AN ROINN OIDEACHAIS AGUS EOLAÍOCHTA

LEAVING CERTIFICATE EXAMINATION, 1998

MATHEMATICS – ORDINARY LEVEL – PAPER 2 (300 marks)

FRIDAY, 12 JUNE – MORNING 9.30 to 12.00

Attempt **5 Questions** from Section **A** and **ONE Question** from Section **B.** Each question carries 50 marks. Marks may be lost if necessary work is not clearly shown or if you do not indicate where a calculator has been used.

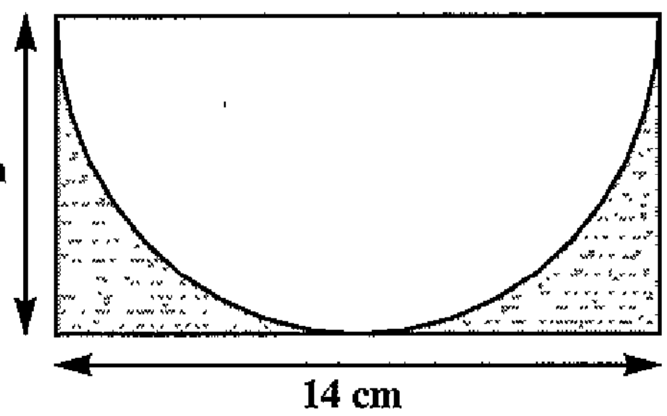
SECTION A

. **(a)** A rectangular piece of metal measures 7 cm by 14 cm.

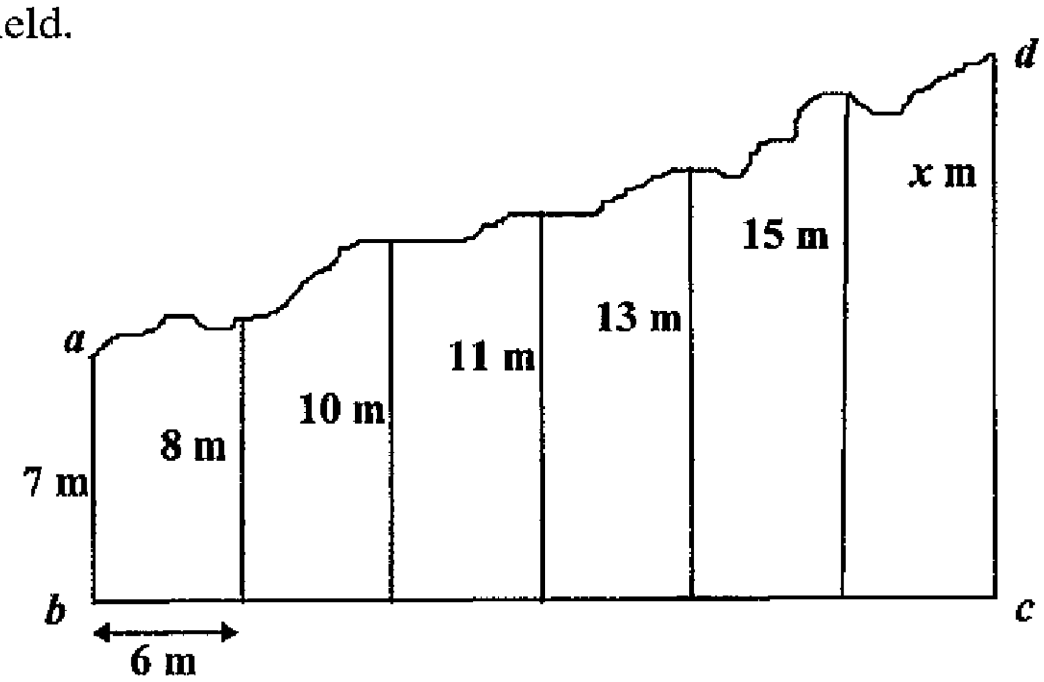
A semi-circular section with radius of length 7 cm is removed.

Calculate the area of the remaining piece of metal.

Take $\pi = \frac{22}{7}$.



(b) The sketch shows a field *abcd* which has one uneven edge. At equal intervals of 6 m along [*bc*] perpendicular measurements of 7 m, 8 m, 10 m, 11 m, 13 m, 15 m and *x* m are made to the top of the field.



Using Simpson's Rule the area of the field is calculated to be 410 m^2.

Calculate the value of x. [See Tables, page 42.]

(c) Find the volume of a solid sphere with a diameter of length 3 cm. Give your answer in terms of π.

A cylindrical vessel with internal diameter of length 15 cm contains water. The surface of the water is 11 cm from the top of the vessel.

How many solid spheres, each with diameter of length 3 cm, must be placed in the vessel in order to bring the surface of the water to 1 cm from the top of the vessel?
Assume that all the spheres are submerged in the water.

2. **(a)** The point (–3, 4) is on the line whose equation is $5x + y + k = 0$.
Find the value of k.

(b) $a(2, -1)$, $b(-2, 3)$, $c(-1, -1)$ and $d(4, -6)$ are points.

(i) Show that ab is parallel to cd.

(ii) Investigate if $abcd$ is a parallelogram.
Give a reason for your answer.

(c) The equation of the line L is $x - 2y + 10 = 0$.

L contains the point $t(2, 6)$.

(i) Find the equation of the line N which passes through t and which is perpendicular to L.

(ii) The line N cuts the x-axis at r and it cuts the y-axis at s.
Calculate the ratio

$$\frac{|rt|}{|ts|}.$$

Give your answer in the form $\frac{p}{q}$, where p and q are whole numbers.

3. **(a)** A circle C, with centre (0, 0), passes through the point (4, –3).

(i) Find the length of the radius of C.

(ii) Show, by calculation, that the point (6, –1) lies outside C.

(b) The equation of the circle K is $(x - 3)^2 + (y + 2)^2 = 29$.

(i) Write down the radius length and the coordinates of the centre of K.

(ii) Find the coordinates of the two points where K intersects the x-axis.

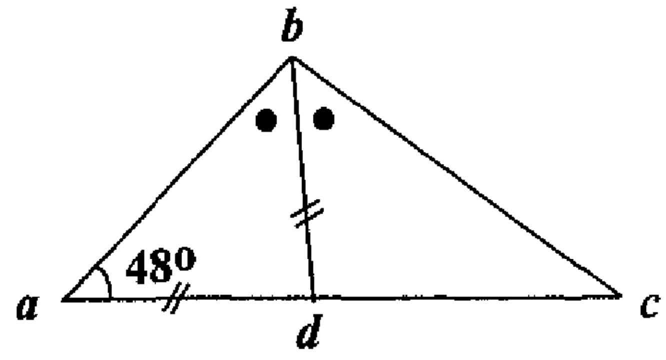
(c) The line with equation $3x - y + 10 = 0$ is a tangent to the circle which has equation $x^2 + y^2 = 10$.

(i) Find the coordinates of a, the point at which the line touches the circle.

(ii) The origin is the midpoint of $[ab]$.
Find the equation of the tangent to the circle at b.

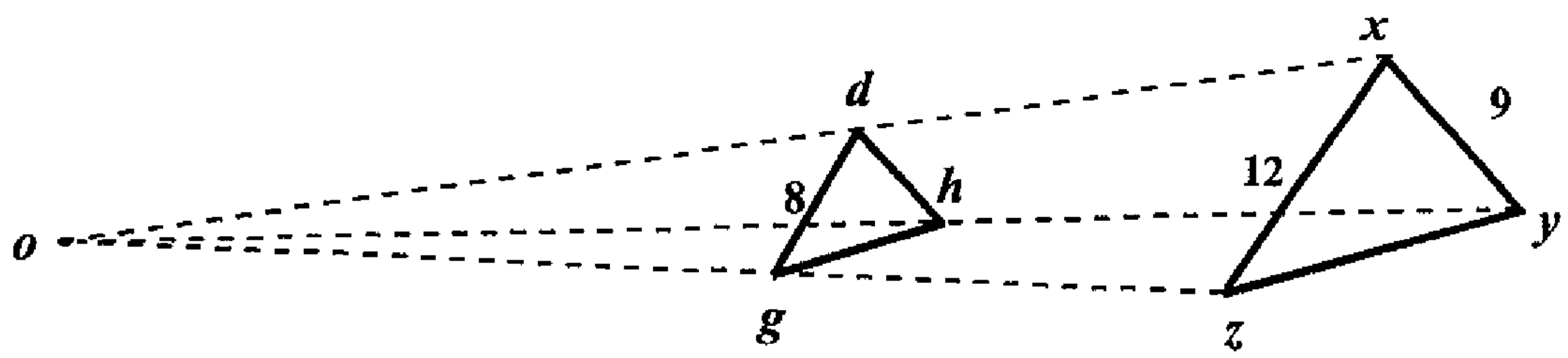
4. **(a)** In the triangle *abc*, $|ad| = |bd|$, $|\angle abd| = |\angle dbc|$ and $|\angle dab| = 48°$.

Find $|\angle dcb|$.



(b) Prove that if the lengths of two sides of a triangle are unequal, then the degree-measures of the angles opposite to them are unequal, with the greater angle opposite to the longer side.

(c) The triangle *xzy* is the image of the triangle *dgh* under the enlargement, centre *o*, with $|dg| = 8$, $|xz| = 12$ and $|xy| = 9$.



(i) Find the scale factor of the enlargement.

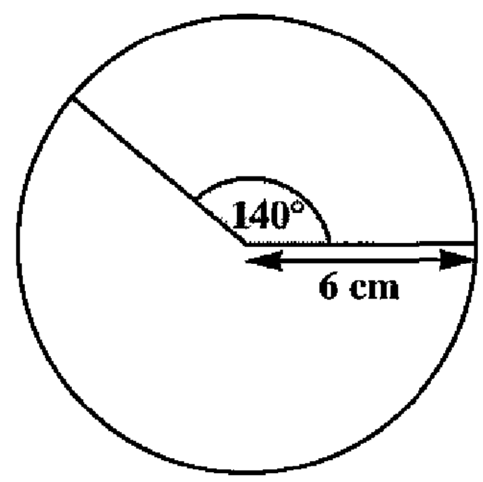
(ii) Find $|dh|$.

(iii) The area of the triangle *xzy* is 27 square units. Find the area of the triangle *dgh*.

5. **(a)** The angle at the centre of a sector of a disc measures 140°.

The radius of the disc measures 6 cm.

Find, in terms of π, the area of the sector.



(b) A is an acute angle such that $\tan A = \frac{21}{20}$.

(i) Find, as fractions, the value of cos A and the value of sin A.

(ii) Find the measurement of angle A, correct to the nearest degree.

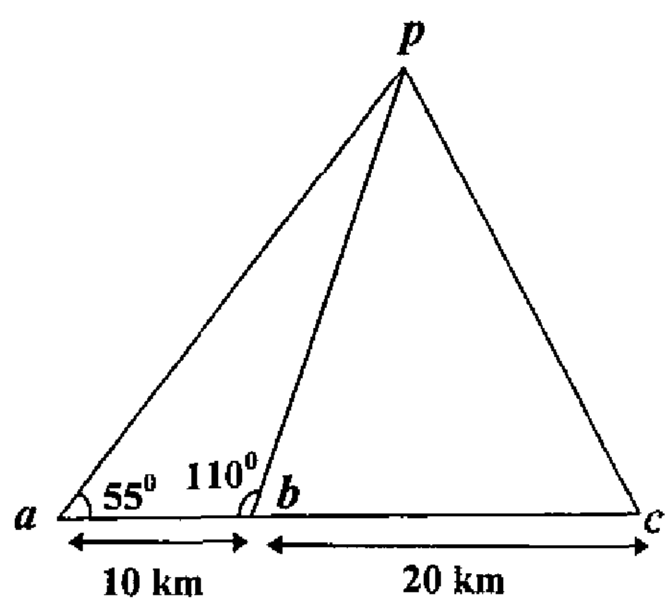
(c) Three ships are situated in a straight line at points *a*, *b* and *c*.

p is a port such that $|\angle bap| = 55°$, $|\angle abp| = 110°$, $|ab| = 10$ km and $|bc| = 20$ km.

Calculate

(i) $|bp|$, correct to the nearest km

(ii) $|cp|$, correct to the nearest km.



6. **(a)** One letter is chosen at random from the letters of the word LEAVING.

(i) Find the probability that the letter chosen is L.

(ii) Find the probability that the letter chosen is a vowel.

(b) A committee of 4 people is to be formed from a group of 7 men and 6 women.

(i) How many different committees can be formed?

(ii) On how many of these committees is there an equal number of men and of women?

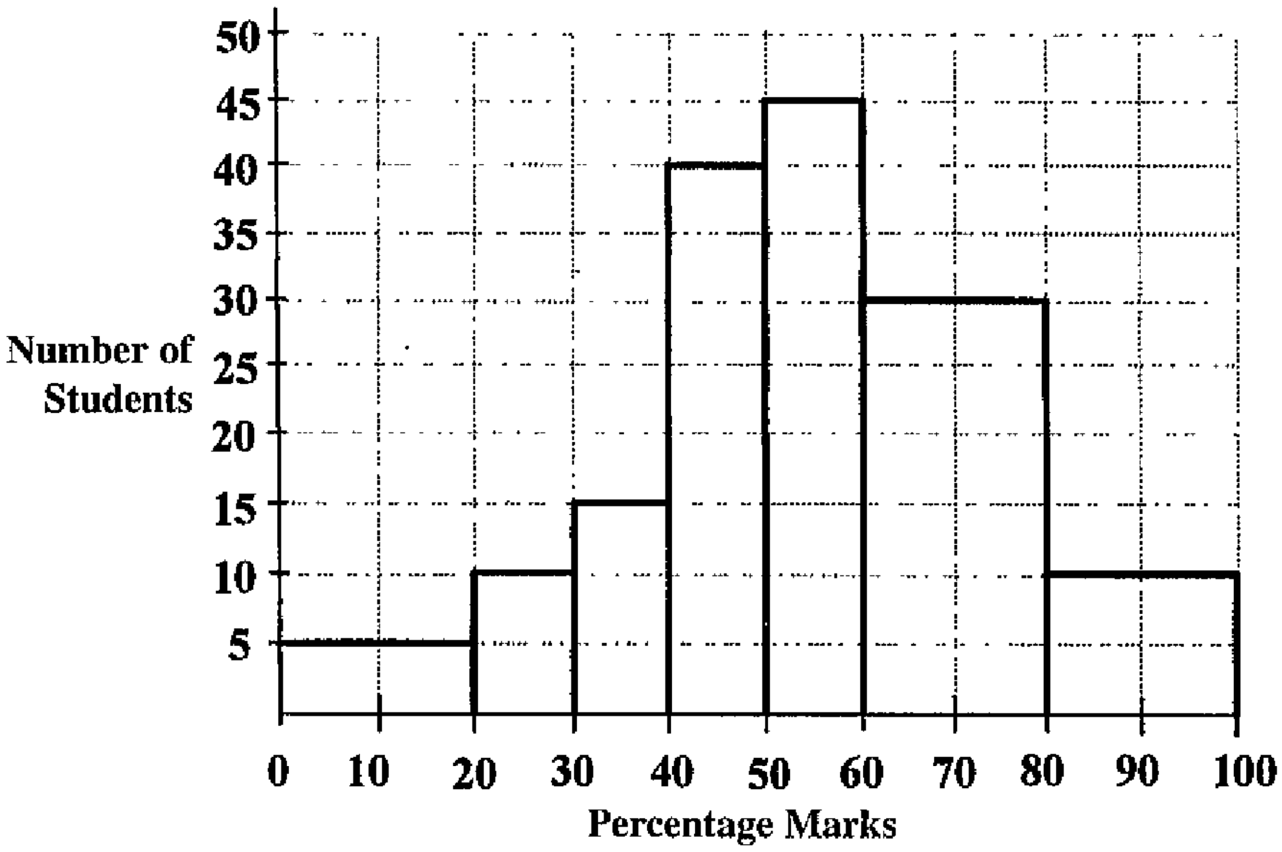
(c) **(i)** How many different numbers, each with 3 digits or less, can be formed from the digits 1, 2, 3, 4, 5? Each digit can be used only once in each number.

(ii) How many of the above numbers are even?

7. **(a)** Find the mean and the median of the following array of numbers:

2, 5, 7, 11, 15, 3, 6.

(b) The distribution of percentage marks awarded to a group of 200 Leaving Certificate students in a particular subject is shown in the histogram below.



(i) If 45 students obtained between 50% and 60%, copy and complete the frequency table below.

Marks (%)	0 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 80	80 - 100
Frequency					45		

(ii) What is the greatest possible number of students who could have obtained a grade C or better (i.e. mark ≥ 55)?

(c) The following table shows the sizes, in hectares, of 20 farms in a particular area:

Number of hectares	15 - 45	45 - 75	75 - 105	105 - 195
Number of farms	1	4	8	7

By taking the data at mid-interval values, calculate

(i) the mean number of hectares per farm

(ii) the standard deviation, correct to the nearest hectare.

[Turn over

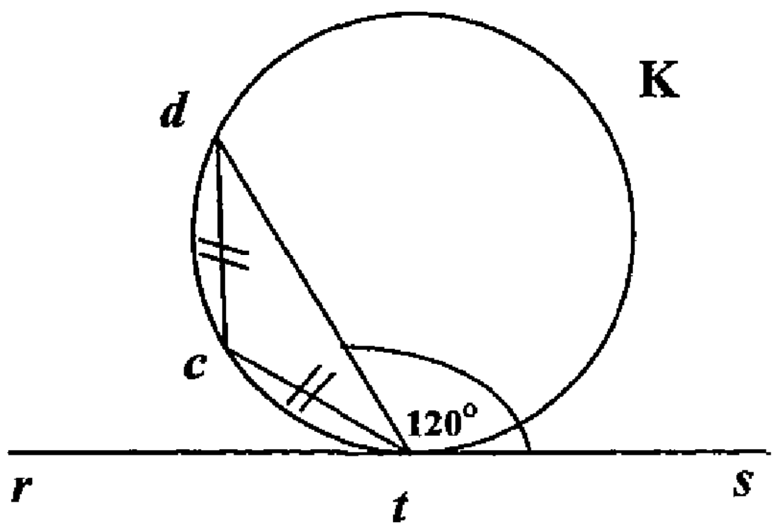
SECTION B
Attempt ONE question

8. **(a)** The line rs is a tangent to the circle K at the point t.

The obtuse angle between the tangent rs and the chord $[td]$ measures 120°.

The point c is on the circle K and $|dc| = |tc|$.

Find $|\angle cdt|$.



(b) Prove that if $[ab]$ and $[cd]$ are chords of a circle and the lines ab and cd meet at the point k where k is outside the circle, then

$$|ak| . |kb| = |ck| . |kd|.$$

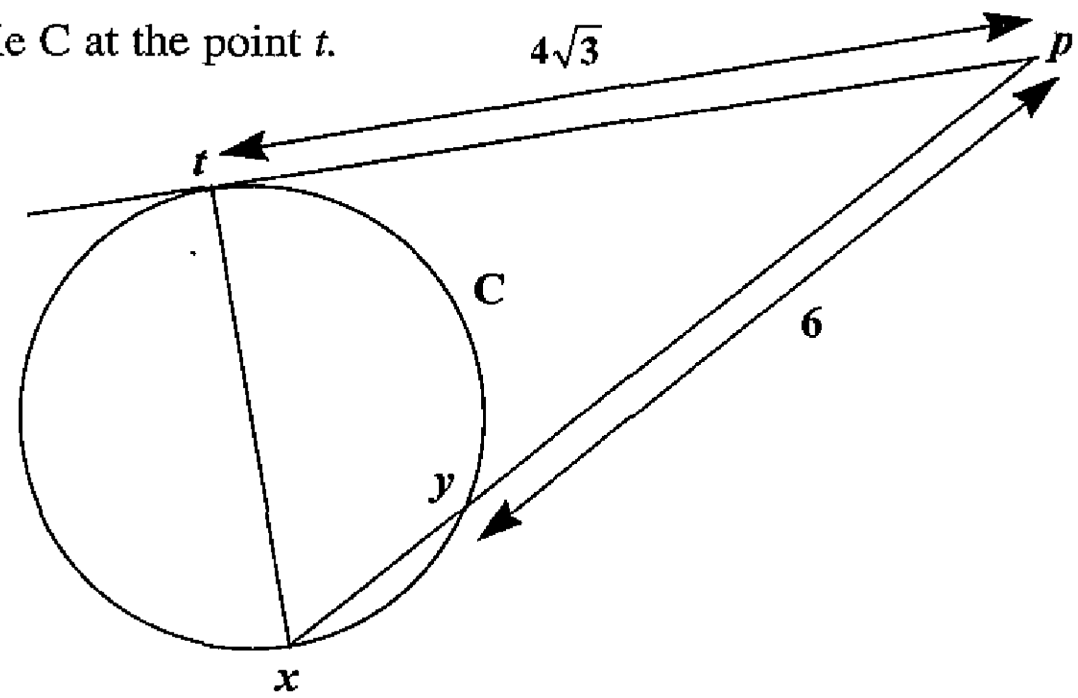
(c) The line pt is a tangent to the circle C at the point t.

$[xt]$ is a diameter of the circle.

$[px]$ cuts the circle at y.

If $|py| = 6$ and $|pt| = 4\sqrt{3}$, calculate $|xy|$.

Hence, calculate $|xt|$.



9. **(a)** Let $\vec{x} = 3\vec{i} + 2\vec{j}$ and $\vec{y} = 4\vec{i} - \vec{j}$.

Express, in terms of $\vec{i}$ and $\vec{j}$,

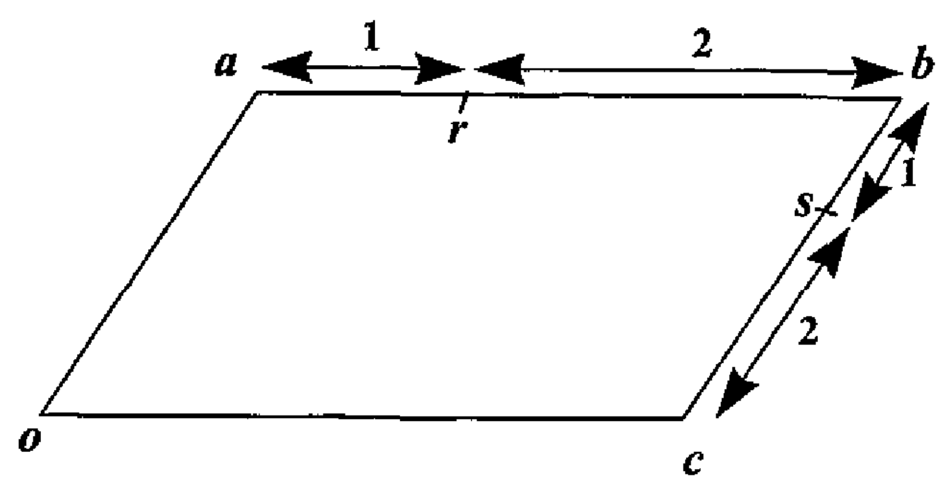
(i) $\vec{x} + \vec{y}$

(ii) $3\vec{y} - 4\vec{x}$.

(b) $ocba$ is a parallelogram where o is the origin. The point r divides $[ab]$ in the ratio 1 : 2 and the point s divides $[bc]$ in the ratio 1 : 2.

Express in terms of $\vec{a}$ and $\vec{c}$,

(i) $\vec{b}$ **(ii)** $\vec{r}$ **(iii)** $\vec{s}$ **(iv)** $\vec{rs}$.



(c) Let $\vec{u} = -2\vec{i} - \vec{j}$ and $\vec{v} = \vec{i} + 3\vec{j}$.

Write $\vec{u}^{\perp}$ and $\vec{v}^{\perp}$ in terms of $\vec{i}$ and $\vec{j}$.

Find the value of the scalar k and the value of the scalar p for which

$$k(\vec{u}^{\perp}) + p(\vec{v}^{\perp}) = 5(\vec{i} - \vec{j}).$$

10. **(a)** Find the sum to infinity of the geometric series

$$8 + 4 + 2 + 1 + \frac{1}{2} + ...$$

(b) The sum of the first 21 terms of an arithmetic series is 735.

The common difference between each term is 1.

Find the first term of the series.

(c) A person invested IR£750 at the beginning of each year for four consecutive years at 6% per annum compound interest.

Find

(i) the value of the first investment of IR£750 at the end of the fourth year, correct to the nearest penny

(ii) the total value of all the investments at the end of the fourth year, correct to the nearest IR£.

11. **(a)** Write down the coordinates of two points on the line $2x + 3y = 18$.

On a diagram, illustrate the set of points (x, y) that satisfy simultaneously the three inequalities

$$2x + 3y \leq 18$$

$$x \geq 3$$

$$y \geq 2.$$

(b) A company produces two products, A and B.

Each unit of the two products must be processed on two assembly lines, the red line and the blue line, for a certain length of time.

Each unit of A requires 3 hours on the red line and 1 hour on the blue line.
Each unit of B requires 1 hour on the red line and 2 hours on the blue line.

Each week, the maximum time available on the red line is 60 hours and the maximum time available on the blue line is 40 hours.

(i) If x represents the number of units of A produced in a week and y represents the number of units of B produced in a week, write down two inequalities in x and y. Illustrate these on graph paper.

(ii) The profit made on each unit of A is twice the profit made on each unit of B. How many units of each product must be manufactured in a week so as to maximise profit?

(iii) If the maximum profit that can be made in a week is IR£1980, calculate the profit made on each unit of A and on each unit of B.