)
B1 Incorrect step.
B2 Steps written in an illogical order (once).

Attempts (7 marks)
A1 Construction and stops (1st step of construction in method β).
A2 States or illustrates a special case, e.g. $^3/_4 = {}^6/_8$.
A3 A relevant theorem stated or proved, e.g. theorem dealing with equal intercepts.
A4 No construction merits, at most, attempt marks.

Worthless (0 marks)
W1 Any irrelevant theorem.

Part (c) **20 (5, 5, 5, 5) marks** **Att 8 (2, 2, 2, 2)**

(c) (i) Draw a square with sides 7 cm and mark *o*, the point of intersection of the diagonals.

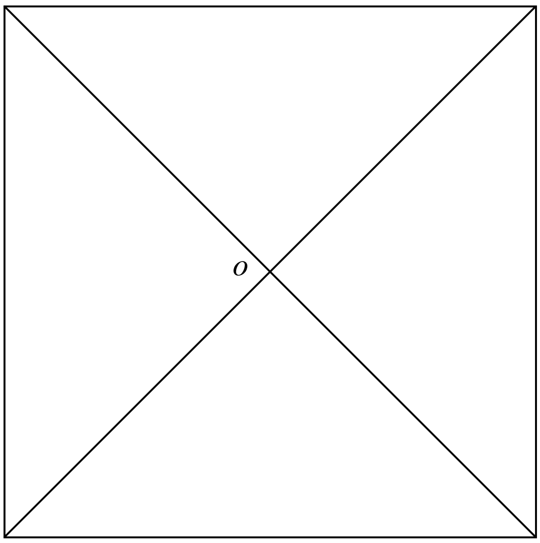
(ii) Draw the image of the square under the enlargement with centre *o* and scale factor ½.

(iii) Calculate the area of the image square.

(iv) Under another enlargement the area of the image of the square with sides 7 cm is 196 cm^2. What is the scale factor of this enlargement?

(c) (i) **5 marks** **Att 2**

(c) (i)



* Accept a quadrilateral with sides within ± 1 cm and angles within ± 5° of required measures.

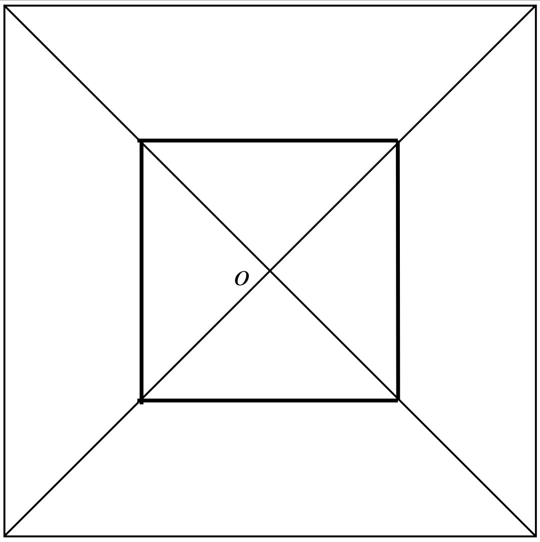
Blunders (- 3)
B1 Rectangle, not a square, with measure of sides outside margin of error, ± 1 cm.
B2 Measure of angles outside margin of error, ± 5°.

Slips (- 1)
S1 Square, but with measure of sides outside margin of error, ± 1 cm.
S2 Sides not straight (no straight edge) but measurements within margins of error. (S1 and S2 may apply).
S3 Does not label the point of intersection, *o*, of the diagonals.

Attempts (2 marks)
A1 Draws a square but does not draw the diagonals.
A2 Draws a quadrilateral, which does not merit more than attempt mark, even if freehand.
A3 Indicates some correct knowledge of a square.

(c) (ii) **5 marks** **Att 2**

(c) (ii)



* Apply Scheme of (c) (i) where appropriate.

Blunders (- 3)
B1 Scale factor ≠ ½ (to the eye), e.g. $k = 2$.

Slips (- 1)
S1 Some point other than *o* taken as centre, e.g. one of the vertices.

Attempts (2 marks)
A1 Midpoint of any line segment indicated (to the eye) or some manipulation with ½ or 2.

(c) (iii) **5 marks** **Att 2**

(c) (iii)	Area of original square = $7 \times 7 = 49$ Area of image = $49 \times (\frac{1}{2})^2 = {}^{49}/_4$ or 12·25

* Accept correct answer without work.
* Accept area of candidate's square, e.g. $k = 2$ ==> area = 196.

Blunders (- 3)
B1 $49 \times \frac{1}{2}$ or ${}^{49}/_{\frac{1}{2}}$ and continues.
B2 ${}^{49}/_{(\frac{1}{2})^2}$ or $\frac{(\frac{1}{2})^2}{49}$ and continues.
B3 $7 \times (\frac{1}{2})^2$ and continues.
B4 Mishandles fractions.

Slips (- 1)
S1 Each obvious numerical slip to a maximum of 3.

Attempts (2 marks)
A1 1st step or area of image = k^2(original) and stops.
A2 States area of a square of side a is a^2 or calculates area of inconsistent square.
A3 Area = L × B and stops.

(c) (iv) **5 marks** **Att 2**

(c) (iv)	Area of original square = $7 \times 7 = 49$ $49\,k^2 = 196$ $k^2 = {}^{196}/_{49} = 4$ $k = \sqrt{4}$ or 2 or - 2

* Accept area of original or small square consistent with previous parts.

Blunders (- 3)
B1 ${}^{49}/_{k^2}$ and continues.
B2 $49 \times k$ and continues.
B3 $7 \times k^2$ and continues or $7 \times k$ and continues.
B4 Transposing error.

Slips (- 1)
S1 Each obvious numerical slip to a maximum of 3.

Misreadings (- 1)
M1 Works from area of small square instead of original one, i.e.$12{\cdot}25k^2 = 196 \Rightarrow k = 4$ or $\sqrt{16}$.

Attempts (2 marks)
A1 1st step or $k^2 = {}^{\text{image area}}/_{\text{original area}}$ or $k = {}^{\text{image length}}/_{\text{original length}}$ and stops.
A2 Correct answer without work or from part (iii) if $k = 2$ in that part.

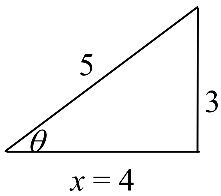
Question 5

Part (a)	**10 marks**	**Att 4**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 6**

Part (a) **10 (5, 5) marks** **Att 4 (2, 2)**

> **(a)** $\sin\theta = {}^3/_5$ where $0^\circ < \theta < 90^\circ$.
> Find, without using the Tables or a calculator, the value of
> **(i)** $\cos\theta$
> **(ii)** $\cos 2\theta$. [Note: $\cos 2\theta = \cos^2\theta - \sin^2\theta$.]

(a) (i) **5 marks** **Att 2**

(a) (i) α	β
$\sin\theta = {}^3/_5$ $5^2 = 3^2 + x^2$ $x^2 = 25 - 9 = 16$ $x = 4$ $\cos\theta = {}^4/_5$ 	$\cos\theta = \sqrt{(1 - \sin^2\theta)}$ $= \sqrt{(1 - {}^9/_{25})}$ $= \sqrt{{}^{16}/_{25}}$ or ${}^4/_5$

* Accept correct answer without work.

Blunders (-3)
B1 Mathematical error dealing with Pythagoras' theorem or transposing error.
B2 Incorrect placing of side lengths in triangle or error in defining sin and/or cos.
B3 Incorrect statement of formula in β and continues.
B4 Mathematical error dealing with fractions in β.

Slips (-1)
S1 Each numerical slip to a maximum of 3.

Attempt (2 marks)
A1 Draws right-angled triangle with or without sides 3, 4, 5 correctly placed.
A2 Indicates some use of Pythagoras' theorem.
A3 Uses Tables or calculator to calculate θ correctly, i.e. $36^\circ 52'$ without work.
A4 Defines sin, cos or tan correctly or some correct mnemonic and stops.

(a) (ii) **5 marks** **Att 2**

(a) (ii) α	β
$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $= ({}^4/_5)^2 - ({}^3/_5)^2$ $= {}^{(16-9)}/_{25}$ $= {}^7/_{25}$	$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $= (\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$ $= ({}^4/_5 - {}^3/_5)({}^4/_5 + {}^3/_5)$ $= {}^1/_5 \times {}^7/_5$ $= {}^7/_{25}$

*Accept candidate's incorrect value from (i) if already penalised,(even if calculator was used)

Blunders (-3)
B1 Incorrect substitution in formula, if not obvious slip.
B2 Incorrect manipulation of fractions.
B3 Difference of squares stated incorrectly and continues.

Slips (-1)
S1 Each numerical slip to a maximum of 3.

Attempts (2 marks)
A1 Some relevant work, e.g. relevant substitution, correct manipulation of fractions, factors.
A2 $\text{Cos}2\theta = 2\cos\theta = {}^{8}/_{5}$ or consistent with (i).
Must multiply fractions or decimals correctly.
A3 Correct answer without work.

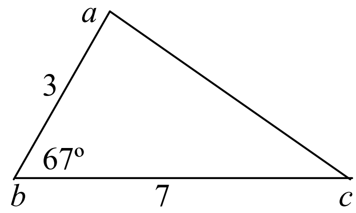
Worthless (0 marks)
W1 Formula (given or from Tables) written down and stops.

Part (b) **20 (10, 10) marks** **Att 6 (3, 3)**

(b) In the triangle *abc*, $|ab| = 3$ units, $|bc| = 7$ units, and $|\angle abc| = 67^\circ$.

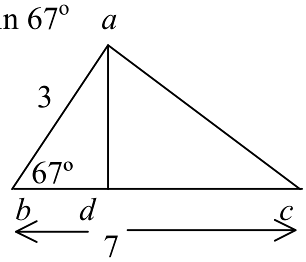
(i) Calculate the area of the triangle *abc* correct to one decimal place.

(ii) Calculate $|ac|$, correct to the nearest whole number.



(b) (i) **10 marks** **Att 3**

α	β
Area $\Delta abc = \frac{1}{2}\,|a|.|c|.\text{Sin}|\angle abc|$	Area $\Delta abc = \frac{1}{2} \times |bc| \times |ad|$ or $|ad| = 3\sin 67^\circ$
Area $= \frac{1}{2} \times 3 \times 7 \times \text{Sin } 67^\circ$	$= \frac{1}{2} \times 7 \times 3 \sin 67^\circ$
or $\frac{1}{2} \times 3 \times 7 \times 0.9205$	or $\frac{1}{2} \times 7 \times 2.76$
$= 9.66525$	$= 9.66$
≈ 9.7 unit²	≈ 9.7 unit²



* May use Area $=\sqrt{s(s-a)(s-b)(s-c)}$

* Case: When only 1st line and answer written

Formula or 1st line correct + 9.7	10 marks
Formula or 1st line correct + 9.6…	9 marks
1st line correct, not 9.6…	3 marks
Formula from Tables, not 9.6…	0 marks

Blunders (-3)
B1 Mathematical error dealing with fractions or decimals (once)
B2 Confuses radians with degrees.
B3 Incorrect function read.
B4 Incorrect substitution, if not obvious misreading or slip.

Slips (-1)
S1 Each numerical slip to a maximum of 3.
S2 Obvious slip in reading Tables or calculator.
S3 Failure to round off, incorrect rounding off, early rounding off affecting answer.

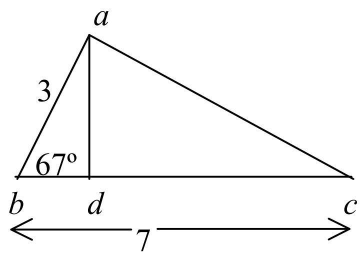
Attempts (3 marks)
A1 Some correct substitution in reasonable formula, e.g. $\frac{1}{2} \times 7 \times 3$.
A2 Some correct use of sine or cosine rule.

A3 Perpendicular drawn from a to d.
A4 Definition of any trigonometrical function and stops.
A5 Answer (9.7 or 9.6…) without work.
A6 Assumes $|\angle bac| = 90^o$ and finishes.

Worthless (0 marks)
W1 Formula taken from Tables and stops.

(b) (ii) **10 marks** **Att 3**

α	β
$\|b\|^2 = \|a\|^2 + \|c\|^2 - 2.\|a\|.\|c\|.\text{Cos}\|\angle abc\|$	$\|ad\| = 2.76$
$\|ac\|^2 = 49 + 9 - 2.3.7.\text{Cos } 67^o$	$\text{Cos } 67^o = {}^{\|bd\|}/_3$
or $49 + 9 - 2 \times 3 \times 7 \times 0.3907$	$\|bd\| = 3.\text{Cos } 67^o$
$= 58 - 42 \times 0.3907$	$= 3 \times 0.3907 = 1.1721$
$= 58 - 16.4094$	$\|dc\| = 7 - 1.1721 = 5.8279$
$\|ac\|^2 = 41.5906$	$\|ac\|^2 = 2.76^2 + 5.83^2$
$\|ac\| = \sqrt{41.5906}$ or 6.44	$\|ac\| = \sqrt{41.6065}$ or 6·45
≈ 6 units	≈ 6 units



Blunders (-3)
B1 Incorrect statement of reasonable formula and continues.
B2 Correct formula, incorrect substitution and continues.
B3 Incorrect reading of function, if not an obvious slip.
B4 Use of incorrect function to calculate $|ad|$ or $|bd|$ in β.
B5 Incorrect application of Pythagoras' theorem and continues.
B6 Method α: 58 - 42 cos 67° = 16 cos 67° and continues.

Slips (-1)
S1 Obvious slip in reading Tables or calculator.
S2 Each error in calculation to a maximum of 3.
S3 Failure to round off, incorrect rounding off, rounds off too soon affecting answer.

Attempt (3)
A1 Correct use of some trigonometric formula.
A2 Assumes $|\angle bac| = 90^o$ and applies Pythagoras' theorem correctly to Δabc.
A3 6 or 6·4… without work.

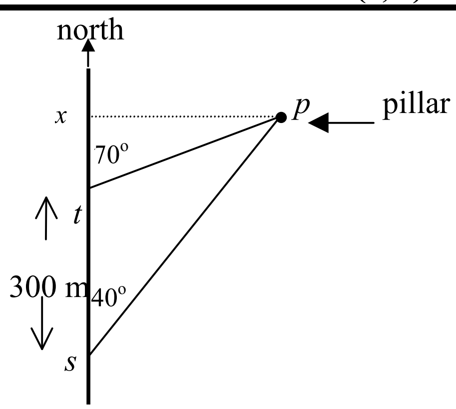
Part (c) **20 (10, 10) marks** **Att 6 (3, 3)**

(c) s and t are two points 300 m apart on a straight path due north.

From s the bearing of a pillar is N 40° E.
From t the bearing of the pillar is N 70° E.

(i) Show that the distance from t to the pillar is 386 m, correct to the nearest metre.

(ii) Find the shortest distance from the path to the pillar, correct to the nearest metre



(c)(i) **10 marks** **Att 3**

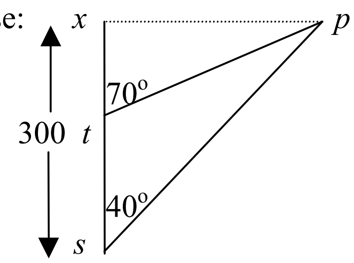
(c) (i) $|\angle tps| = 70^o - 40^o = 30^o$ or $180^o - (110 + 40^o) = 30^o$

$^{|tp|}/_{\text{Sin}40^o} = {}^{300}/_{\text{Sin}30^o}$

or $|tp| = {}^{300\text{Sin}40^o}/_{\text{Sin}30^o} = {}^{300 \times 0.6428}/_{0.5}$

$= 385.68$ or 386 m

* Case:



Assumes $|xs| = 300$ m and finds $|px|$ first.

(ii) $^{|px|}/_{300} = \tan 40^o$ or $|px| = 300 \tan 40^o$ 4 marks

$|px| = 251{\cdot}73$ 6 marks

$|px| \approx 252$ m Max. 7 marks

(i) $\sin 70^o = {}^{|px|}/_{|pt|}$ Att 3 marks, even if finished.

* Accept verification, e.g. $^{386}/_{\sin 40} = {}^{300}/_{\sin 30} \Longrightarrow {}^{386}/_{0{\cdot}6428} = {}^{300}/_{0{\cdot}5} \Longrightarrow 600{\cdot}49 \approx 600$.

Blunders (-3)

B1 Incorrect statement of Sine Rule and continues.
B2 Incorrect calculation of $|\angle tps|$ and continues, if not an obvious slip.
B3 Incorrect manipulation of fractions.
B4 Incorrect use of Tables or calculator.

Slips (–1)

S1 Each numerical slip to a maximum of 3.
S2 Obvious slip in reading Tables or calculator.
S3 Incorrectly rounds off or rounds off too soon, if it affects the answer.

Attempts (3)

A1 Some correct application of sin/cos/tan to right angled triangle.
A2 Correct statement and/or some application of Cosine Rule to diagram.
A3 385·… without work.

(c) (ii) **10 marks** **Att 3**

(c) (ii) α β Shortest distance is $|px|$ γ

$^{|px|}/_{386} = \sin 70^o$ or $^{|px|}/_{\sin 70^o} = {}^{|tp|}/_{\sin 90^o}$

or $^{|px|}/_{386} = 0.9397$ or $^{|px|}/_{0.9397} = {}^{386}/_{1}$ or $|px| = 386 \sin 70^o$

$|px| = 362.72\ldots$

≈ 363 m

γ: $^{|px|}/_{385.68} = \sin 70^o$

`` `` `` `` `` ``

$|px| = 362.42$

≈ 362 m

* Accept $^{|px|}/_{70} = {}^{|tp|}/_{90} \rightarrow |px| \approx 363$ or 362.

Blunders (-3)

B1 Incorrect use of ratio and continues.
B2 Incorrect manipulation of fractions.
B3 Incorrect reading of functions.

Slips (-1)

S1 Each numerical slip to a maximum of 3.
S2 Obvious slip in reading Tables or calculator.
S3 Failure to round off, incorrectly rounds off, rounds off too soon affecting answer.

Attempts (3 marks)

A1 Recognises shortest distance.

A2 Some correct use of sin/cos/tan.

A3 States and makes some correct use of Pythagoras' theorem.

A4 $|px|/_{70} = |tp|/_{90}$ and stops or continues without using sine.

A5 362·… or 363 without work.

Question 6

Part (a)	**10 marks**	**Att 4**
Part (b)	**20 marks**	**Att 8**
Part (c)	**20 marks**	**Att 6**

Part (a)	**10 (5, 5)marks**	**Att 4 (2, 2)**

(a) Sarah and Jim celebrate their birthdays in a particular week (Monday to Sunday inclusive) Assuming that the birthdays are equally likely to fall on any day of the week, what is the probability that

(i) Sarah's birthday is on a Friday

(ii) Sarah's birthday and Jim's birthday are both on a Friday?

(a) (i)	**5marks**	**Att2**

(a) (i)	α	β	γ
	$^1/_7$	0.143	$1 - {^6/_7}$

(a) (ii)	**5marks**	**Att2**

(a)(ii) α	β	γ	δ	ε
$^1/_7 \times {^1/_7}$	$^1/_{49}$	0.020	Sample space	$[\text{Ans (i)}]^2$

* If a list of numbers is written and it is not obvious to which part of the question the numbers apply, assume that they belong to the earliest part that has not yet been examined.

* Accept correct or consistent answers without work.

* Accept 2 decimal places.

mon	tue	wed	thur	fri	sat	sun
S, J	S,--	S,--	S,--	S,--	S,--	S,--
S,--	**S, J**	S,--	S,--	S,--	S,--	S,--
S,--	S,--	**S, J**	S,--	S,--	S,--	S,--
S,--	S,--	S,--	**S, J**	S,--	S,--	S,--
S,--	S,--	S,--	S,--	S,J	S,--	S,--
S,--	S,--	S,--	S,--	S,--	**S, J**	S,--
S,--	S,--	S,--	S,--	S,--	S,--	**S, J**

Blunders (-3)

B1 $\binom{7}{1}$ and stops (i).

B2 $\binom{7}{1} \times \binom{7}{1}$ and stops (ii), if not already penalised.

B3 Addition instead of multiplication (ii), e.g. $^2/_7$.

B4 Inverted fraction $^7/_1$, (i) or $^7/_1 \times {^7/_1}$ or $^{49}/_1$, (ii), if not already penalised in (i).

Slips (-1)

S1 Each obvious slip to a maximum of 3 in each part.

Attempts (2 marks each part)

A1 Relevant integer written down and stops.
- (i) 1, 6, 7.
- (ii) 1, 7, 49, subject to ε method.

A2 Any definition of probability, or statement of probability theorem.

A3 Incomplete sample space only and stops, i.e. 1 × Att2
Complete sample space and stops. Att 2 + Att 2.

A4 One correct step, e.g. any incorrect ratio without work, e.g. $^5/_1$. [5 or 5·3 not accepted.]

Part (b)	**20 (5, 5, 5, 5) marks**	**Att 8 (2, 2, 2, 2)**

(b) (i) How many different arrangements can be made using all the letters of the word IRELAND?
(ii) How many arrangements begin with the letter I?
(iii) How many arrangements end with the word LAND?
(iv) How many begin with I and end with LAND?

(b) (i)	**5 marks**	**Att 2**

(b) (i)	α	β	γ	δ	ε
	7!	7.6.5.4.3.2.1	$^7P_7$	5040	List

(b)(ii)	**5 marks**	**Att 2**

(b) (ii)	α	β	γ	δ	ε	ζ	θ
	1 × 6!	6!	6.5.4.3.2.1	720	$^6P_6$	List	$^1/_7 \times$ (i)

(b) (iii)	**5 marks**	**Att 2**

(b) (iii)	α	β	γ	δ	ε	ζ
	3! × 1	3!	3 × 2 × 1	6	$^3P_3$	List

(b) (iv)	**5 marks**	**Att 2**

(b) (iv)	α	β	γ	δ	ε	ζ
	1 × 2! × 1	2!	2	$^2P_2$	List	$^1/_3 \times$ (iii)

* Accept correct or consistent answer without work.
* Multiplication must be clearly indicated, i.e. 3, 2, 1 or [3] [2] [1] and stops merits Att 2 marks
* If parts of (b) are not identified, and if it is not obvious which part is being attempted treat each part in order.
* Penalise repeated errors in each section, but allow the candidate to use results of one section in a later one.

Blunders (-3)
B1 Each ‘box’ omitted, (but allow omission of 1 e.g. 7.6.5.4.3.2).
B2 Each incorrect ‘box’.
B3 Addition instead of multiplication.

Misreadings (-1)
M1 3! × 4! or 144 or equivalent (iii), and/or 2! × 4! or 48 or equivalent (iv),
i.e. arranged the letters in LAND. [Misreading in each part.]

Slips (-1)
S1 List method: (-1) each entry omitted, subject to attempt.

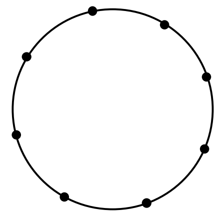
Attempts (2 marks)
A1 One correct step and stops.
A2 At least one correct permutation listed, e.g. I, R, E, L, A, N, D (not IRELAND).
A3 Answers (ii) + (iii) or similar in (iv).
A4 Any relevant <u>integer</u> written down and stops
in (i), (ii) 1 to 7 [$^1/_5$ not acceptable.]
in (iii) 1, 2, 3, 6
in (iv) 1, 2, 3.
A5 Fraction $^1/_7$ [in part(ii)] or $^1/_3$ [in part(iv)].
A6 Writes any permutation, combination or factorial and stops, e.g. $^8P_3$ or 8!, say.

A7 Writes any permutation, combination or factorial symbol and stops.

Worthless (0 marks)
W1 Answers, other than above, which are incorrect and inconsistent, without work.

Part (c) **20 (10, 10) marks** **Att 6 (3, 3)**

(c) **(i)** Eight points lie on a circle, as in the diagram.
How many different lines can be drawn by joining any two of the eight points?



(ii) Find the value of the natural number n such that

$$\binom{n}{2} = 105. \quad \text{[Note: } \binom{n}{2} \text{ may also be written as } {}^{n}C_{2}.\text{]}$$

(c) (i) **10 marks** **Att 3**

(c)(i) α	β	γ	δ	ε	ζ	η	θ
$\binom{8}{2}$	${}^{8}C_{2}$	$\frac{8.7}{1.2}$ or ${}^{56}/_{2}$	$\frac{{}^{8}P_{2}}{2!}$	$\frac{8!}{6!2!}$	28	7+6+5+4+3+2+1	Diagram
$\binom{8}{6}$	${}^{8}C_{6}$	$\frac{8.7.6.5.4.3}{1.2.3.4.5.6}$	$\frac{{}^{8}P_{6}}{6!}$	or equivalent			

* Accept correct answer and no work.

* No penalty for $\binom{8}{2}$ but $\frac{8}{2}$ is B2. No penalty for $\binom{8}{6}$ but $\frac{8}{6}$ is B2 + B3.

* If only top part **or** bottom part is correct, e.g. 8×7 or 8! or ${}^{12\text{ say}}/_{2}$, award 7 marks.

Blunders (-3)
B1 ${}^{8}P_{2}$ or 8×7 or 56 and stops.
B2 'Top' section incorrect (α to ε), ${}^{9}C_{2}$ or ${}^{7}/_{2}$ or ${}^{(7 \times 6)}/_{(2 \times 1)}$.
B3 'Bottom' section incorrect (α to ε), ${}^{8}C_{3}$ or ${}^{(8 \times 7 \times 6)}/_{(3 \times 2 \times 1)}$.
B4 Inverted, e.g. ${}^{2}C_{8}$ (α and β).
B5 Addition instead of multiplication, [or vice-versa (η)].
B2 and B3 may apply, e.g. ${}^{7}C_{4}$. [Note: ${}^{7}C_{2}$ is only one blunder, i.e. B2.]

Slips (-1)
S1 (-1) for each line omitted, subject to attempt (θ method).

Attempts (3marks)
A1 One correct step and stops.
A2 Any integer 1 to 8 and stops.
A3 Writes any permutation, combination or factorial symbol and stops.

Worthless (0 marks)
W1 Incorrect answer and no work (except as above).

(c) (ii) **10 marks** **Att 3**

(c) (ii) α		β
$\frac{n(n-1)}{1.2}$	$= 105 \Rightarrow n^2 - n - 210 = 0$ or $(n-15)(n+14) = 0$	$\Rightarrow n = 15$ 15

* Accept correct answer without work.
* Allow 15 and -14 for 10 marks.
* Accept $^{15}C_2$.

Blunders (-3)
B1 Error in numerator.
B2 Error in denominator.
B3 Transposition error.
B4 Sign error.
B5 Incorrect factors (or error in using quadratic formula).

Attempts (3 marks)
A1 $^{n(n-1)}/_2$ and stops.
A2 One correct step and stops, e.g. $^{\text{any number}}/_2$ or 210.
A3 $^{n}/_2$ =105 even if continues, i.e. an error which leads to a linear equation.
A4 One incorrect test, e.g. $^{105 \times 104}/_{2 \times 1}$.

Worthless (0 marks)
W1 Incorrect answer without work.

Question 7

Part (a)	**20 marks**	**Att 6**
Part (b)	**30 marks**	**Att 10**

Part (a) **20 (10, 10) marks** **Att 6 (3, 3)**

(a) **(i)** Calculate the mean of the following numbers

2, 3, 5, 7, 8

(ii) Hence, calculate the standard deviation of the numbers correct to one decimal place.

(a) (i) **10 marks** **Att 3**

(a) (i) α: $\dfrac{2+3+5+7+8}{5} = \dfrac{25}{5}$ or 5

β: $0{\cdot}4 + 0{\cdot}6 + 1 + 1{\cdot}4 + 1{\cdot}6 = 5$

γ: Assumed mean = 4 (say)

$\Rightarrow \dfrac{-2-1+1+3+4}{5} + 4 = 1 + 4$ or 5

δ: Sum of numbers is 25 ==> Mean = $^{25}/_5$ or 5

* Accept 5 without work.

Blunders (- 3)

B1 Multiplies instead of adds (336).

B2 Incorrect divisor.

B3 Omits a variable, e.g. $^{(2+3+5+7)}/_4$ or $^{(2+3+5+7)}/_5$.

Slips (- 1)

S1 Each arithmetical slip to a maximum of 3.

Misreadings (- 1)

M1 Misreading which does not change or simplify task, e.g. $^{(2+3+5+7+6)}/_5$ (say).

Attempts (3 marks)

A1 Partial addition and stops.

A2 Idea of mean indicated, e.g. $^{\Sigma x}/_n$.

A3 States that **median** is 5 and stops.

(a) (ii) **10 marks** **Att 3**

(a) (ii) α: Mean = 5

x	d	d^2
2	3	9
3	2	4
5	0	0
7	2	4
8	3	9

$\sigma = \sqrt{\dfrac{26}{5}} = \sqrt{5{\cdot}2} = 2{\cdot}280 \approx 2{\cdot}3$

β:

x	f	fx	d	d^2	fd^2
2	k	$2k$	3	9	$9k$
3	k	$3k$	2	4	$4k$
5	k	$5k$	0	0	0
7	k	$7k$	2	4	$4k$
8	k	$8k$	3	9	$9k$
	5k	25k			

Mean = $^{25k}/_{5k} = 5$

$\sigma = \sqrt{\dfrac{26k}{5k}} = \sqrt{5{\cdot}2} = 2{\cdot}280 \approx 2{\cdot}3$

Accept any consistent k.

*Without work:2·3 or answer consistent with (i) merits 10 marks. [Accept mean from (i).]

* $\sqrt{\frac{26}{5}}$ or $\sqrt{5 \cdot 2}$ or 2·28 or value to two decimal places consistent with (i) merits 9 marks

d^2 column correct or consistent with (i) merits 7 marks
d column correct or consistent with (i) merits 3 marks
other answers merit 0 marks (subject to attempt).

Blunders (- 3)

B1 No d^2 column ==> $\sqrt{10/5} = \sqrt{2} = 1 \cdot 4$.
B2 No √ indicated. [Note: Mean deviation: $^{10}/_5 = 2$. Apply B1 and B2.]
B3 Uses inconsistent mean. [If mean is calculated better in (ii) than (i), award marks for (ii).]
B4 Denominator ≠ 5, e.g. $\sqrt{26} = 5 \cdot 09 \approx 5 \cdot 1$ (unless an obvious slip).
B5 Mathematical error, e.g. $3^2 = 6$ or $(-3)^2 = -9$ or - 6 (once).
B6 Inconsistent k, if used or confuses x with f.

Slips (- 1)

S1 Each arithmetical slip to a maximum of 3.
S2 Failure to round off.

Attempts (3 marks)

A1 Correct relevant formula and stops, e.g. $\sqrt{\frac{\sum fd^2}{\sum f}}$ *or* $\frac{\sum fx}{\sum f}$ (latter written for this part).

A2 Writes d or finds one deviation and stops.

Part (b) 30 (10, 5, 10, 5) Att 10 (3, 2, 3, 2)

(b) The following table shows the distribution of the amounts spent by 40 customers in a shop:

Amount Spent	IR£0 - IR£8	IR£8 - IR£12	IR£12 - IR£16	IR£16 - IR£20	IR£20 - IR£32
Number of Customers	2	9	13	10	6

[Note: IR£8 - IR£12 means IR£8 or over but less than IR£12 etc.]

(i) Taking the mid-interval values, estimate the mean amount spent by the customers.

(ii) Copy and complete the following cumulative frequency table:

Amount Spent	<IR£8	<IR£12	< IR£16	< IR£20	< IR£32
Number of Customers					

(iii) Draw a cumulative frequency curve (ogive).

(iv) Use your curve to estimate the number of customers who spent IR£25 or more.

(b) (i) 10 marks Att 3

(b) (i) Mid-intervals 4, 10, 14, 18, 26

$$\text{Mean} = \frac{(4 \times 2) + (10 \times 9) + (14 \times 13) + (18 \times 10) + (26 \times 6)}{2 + 9 + 13 + 10 + 6}$$

$$\text{or} \quad \frac{8 + 90 + 182 + 180 + 156}{40} = \frac{616}{40} \quad \text{or } 15 \cdot 4$$

* Accept $^{616}/_{40}$ or 15·4 without work. $^{40}/_{616}$ without work merits 7 marks.

* With the exception of ${}^{40}/_{616}$, incorrect answers without work merit 0 marks (subject to Att).

Blunders (- 3)
B1 Mid-interval values not used [lower → 12·7, upper → 18·1 with work].
B2 Multiplies instead of adds in denominator [${}^{616}/_{14040}$ = 0·04387… with work].
B3 Adds instead of multiplies in numerator [${}^{112}/_{40}$ = 2·8 with work].
B4 Uses 5 as denominator [${}^{616}/_{5}$ = 123·2 with work] or no denominator [616 with work].
B5 Inverts, i.e. ${}^{40}/_{616}$ = 0·0649… with work.
B6 No frequencies, i.e. $\frac{4+10+14+18+26}{40} \Rightarrow \frac{72}{40} \Rightarrow 1\cdot 8$ with work.
B7 Omits a class.
B2 + B3 ==> ${}^{112}/_{14040}$ ==> 0·00797 with work.
B4 + B6 ==> $\frac{4+10+14+18+26}{5} \Rightarrow \frac{72}{5} \Rightarrow 14\cdot 4$ with work.
B1 + B6 ==> ${}^{(8+4+4+4+12)}/_{40}$ ==> ${}^{32}/_{40}$ ==> 0·8 with work.

Slips (- 1)
S1 Each arithmetical slip to a maximum of 3.
S2 Each incorrect mid-interval value to a maximum of 3.

Attempts (3 marks)
A1 Mean = ${}^{\Sigma fx}/_{\Sigma f}$ and stops.
A2 A correct mid-interval value and stops.
A3 A correct relevant multiplication and stops.
A4 Σf = 40 and stops. [40 on its own merits 0 marks.]

(b) (ii) **5 marks** **Att 2**

(b) (ii) Amount Spent	<IR£8	<IR£12	< IR£16	< IR£20	< IR£32
Number of Customers	2	11	24	34	40

*If a blunder has not been made, award 1 mark for each correct or (correctly) consistent entry.

Blunders (- 3)
B1 Cumulative cumulative table, i.e.

<IR£8	<IR£12	<IR£16	<IR£20	<IR£32
2	13	37	71	111

B2 Subtracts instead of adds, i.e.

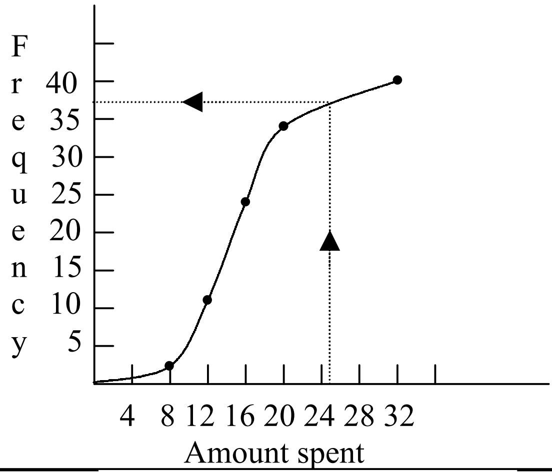
<IR£8	<IR£12	<IR£16	<IR£20	<IR£32
2	7	6	4	2

Slips (- 1)
S1 Each arithmetical slip to a maximum of 3.
S2 Each frequency omitted to a maximum of 3.

Attempts (2 marks)
A1 One correct frequency and stops.
A2 Copies either given table (for this part) and stops.

(b) (iii) **10 marks** **Att 3**

(b)(iii)



* A correct ogive presupposes a correct table. [If table is not made out, award 5 + 10.]
* Accept frequency on the horizontal.
* Accept Cumulative Frequency Polygon.
* No penalty for not joining (0, 0) to (8, 2).
* Accept curve consistent with table offered for part (ii), even if it is original frequency table.
* If part (ii) is not attempted and curve consistent with original frequency table is drawn, award 0 + Att 3 marks.
* Allow interval 0 to 8 on the horizontal axis to have a different scale to the other intervals.

Blunders (- 3)
B1 Scale irregular (once).
B2 Plots points but does not join them.
B3 Draws cumulative frequency histogram correctly instead of ogive. (Apply S. and B.)
B4 Draws cumulative cumulative curve, if not already penalised. (Apply S. and B.)

Slips (- 1)
S1 Each point omitted or incorrectly plotted (to the eye). [Blunders also apply.]

Attempts (3 marks)
A1 Draws a frequency curve or polygon not consistent with table offered for part (ii).
A2 Draws the axes and stops.

(b) (iv) **5 marks** **Att2**

(b) (iv)	3 or 4 customers or 40 - 37 or similar

* Accept "correct" answer ± 2 consistent with candidate's ogive even without lines, even 3½.
* Histogram or frequency polygon in (iii) ==> ≤ Att2 in (iv).

Blunders (- 3)
B1 Draws one or both relevant perpendiculars and stops.
B2 Does not subtract from 40, i.e. 36 or 37.
B3 One perpendicular incorrectly located (in the wrong interval).
B4 Relevant work but answer outside tolerance, subject to S1.

Slips (- 1)
S1 Indicates space between 40 and candidate's horizontal and stops.

Attempts (2 marks)
A1 Draws a vertical or horizontal line.
A2 40 - *k* where *k* is an arbitrary number, e.g. 40 - 25.
A3 Calculates answer, e.g. $^7/_{12} \times 6$ = number greater than 25 ==> 3½.

Worthless (0 marks)
W1 Incorrect and inconsistent answer without work, e.g. 15.

Question 8

Part (a)	**10 marks**	**Att 4**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 7**

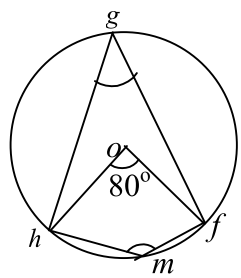
Part (a) **10 (5, 5) marks** **Att 4 (2, 2)**

(a) The points *f, g, h* and *m* lie on a circle with centre *o*.

Given that $|\angle foh| = 80^o$, find

(i) $|\angle fgh|$

(ii) $|\angle fmh|$.



(a) (i) **5 marks** **Att 2**

(a) (i)	α: $|\angle foh| = 2\,|\angle fgh|$ $|\angle fgh| = 40^o$	β: Reflex $|\angle foh| = 280^o \Rightarrow |\angle fmh| = 140^o$ $|\angle fgh| = 40^o$

* Accept correct answer without work.

Blunders (- 3)
B1 $|\angle fgh| = 2|\angle foh|$.
B2 Reflex $|\angle foh| = 280^o$ and stops.

Attempts (2 marks)
A1 States relevant theorem.
A2 Any correct relevant step and stops.

(a) (ii) **5 marks** **Att 2**

(a) (ii)	α: $|\angle fmh| = 180^o - |\angle fgh|$ $= 180^o - 40^o$ or 140^o	β: Reflex $|\angle foh| = 280^o = 2|\angle fmh|$ $\Rightarrow |\angle fmh| = 140^o$

* Accept correct or consistent answer without work.
* Accept answer consistent with part (i).

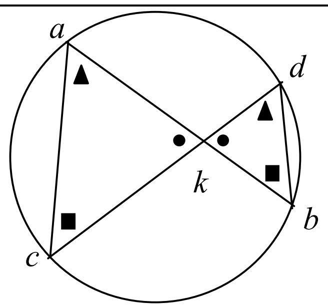
Blunders (- 3)
B1 $|\angle fmh| + |\angle fgh| \neq 180^o$.
B2 Transposing error.

Attempts (2 marks)
A1 States relevant theorem.
A2 Any correct relevant step and stops.

Part (b) **20 marks** **Att 7**

(a) Prove that if [*ab*] and [*cd*] are chords of a circle and the lines *ab* and *cd* meet at the point *k* which is inside the circle, then $|ak|.|kb| = |ck|.|kd|$.

(b) **20 marks** **Att 7**



Construction:	Join *a* to *c* and *d* to *b*	5 m
Proof:	$|\angle akc| = |\angle bkd|$	5, 2
	$|\angle ack| = |\angle dbk|$	
	$|\angle cak| = |\angle kbd|$ or triangles *akc* and *dkb* are similar	5, 2
	$\frac{|ak|}{|kd|} = \frac{|ck|}{|kb|}$	5, 2
	$(|ak|.|kb| = |ck|.|kd|)$	

* Accept steps stated or clearly indicated. [Diagram as shown merits 15 marks.]
* If a candidate obtains more than 0 marks, he/she must be awarded at least Att 7 marks.

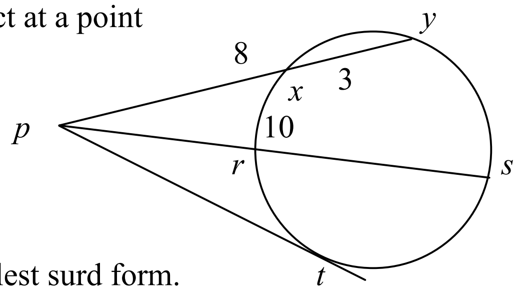
Blunders (- 3)
B1 Incorrect step or part of a step. [Award 0 marks for a step which is omitted.]
B2 Proves the external case.
B3 Steps in incorrect order.
B4 $|\angle kac| = |\angle kbd|$ [further blunders may apply.]

Attempts (7 marks)
A1 Outline diagram and stops (Circle with two intersecting chords).

Part (c) **20 (5, 5, 10) marks** **Att 7 (2, 2, 3)**

(c) [*xy*] and [*rs*] are chords of a circle which intersect at a point *p* outside the circle.
pt is a tangent to the circle at the point *t*.

Given that $|py| = 8$, $|xy| = 3$ and $|ps| = 10$,
(i) write down $|px|$
(ii) calculate $|rs|$
(iii) calculate $|pt|$, giving your answer in its simplest surd form.



(c) (i) **5 marks** **Att 2**

(c) (i)	$|px| = 8 - 3$ or 5

* Accept 5 without work.

Blunders (- 3)
B1 $|px| = 8$.
B2 Mathematical error in calculations.

Slips (- 1)
S1 Obvious slip in calculations.

Attempts (2 marks)
A1 Some work with 3, 8 or 11.

(c) (ii) **5 marks** **Att 2**

(c) (ii)	$\lvert ps \rvert . \lvert pr \rvert = \lvert py \rvert . \lvert px \rvert$ $10 \times \lvert pr \rvert = 8 \times 5$ $\lvert pr \rvert = 4$ $\lvert rs \rvert = 6$

* Accept $\lvert px \rvert$ from part (i).

Blunders (- 3)
B1 Incorrect substitution, e.g. $\lvert rs \rvert = 10$.
B2 Incorrect product, e.g. $\lvert px \rvert . \lvert xy \rvert$.
B3 Transposing error.
B4 Incorrect ratios instead of products.

Slips (- 1)
S1 No subtraction, i.e. leaves answer as $\lvert pr \rvert = 4$.
S2 Obvious slip in calculations.

Attempts (2 marks)
A1 Correct answer without work.
A2 Correct statement of theorem.
A3 Some product of lengths of line segments.

(c) (iii) **10 marks** **Att 3**

(c) (iii)	α:	$\lvert ps \rvert . \lvert pr \rvert = \lvert pt \rvert^2$ $10 \times 4 = \lvert pt \rvert^2$ $40 = \lvert pt \rvert^2$ $\sqrt{40} = \lvert pt \rvert$ $2\sqrt{10} = \lvert pt \rvert$	or β:	$\lvert py \rvert . \lvert px \rvert = \lvert pt \rvert^2$ $8 \times 5 = \lvert pt \rvert^2$ $40 = \lvert pt \rvert^2$ $\sqrt{40} = \lvert pt \rvert$ $2\sqrt{10} = \lvert pt \rvert$

* Accept results from previous parts.

Blunders (- 3)
B1 Incorrect substitution.
B2 Incorrect product, e.g. $\lvert px \rvert . \lvert xy \rvert$.
B3 Transposing error.

Slips (- 1)
S1 $\sqrt{40}$ and stops or 6.3 etc.

Attempts (3 marks)
A1 Correct answer without work.
A2 Some correct relevant step.

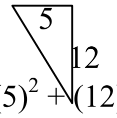
Question 9

Part (a)	**10 marks**	**Att 4**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 6**

Part (a) **10 (5, 5) marks** **Att 4 (2, 2)**

> **(a)** Given that $\vec{p} = 5\vec{i} - 12\vec{j}$,
>
> **(i)** calculate $|\vec{p}|$
>
> **(ii)** write down $\vec{p}^{\perp}$ in terms of $\vec{i}$ and $\vec{j}$.

(a) (i) **5 marks** **Att 2**

(a) (i)	α	β	γ
	$\sqrt{x^2 + y^2}$	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	
	or $\sqrt{(5)^2 + (-12)^2}$	or $\sqrt{(5 - 0)^2 + (-12 - 0)^2}$	or $\|op\|^2 = (5)^2 + (12)^2$
	$\|\vec{p}\| = \sqrt{25 + 144}$	$\|\vec{p}\| = \sqrt{25 + 144}$	$\|op\| = \sqrt{25 + 144}$
	or $= \sqrt{169}$ or 13	or $= \sqrt{169}$ or 13	or $= \sqrt{169}$ or 13

* Accept $\sqrt{(-12)^2 + (5)^2}$ or $\sqrt{(12)^2 + (5)^2} = \sqrt{169}$ or 13.
* Accept $\sqrt{169}$ or 13 or $\sqrt{25 + 144}$ without work.

Blunders (- 3)
B1 Incorrect relevant formula, e.g. $\sqrt{x^2 - y^2}$ or $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$, central sign incorrect.
B2 Incorrect signs $(x_2 + x_1)$ **and** $(y_2 + y_1)$ in method β.
B3 Incorrect use of Pythagoras' Theorem.
B4 Mathematical error, e.g. $5^2 = 10$ or $\sqrt{(5)^2 + (-12)^2} = \sqrt{25 - 144} = \sqrt{-119}$.
B5 $\sqrt{25\vec{i}^{\,2} + 144\vec{j}^{\,2}}$ and stops, i.e. $\vec{i}^{\,2} \neq 1$ and $\vec{j}^{\,2} \neq 1$.

Slips (- 1)
S1 Each numerical slip to a maximum of 3.
S2 Incorrect signs $(x_2 + x_1)$ **or** $(y_2 + y_1)$ but not both in method β.

Attempts (2 marks)
A1 Correct formula or states Pythagoras' Theorem and stops or diagram in γ method.
A2 $(5)^2 = 25$ and/or $(-12)^2 = 144$ and stops.
A3 $\sqrt{25\vec{i} + 144\vec{j}}$.
A4 Some correct relevant step, e.g. $|\vec{p}|^2 = \vec{p}^{\,2}$ or $\vec{p}^{\,2} = 25 + 144$ and stops.
A5 $|\vec{p}| = \sqrt{a + b} = \sqrt{5 - 12} = \sqrt{-7}$.
A6 Plots p correctly.

(a) (ii) **5 marks** **Att 2**

(a) (ii)	$\vec{p}^{\perp} = 12\vec{i} + 5\vec{j}$

* Accept correct answer without work.

Blunders (- 3)
B1 Switches coefficients correctly but one or both signs incorrect.

Slips (- 1)
S1 Each obvious slip to a maximum of 3.
S2 Plots $\vec{p}^{\perp}$ correctly but does not write it in terms of $\vec{i}$ and $\vec{j}$.

Attempts (2 marks)
A1 Plots $\vec{p}$ correctly for this part and stops.
A2 Does not change both coefficients, i.e. symmetries in origin or axis.
A3 Mentions rotation.

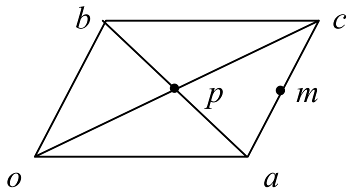
Worthless (0 marks)
W1 $\vec{p}^{\perp} = 5\vec{i} - 12\vec{j}$, i.e. what was given.

Part (b) **20 (10, 5, 5) marks** **Att 7 (3, 2, 2)**

(b) (i) Find the scalars k and t such that $2(3\vec{i} - t\vec{j}) + k(-\vec{i} + 2\vec{j}) = t\vec{i} - 8\vec{j}$.

(ii) *oacb* is a parallelogram where *o* is the origin.
p is the point of intersection of the diagonals.
m is the midpoint of [*ac*].

Express $\vec{p}$ and $\vec{m}$ in terms of $\vec{a}$ and $\vec{b}$.



(b) (i) **10 marks** **Att 3**

(b) (i)

$$6\vec{i} - 2t\vec{j} - k\vec{i} + 2k\vec{j} = t\vec{i} - 8\vec{j}$$
$$6 - k = t$$
$$-2t + 2k = -8$$
$$k = 1 \text{ and } t = 5$$

* Accept both $k = 1$ and $t = 5$ verified correctly by substitution.

Blunders (- 3)
B1 Removes brackets incorrectly, e.g. $6\vec{i} - t\vec{j}$ or $6\vec{i} + 2t\vec{j}$ (once, if it does not oversimplify)
B2 Treats $t\vec{j}$ as $\vec{tj} = \vec{j} - \vec{t}$ and continues.[Other blunders may apply.]
B3 Equates coefficients of $\vec{i}$ with coefficients of $\vec{j}$, e.g. $6 - k = -8$ (once).
B4 Sets up both equations correctly but does not solve for k and t.
B5 Transposing error, e.g. $2k = 2 \Rightarrow k = 2 - 2$ or $1 + t = 6 \Rightarrow t = 6 + 1$.
B6 Finds the value of only one variable.

Slips (- 1)
S1 Each obvious arithmetical slip to a maximum of 3.

Attempts (3 marks)
A1 Correct answer without work.
A2 Correct relevant step, e.g. $6\vec{i}$.
A3 Writes "$\vec{i}$ s = $\vec{i}$ s" or similar.
A4 Tests arbitrary values for k and t correctly.

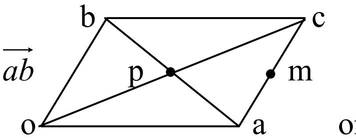
(b) (ii) $\vec{p}$ **5 marks** **Att 2**

$\vec{m}$ **5 marks** **Att 2**

(b) (ii) $\vec{p}$

α: $\vec{p} = \frac{1}{2}\vec{c}$ $= \frac{1}{2}(\vec{a} + \vec{b})$ or $\frac{1}{2}\vec{a} + \frac{1}{2}\vec{b}$

β: $\vec{p} = \vec{a} + \frac{1}{2}\overrightarrow{ab}$ $= \vec{a} + \frac{1}{2}(\vec{b} - \vec{a})$



or equivalent in $\vec{a}$ and $\vec{b}$

$\vec{m}$

α: $\vec{m} = \vec{a} + \frac{1}{2}\overrightarrow{ac}$ $= \vec{a} + \frac{1}{2}\vec{b}$

β: $\vec{m} = \vec{a} + \frac{1}{2}\overrightarrow{ac} = \vec{a} + \frac{1}{2}(\vec{c} - \vec{a})$ $= \vec{a} + \frac{1}{2}(\vec{a} + \vec{b} - \vec{a})$

γ: $\vec{m} = \vec{c} + \frac{1}{2}\overrightarrow{ca} = \vec{c} + \frac{1}{2}(\vec{a} - \vec{c})$ $= \vec{a} + \vec{b} + \frac{1}{2}[\vec{a} - (\vec{a} + \vec{b})]$

δ: $\vec{m} = \vec{p} + \overrightarrow{pm}$ $= \frac{1}{2}(\vec{a} + \vec{b}) + \frac{1}{2}\vec{a}$

or equivalent in $\vec{a}$ and $\vec{b}$

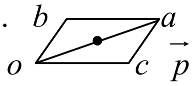
* Accept correct answers without work.

* Accept candidate's $\vec{p}$, if used to find $\vec{m}$.

* If a candidate reaches an acceptable answer, do not penalise for subsequent work,
e.g. $\vec{p} = \frac{1}{2}(\vec{a} + \vec{b}) = \frac{1}{2}\vec{a} + \vec{b}$ merits 5 marks but $\frac{1}{2}\vec{a} + \vec{b}$ only merits 2 marks.

Blunders (- 3)
B1 Incorrect direction (each part).
B2 Irrelevant distance (each part).
B3 Error in using triangle or parallelogram law, e.g. $\vec{c} = \vec{a} - \vec{b}$ or $\overrightarrow{ac} = \vec{a} + \vec{c}$, each part.
B4 Does not use origin correctly, e.g. $\overrightarrow{am} = \vec{m}$.
B5 Does not simplify to $\vec{a}$ and $\vec{b}$, e.g. $\vec{p} = \vec{a} + \frac{1}{2}\overrightarrow{ab}$ and stops
.
Misreadings (- 1)
M1 Labels switched in original parallelogram, if it does not simplify the task.

Attempts (2 marks each)
A1 Correct relevant step towards finding $\vec{p}$ **or** $\vec{m}$,
e.g. relevant arrow added to given diagram (1 × Att 2).
A2 Separate correct relevant steps towards finding $\vec{p}$ **and** $\vec{m}$,
i.e. obvious correct attempt at each, award 2 × Att 2, e.g. arrow *o* to *p* and arrow *o* to *m*.
A3 Some correct relevant application of vectors, e.g. $\overrightarrow{ac} = \vec{b}$ (1×Att2).
A4 Labels switched in original parallelogram, and task simplified, e.g.



$\vec{p} = \frac{1}{2}\vec{a}$.
A5 Relevant statement, e.g. "opposite sides of parallelograms are equal in measure"(1×Att2).

Part (c) **20 (10, 10) marks** **Att 6 (3, 3)**

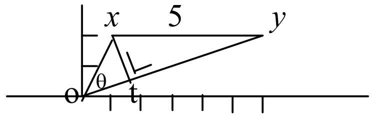
(c) Let $\vec{x} = \vec{i} + 2\vec{j}$ and $\vec{y} = 6\vec{i} + 2\vec{j}$.

(i) Calculate $\vec{x} \cdot \vec{y}$.

(ii) Hence, find the measure of the angle between $\vec{x}$ and $\vec{y}$.

(c) (i) **10 marks** **Att 3**

(c) (i)



α	β	γ
$\vec{x}.\vec{y} = (\vec{i} + 2\vec{j})(6\vec{i} + 2\vec{j})$	Using cosine formula, $\cos\theta = {}^{1}/_{\sqrt{2}}$	$\lvert ot \rvert = {}^{\sqrt{5}}/_{\sqrt{2}}$
or $6\vec{i}^{\,2} + 14\vec{i}.\vec{j} + 4\vec{j}^{\,2}$	or $\vec{x}.\vec{y} = \lvert\vec{x}\rvert.\lvert\vec{y}\rvert\cos\theta$	or $\vec{x}.\vec{y} = \lvert ot \rvert.\lvert oy \rvert$
$6\vec{i}^{\,2} + 4\vec{j}^{\,2}$ or $6 + 14\vec{i}.\vec{j} + 4$	$\sqrt{5} \times \sqrt{40} \times {}^{1}/_{\sqrt{2}}$	${}^{\sqrt{5}}/_{\sqrt{2}} \times \sqrt{40}$
6 + 4 or 10	10	10

* Accept correct answer without work.
* Each step presupposes previous steps.

Blunders (- 3)
B1 Incorrect relevant formula (task not simplified), e.g. $\lvert\vec{x}\rvert = \sqrt{a^2 - b^2}$ or $\vec{x}.\vec{y} = \lvert\vec{x}\rvert.\lvert\vec{y}\rvert\sin\theta$.
B2 Omits last step, e.g. $6\vec{i}^{\,2} + 4\vec{j}^{\,2}$ merits 7 marks. [$6\vec{i}^{\,2} + 4\vec{j}^{\,2} = \sqrt{52}$ still merits 7 marks.]
B3 $\vec{i}^{\,2} \neq 1$ and/or $\vec{j}^{\,2} \neq 1$, e.g.6 - 4, even without work merits 7 marks. [2 without work 0 m]
B4 $\vec{i}.\vec{j} \neq 0$.
B5 Mathematical error, e.g. mishandles surds or fractions or blunder in removing brackets.

Slips (- 1)
S1 Each arithmetical slip to a maximum of 3.

Attempts (3 marks)
A1 Correct formula and stops.
A2 Some correct work in multiplication, e.g. $6\vec{i}^{\,2}$ or 6 or $6\vec{i} + 4\vec{j}$.
A3 $\vec{i}^{\,2} = 1$ and/or $\vec{j}^{\,2} = 1$ and/or $\vec{i}.\vec{j} = 0$ and stops.
A4 $\lvert\vec{x}\rvert = \sqrt{5}$ and/or $\lvert\vec{y}\rvert = \sqrt{40}$ and stops
A5 $\vec{x}.\vec{y} = \sqrt{5} \times \sqrt{40} = \sqrt{200}$.

Worthless (0 marks)
W1 $\vec{x}.\vec{y} = \vec{y} - \vec{x} = 5\vec{i}$ or $\vec{x}.\vec{y} = \vec{x} + \vec{y} = 7\vec{i} + 4\vec{j}$.

(c) (ii) **10 marks** **Att 3**

(c) (ii)			
	$\vec{x}.\vec{y} = \lvert\vec{x}\rvert.\lvert\vec{y}\rvert\cos\theta$	or	$\cos\theta = \dfrac{\vec{x}.\vec{y}}{\lvert\vec{x}\rvert.\lvert\vec{y}\rvert}$
	$10 = \sqrt{5} \times \sqrt{40}\cos\theta$	or	$\cos\theta = {}^{10}/_{\sqrt{5}\times\sqrt{40}}$ or ${}^{1}/_{\sqrt{2}}$
	$\theta = 45^o$ or ${}^{\pi}/_{4}$		$\theta = 45^o$ or ${}^{\pi}/_{4}$

* Accept candidate's $\vec{x}.\vec{y}$ from part (i).

Blunders (- 3)

B1 Incorrect relevant formula and continues, e.g. $|\vec{x}| = \sqrt{a^2 - b^2}$ or $\vec{x} \cdot \vec{y} = |\vec{x}|.|\vec{y}| \sin\theta$ (once).

B2 Uses trigonometrical or coordinate geometry method correctly,

e.g. $\cos\theta = \dfrac{(\sqrt{5})^2 + (\sqrt{40})^2 - (5)^2}{2(\sqrt{5})(\sqrt{40})} \Rightarrow \theta = 45^\circ$.

B3 Transposing error.

B4 Omits last step.

B5 Mathematical error, e.g. mishandles surds or fractions.

B2 + B4 Uses trigonometrical method and stops at $\cos\theta = {}^{1}/_{\sqrt{2}}$. [Merits 4 marks.]

Slips (- 1)

S1 Each arithmetical slip to a maximum of 3.

Attempts (3 marks)

A1 Correct answer without work.

A2 Correct relevant formula and stops (written for this part).

A3 $|\vec{x}| = \sqrt{5}$ and/or $|\vec{y}| = \sqrt{40}$.

A4 $\vec{x} \cdot \vec{y} = |\vec{x}|.|\vec{y}| \Longrightarrow \theta = {}^{10}/_{\sqrt{5} \times \sqrt{40}}$.

A5 Slope of *ox* and/or *oy*.

Worthless (0 marks)

W1 Tries to use trigonometry or coordinate geometry incorrectly to find θ.

Question 10

Part (a)	**15 marks**	**Att 6**
Part (b)	**15 marks**	**Att 6**
Part (c)	**20 marks**	**Att 7**

Part (a) **15 (5, 5, 5) marks** **Att 6 (2, 2, 2)**

(a) Expand $(1 + x)^3$ fully.
Expand $(1 - x)^3$ fully.

Hence, find the real numbers a and k such that
$$(1 + x)^3 + (1 - x)^3 = a + kx^2.$$

(a) $(1 + x)^3$ **5 marks** **Att 2**
$(1 - x)^3$ **5 marks** **Att 2**

	α	β	γ
(a) $(1 + x)^3$:	$(1 + 2x + x^2)(1 + x)$ $1 + 3x + 3x^2 + x^3$	$1 + {}^3C_1\, x + {}^3C_2\, x^2 + x^3$ $1 + 3x + 3x^2 + x^3$	1 1 1 1 2 1 1 3 3 1 $1 + 3x + 3x^2 + x^3$
$(1 - x)^3$:	$(1 - 2x + x^2)(1 - x)$ $1 - 3x + 3x^2 - x^3$	$1 + {}^3C_1\,(-x) + {}^3C_2\,(-x)^2 + (-x)^3$ $1 - 3x + 3x^2 - x^3$	$1 + 3(-x) + 3(-x)^2 + (-x)^3$ $1 - 3x + 3x^2 - x^3$

* Accept correct answer without work.
* One expression should not merit both Att 2.

Blunders (- 3)
B1 Error in powers (apply once for each different type of error).
B2 Error in sign (apply once for each different type of error).
B3 Puts power of x as a denominator (once).

Slips (- 1)
S1 Each arithmetical slip to a maximum of 3.

Misreadings (- 1)
M1 Expands $(x - 1)^3$ correctly.

Attempts (2 marks for each expansion)
A1 Any term written correctly or part of Pascal's triangle or a correct coefficient.
A2 ${}^3C_1$ or similar and stops. [May merit both Att 2, if written for each expansion.]
A3 Any correct step towards long multiplication method, e.g. $(1 + x)(1 + x)$.

Worthless (0 marks)
W1 $(1 + x)^3 = 3(1 + x)^2$.

(a) $(1 + x)^3 + (1 - x)^3$ **5 marks** **Att 2**

(a) $(1 + x)^3 + (1 - x)^3$:	$(1 + 3x + 3x^2 + x^3) + (1 - 3x + 3x^2 - x^3) = 2 + 6x^2$

* Accept correct answer without work, if expansions for $(1 + x)^3$ and $(1 - x)^3$ are written.
* Accept $2 + 6x^2$. It is not necessary to specify $a = 2$ or $k = 6$.
* Accept answer consistent with candidate's expansions, if it is not over simplified.

Blunders (- 3)
B1 Blunder in sign.
B2 Algebraic blunder, e.g. $3x^2 + 3x^2 = 6x^4$.
B3 Coefficient blunder.
B4 Not hence: $[(1 + x)+(1 - x)][(1 + 2x + x^2)-(1 - x^2)+(1 - 2x + x^2)] = 2 + 6x^2$.

Slips (- 1)
S1 Each arithmetical slip to a maximum of 3.

Attempts (2 marks)
A1 Any correct relevant addition (or subtraction).
A2 Correct relevant step in factors, e.g. $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$.
A3 $1 + x^3 + 1 - x^3 = 2$ (oversimplified).

Worthless (0 marks)
W1 $(1 + \not{x})^3 + (1 - \not{x})^3 = (1 + 1) + (3 + 3) \Longrightarrow a = 2, k = 6$.

Part (b) **15 (5, 5, 5) marks** **Att 6 (2, 2, 2)**

(b) The nth term of a geometric series is given by $T_n = 27({}^2/_3)^n$.

(i) Write out the first three terms of the series.

(ii) Find an expression for the sum of the first five terms.

(iii) Find the sum to infinity of the series.

(b) (i) **5 marks** **Att 2**

(b) (i) α: $T_1 = 27({}^2/_3)$ or 18,	$T_2 = 27({}^2/_3)^2$ or 12	$T_3 = 27({}^2/_3)^3$ or 8
β: $T_1 = 27(0\cdot\dot{6})$ or $17\cdot\dot{9}$ or 18	$T_2 = 27(0\cdot\dot{6})^2$ or $11\cdot\dot{9}$ or 12	$T_3 = 27(0\cdot\dot{6})^3$ or $7\cdot\dot{9}$ or 8

Blunders (- 3)
B1 Error in index (not required to multiply out).
B2 Uses $T_n = 9(2)^n$ for T_2 and T_3.
B3 Omits a term.
B4 Incorrect r, e.g. $r = {}^3/_2$, if not an obvious misreading.
B5 Incorrect a, e.g. $a = 27$.

Slips (- 1)
S1 Each arithmetical slip to a maximum of 3.
S2 Decimal form (without recurrence),
e.g. $T_1 = 27(0\cdot6)$ or $16\cdot2$, $T_2 = 27(0\cdot6)^2$ or $9\cdot7$, $T_3 = 27(0\cdot6)^3$ or $5\cdot8$ (one slip).

Attempts (2 marks)
A1 One term and stops, e.g. $27({}^2/_3)$. [Arbitrary list with one correct term included.]
A2 $r = {}^2/_3$ and stops.
A3 $T_n = a.r^{n-1}$ and stops.

Worthless (0 marks)
W1 Writes T_1, T_2, T_3 and stops.

(b) (ii) **5 marks** **Att 2**

(b) (ii) α: $S_5 = \frac{a[1-r^n]}{1-r} = \frac{18[1-(\frac{2}{3})^5]}{1-\frac{2}{3}}$ or $\frac{18[(\frac{2}{3})^5-1]}{\frac{2}{3}-1}$ or $54[1-(\frac{2}{3})^5]$ or $54[1-\frac{32}{243}]$ or $54[\frac{211}{243}]$ or $\frac{11394}{243}$ or $46\frac{8}{9}$ or $46{\cdot}\dot{8}$.
β: $T_1 = 27(\frac{2}{3})$, $T_2 = 27(\frac{2}{3})^2$, $T_3 = 27(\frac{2}{3})^3$, $T_4 = 27(\frac{2}{3})^4$, $T_5 = 27(\frac{2}{3})^5$ $S_5 = 27(\frac{2}{3}) + 27(\frac{2}{3})^2 + 27(\frac{2}{3})^3 + 27(\frac{2}{3})^4 + 27(\frac{2}{3})^5$ or $18 + 12 + 8 + \frac{16}{3} + \frac{32}{9}$ or equivalent

Blunders (- 3)
B1 Error in formula, e.g. $\frac{a[1+r^n]}{1+r}$ (**both** signs incorrect in this case).
B2 Incorrect a or r.
B3 Incorrect relevant formula, e.g. ar^{n-1}, and continues.
B4 Index error.
B5 Omits a term or terms, e.g. 18 + 12 + 8.

Slips (- 1)
S1 Each arithmetical slip to a maximum of 3.

Attempts (2 marks)
A1 Correct formula for Sn and stops.
A2 Correct T_4 and/or T_5, e.g. $5\frac{1}{3}$ and/or $3\frac{5}{9}$.
A3 Correct answer without work.
A4 Correct a or r or $r = \frac{T_2}{T_1}$.

Worthless (0 marks)
W1 Formula for an arithmetic series, subject to A4.
W2 S_5 and stops.

(b) (iii) **5 marks** **Att 2**

(b) (iii) α: $S_\infty = \frac{a}{1-r} = \frac{18}{(1-\frac{2}{3})} = \frac{18}{\frac{1}{3}} = 54$
β: $S_n = \frac{a[1-r^n]}{(1-r)} = \frac{18[1-(\frac{2}{3})^n]}{(1-\frac{2}{3})} = 54[1-(\frac{2}{3})^n] = 54$ as $n \to \infty$
γ: $S_n = 27(\frac{2}{3}) + 27(\frac{2}{3})^2 + 27(\frac{2}{3})^3 + \ldots = 27[\frac{\frac{2}{3}[1-(\frac{2}{3})^n]}{(1-\frac{2}{3})}] = 27[2\{1-(\frac{2}{3})^n\}] = 54$ as $n \to \infty$
δ: $S_\infty = \frac{a}{1-r} = \frac{27(\frac{2}{3})}{(1-\frac{2}{3})} = \frac{18}{\frac{1}{3}} = 54$
ε: $S_\infty = \frac{a}{1-r} = \frac{27(\frac{2}{3})[1-(\frac{2}{3})^\infty]}{(1-\frac{2}{3})} = \frac{18[1-0]}{\frac{1}{3}} = 54$
θ: $S_\infty = \frac{a}{1-r} = \frac{18}{(1-0{\cdot}\dot{6})} = 54$

Blunders (- 3)
B1 Incorrect relevant formula, e.g. $S_\infty = \frac{a}{(1+r)}$.
B2 Incorrect a or r.
B3 Mathematical error dealing with fractions or decimals, e.g. misplaced decimal point.
B4 Fails to finish, i.e. leaves answer as $\frac{18}{\frac{1}{3}}$.

Slips (- 1)
S1 Each arithmetical slip to a maximum of 3.

Attempts (2 marks)
A1 $S_\infty = {}^{a}/_{(1-r)}$ and stops.
A2 Mentions $T_6 = 27(^2/_3)^6$ or $^{64}/_{27}$ or 2·370 or subsequent term and stops.
A3 Correct a and/or r and stops.
A4 Correct answer without work.

Worthless (0 marks)
W1 Arithmetic sequence or series formula and stops.

Part (c) **20 marks** **Att 7**

(c) IR£100 was invested at the beginning of each year for twenty consecutive years at 4% per annum compound interest.

Calculate the total value of the investment at the end of the twenty years, correct to the nearest IR£.

(c) **20 marks** **Att 7**

(c) α: Each investment calculated separately

20th 104·0000	16th 121·6652	12th 142·3311	8th 166·5073	4th 194·7900
19th 108·1600	15th 126·5319	11th 148·0244	7th 173·1676	3rd 202·5816
18th 112·4864	14th 131·5931	10th 153·9454	6th 180·0943	2nd 210·6849
17th 116·9858	13th 136·8569	9th 160·1032	5th 187·2981	1st 219·1123

Total = 3096·9195 (or 3096·9201…) ≈ IR£3097

β:

104	563·31	1100·62	1729·20	2464·55
212·16	689·84	1248·64	1902·37	2667·13
324·65	821·43	1402·59	2082·46	2877·82
441·64	958·29	1562·69	2269·76	3096·93 ≈ IR£3097

γ: $A = P[1 + {}^{r}/_{100}]^n$

$A = 100[1 + {}^{4}/_{100}]^{20} + 100[1 + {}^{4}/_{100}]^{19} + 100[1 + {}^{4}/_{100}]^{18} + \ldots + 100[1 + {}^{4}/_{100}]$

$A = 100(1 + {}^{4}/_{100})[(1 + {}^{4}/_{100})^{19} + (1 + {}^{4}/_{100})^{18} + (1 + {}^{4}/_{100})^{17} + \ldots + 1]$

$A =$ 104[S_{20} of geometric series where $a = 1$ and $r = 1·04$]

$A = 104[{}^{1(1·04^{20} - 1)}/_{(1·04 - 1)}]$

$A = 104[{}^{(2·191123143 - 1)}/_{0·04}] = 104[29·77807858..]$

$A = 3096·920172$

$A \approx$ IR£3097

δ: $A = P[1 + {}^{r}/_{100}]^n$

$A = 100[1 + {}^{4}/_{100}]^{20} + 100[1 + {}^{4}/_{100}]^{19} + 100[1 + {}^{4}/_{100}]^{18} + \ldots + 100[1 + {}^{4}/_{100}]$

$A = 100[(1 + {}^{4}/_{100})^{20} + (1 + {}^{4}/_{100})^{19} + (1 + {}^{4}/_{100})^{18} + \ldots + 1(1 + {}^{4}/_{100})]$

$A =$ 100[S_{20} of geometric series where $a = 1(1 + {}^{4}/_{100})$ or 1·04 and $r = 1·04$]

$A = 100[{}^{1·04(1·04^{20} - 1)}/_{(1·04 - 1)}]$

$A = 100[{}^{1·04(2·191123143 - 1)}/_{0·04}] = 100[1·04(29·77807858]$

$A = 3096·920172$

$A \approx$ IR£3097

* α method: Any one year correct merits 7 marks, an extra one mark for a 2nd year correct, an extra one mark for every *two* terms after the 2nd and the final 3 marks for adding and rounding off.

* If a candidate merits a mark greater than 0, he/she must be awarded at least Att 7 marks.
* No penalty for work subsequent to A = £3097, e.g. £3097 - £2000 = £1097 merits 20 marks.

Blunders (- 3)
B1 Error in relevant formula for compound interest.
B2 Error in relevant formula for geometric series.
B3 Index error.
B4 Mathematical error in fraction manipulation (once).
B5 Misplaced decimal point (once).
B6 Incorrect use of logarithms (once).
B7 Incorrect *a* and continues.
B8 Incorrect and inconsistent *r*.
B9 Rounds off to the nearest IR£ prematurely, i.e. it simplifies the task.
B10 Fails to add the 20 terms in α method.

Slips (- 1)
S1 Each arithmetical slip to a maximum of 3.
S2 Failure to round off at the end of the answer or rounds off prematurely with decimals, if it affects the answer and if B9 has not been applied.

Attempts (7 marks)
A1 Correct compound interest formula or correct formula for a geometric series and stops.
A2 Finds 4% of some number and stops.(7 m)
A3 Some correct relevant work, e.g. 1·04, and stops (7 m)
A4 Correct answer without work.
A5 Uses simple interest or states simple interest formula, e.g. 104×20 = IR£2080. (7marks)

Worthless (0 marks)
W1 IR£2080 without work.

Question 11

Part (a)	**15 marks**	**Att 6**
Part (b)	**35 marks**	**Att 14**

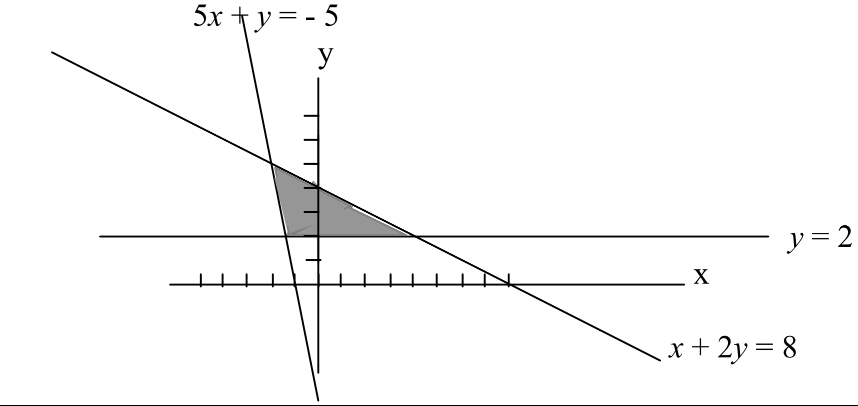
Part (a) **15 (5, 5, 5) marks** **Att 6 (2, 2, 2)**

(a) Using graph paper, illustrate the set of points (x, y) that simultaneously satisfy the three equations:

$$y \geq 2$$
$$x + 2y \leq 8$$
$$5x + y \geq -5.$$

(a) **15 (5, 5, 5) marks** **Att 6 (2, 2, 2)**

(a)



* Each half-plane merits 5 marks, attempt 2 marks.
* Points or scale required.
* Half-planes required but no penalty for not indicating intersection if half-planes are indicated. If half-planes are indicated correctly, do not penalise for incorrect shading.
* Accept correct shading of intersection for half-planes but candidates may shade out areas that are not required and leave intersection blank
* Correct shading over-rules arrows.
* Three lines drawn and intersection not indicated apply only one of the following:

Case 1: Three sets of arrows in expected direction -- (full marks)		
Case 2: Three sets of arrows in unexpected direction (full marks)		
Case 3: Three sets of arrows, not all consistent	(1×-3)	(one set is incorrect)
Case 4: Two sets of arrows, consistent	(1×-3)	(one line without arrows)
Case 5: Two sets of arrows, inconsistent.	(2×-3)	(one incorrect, one line without)
Case 6: One set of arrows	(2×-3)	(two lines without)
Case 7: No arrows	(3×-3)	(three lines without)

Blunders (- 3)
B1 No half-plane indicated (each time).
B2 Blunder in plotting a line or in calculations (each line).
B3 Incorrect shading, i.e. 2 or 3 sets of arrows but not all “acceptable” as described above.

Attempts (2 marks each half-plane)
A1 Some relevant work towards a point on a line, i.e. 2 marks for each line attempted.
A2 Draws axes or axes and one line (1 × attempt 2).

Part (b) **35 (20, 15)** **Att 14 (8, 6)**

(a) Houses are to be built on 9 hectares of land.
Two types of houses, bungalows and semi-detached houses, are possible.

Each bungalow occupies one fifth of a hectare.
Each semi-detached house occupies one tenth of a hectare.

The cost of building a bungalow is IR£80 000.
The cost of building a semi-detached house is IR£50 000.
The total cost of building the houses cannot be greater than IR£4 million.

(i) Taking x to represent the number of bungalows and y to represent the number of semi-detached houses, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The profit on each bungalow is IR£10 000. The profit on each semi-detached house is IR£7 000. How many of each type of house should be built so as to maximise profit?

(b) (i) Inequalities **10 (5, 5) marks** **Att 4 (2, 2)**

(b) (i)
Area: $\frac{1}{5}x + \frac{1}{10}y \le 9$
Cost: $80\,000x + 50\,000y \le 4\,000\,000$

Table	Bungalow	Semi-detached	Max.
Area	$\frac{1}{5}x$	$\frac{1}{10}y$	9
Cost	$80\,000x$	$50\,000y$	4 000 000

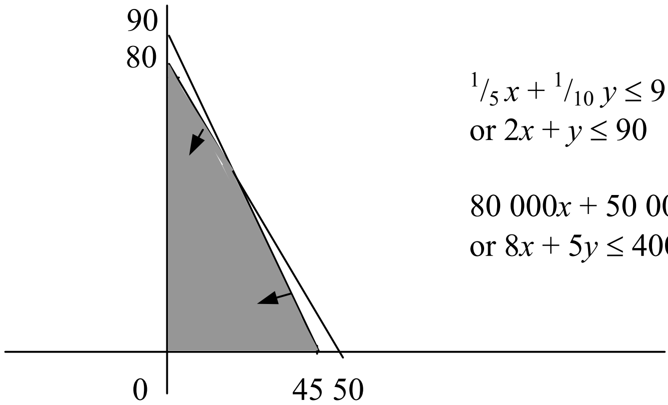
* Accept correct multiples of inequalities or different letters.
* Do not penalise here for incorrect or no inequality sign. Penalise in graph if used.
* Case: $\frac{1}{5}$ $\frac{1}{10}$ 9
80 000 50 000 4 000 000. Award 10 marks. Penalise in graph, if linkup is incorrect.

Blunders (-3)
B1 Mixes up x's and y's (once only, if consistent error).
B2 Confuses rows and columns in table, e.g. $\frac{1}{5}x + 80\,000y \le 9$.
B3 Misplaced decimal point (once).[Accept $8x + 5y \le 400$.]
B4 Blunder dealing with fractions, if different error to B3 (once), e.g. $x + y \le 9$.

Attempts (2 marks for each inequality)
A1 Incomplete relevant data in table and stops (each inequality).
A2 Any other correct inequality and stops, e.g. $x \ge 0, y \ge 0$ (each time).
A3 Some variable ≤ 9, some variable $\le 4\,000\,000$ each merits attempt 2.
A4 $\frac{1}{5}x$ and/or $\frac{1}{10}y$ and stops. [1 × Att 2.]
A5 $80\,000x$ and/or $50\,000y$ and stops. [1 × Att 2.]

(b) (i) Graph **10 (5, 5)** **Att 4 (2, 2)**

(b) (i)



$^1/_5\,x + {}^1/_{10}\,y \le 9$
or $2x + y \le 90$

$80\ 000x + 50\ 000y \le 4\ 000\ 000$
or $8x + 5y \le 400$

* Points or scale required.
* Half-planes required. No penalty for not indicating intersection, if half-planes are indicated.
* Correct shading pre-supposes correct half-planes.
* Candidates may shade out areas that are not required and leave intersection blank. Accept the two narrow triangles shaded.
* Correct graph without inequalities merits 10 marks for the graph and + 2 + 2 for the inequalities.
* Without work, correct shading overrules arrows.
* Cases: Lines drawn and no shading, only one of the following applies:

Case 1	Both sets of arrows in expected direction	10 marks
Case 2	Both sets of arrows in unexpected direction	10 marks
Case 3	One set of arrows "correct" and the other "incorrect"	7 marks (5 + Att 2)
Case 4	No arrows	4 marks (Att 2, Att2)
Case 5	One line with and the other without arrows	5 + Att 2

Blunders (- 3)
B1 No half-plane indicated (each time).
B2 Incorrect or no line drawn (each time, if not a slip).
B3 Plots incorrect point, e.g. (45, 90).
B4 Shades one of narrow triangles.
B5 Transposing error.

Slips (-1)
S1 Each arithmetic slip to a maximum of 3.

Attempts (2 marks each half-plane)
A1 Some relevant work towards a point on a line.
A2 Draws axes or axes and one line (award 1 × Att 2).
A3 Draws axes and two lines (award Att 2 + Att 2).

(b) (ii) Intersection of lines **5 marks** **Att 2**

(b) (ii) $^1/_5\,x + {}^1/_{10}\,y = 9$ $\quad 80\ 000x + 50\ 000y = 4\ 000\ 000$ or $2x + y = 90$ $\quad 8x + 5y = 400 \Rightarrow x = 25, \; y = 40$

* Accept candidate's own equations from previous parts.
* If x is calculated accept consistent value for y without further work and vice versa.

Blunders (- 3)
B1 Fails to multiply/divide both sides of equation(s) correctly when eliminating variable.
B2 Sign error.

B3 x **or** y value only.
B4 Transposing error.

Slips (- 1)
S1 Each arithmetic slip to a maximum of 3.

Attempts (2 marks)
A1 Correct answer without work or answer consistent with earlier work or graph.
[Should get same values from graph as if they had been found algebraically.]
A2 Any relevant step towards solving equations.

Worthless (0 marks)
W1 Incorrect answer without work and inconsistent with graph.

(b) (ii) Expression for Profit 5 marks Att 2

(b) (ii) $10\,000x + 7\,000y$ or multiple/fraction, e.g. $10x + 7y$

Blunders (- 3)
B1 Confuses x's and y's (no penalty, if consistent with previous work).

Attempts (2 marks)
A1 Any relevant work involving x and/or y and/or 10 000, 7 000, 10, 7 or similar.

(b) (ii) Number of houses 5 marks Att 2

(b) (ii)

	Vertices	$10\,000x + 7\,000y$	Profit
Step 1	(0, 80)	0 + 560 000	560 000
Step 2	(25, 40)	250 000 + 280 000	530 000
Step 3	(45, 0)	450 000 + 0	450 000

Conclusion: No bungalow and 80 semi-detached houses

* Accept point of intersection from previous part.
* Information does not have to be in table form.
* One mark for each consistent step (1 to 3), subject to the attempt mark.
* Accept only vertices consistent with candidate's previous accepted work, not arbitrary vertices.
* If no marks have been awarded for intersection of lines and this point is now written here award 2 marks for previous part and also 1 mark for this part if step is correct.
* Correct steps 1, 2 and 3 presuppose *Expression for Profit*.
* Answer must be explicit for full marks, i.e. – 1 for step indicated but number of bungalows and semi-detached houses not explicitly written.

Slips (- 1)
S1 Each arithmetic slip to a maximum of 3.
S2 Step 1, 2 or 3 omitted or incorrect (each time), e.g. (80, 0)

Attempt (2 marks)
A1 Any attempt at substituting coordinates into some expression.
A2 Correct answer and no work.
A3 Step 2 only ==> 25 bungalows and 40 semi-detached houses.