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LEAVING CERTIFICATE EXAMINATION, 1998 58604

MATHEMATICS — ORDINARY LEVEL — PAPER 1 (300 marks)

THURSDAY, 11 JUNE – MORNING, 9.30 to 12.00

Attempt **SIX QUESTIONS** (50 marks each).

Marks may be lost if necessary work is not clearly shown or if you do not indicate where a calculator has been used.

1. **(a)** When a cyclist had travelled a distance of 12.6 km he had completed $\frac{3}{7}$ of his journey.

What was the length of the journey?

(b) **(i)** At what rate of interest will IR£2000 amount to IR£2065 after one year?

(ii) Divide 357 grammes in the ratio $\frac{1}{2} : \frac{1}{4} : 1$.

(c) A supplier agrees to buy 300 computer parts for 1060 Deutschmarks (DM) each.

He plans to sell them for a total of IR£138 000.

(i) Calculate the percentage profit (on the cost price) he will make if the exchange rate is IR£1 = DM 2.65.

(ii) By how much will the percentage profit (on the cost price) change if the exchange rate becomes IR£1 = DM 2.50? Give your answer correct to one place of decimals.

2. **(a)** Solve

$$5x - 2y = 13$$
$$3(x - 4) = 4y.$$

(b) Find the value of

$$\frac{a - b + 1}{a + b + 1}$$

when $a = \frac{1}{8}$ and $b = 2$.

(c) **(i)** Write $\sqrt{125}$ as a power of 5.

(ii) Solve for x the equation

$$\frac{5^{2x+1}}{\sqrt{5}} = \left(\frac{1}{\sqrt{125}}\right)^3.$$

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3. **(a)** Express p in terms of q and t when

$$q + \frac{p}{5t} = 3, \quad t \neq 0.$$

(b) **(i)** If $(x - 2)$ is a factor of $3x^3 + x^2 + kx + 6$, find the value of k.

(ii) Write down an equation which has three roots of value -3, 1 and 5.

(c) **(i)** Write $\frac{1}{x+1} + \frac{2}{x-3}$ as a single fraction where $x \neq -1$ and $x \neq 3$.

(ii) Hence, or otherwise, find, correct to one place of decimals, the two solutions of

$$\frac{1}{x+1} + \frac{2}{x-3} = 1, \quad x \neq -1, \; x \neq 3.$$

4. **(a)** Let $w = 2i$, where $i^2 = -1$. Plot

(i) w^2

(ii) w^3

on an Argand diagram.

(b) **(i)** Verify that $4 - 3i$ is a root of

$$z^2 - 8z + 25 = 0$$

and write down the other root.

(ii) Investigate if

$$|2 + 14i| = |10(1 - i)|.$$

(c) Let $u = 2 - i$.

(i) Express $u + \frac{1}{u}$ in the form $a + bi$, $a, b \in \mathbf{R}$.

(ii) Hence, solve

$$k\left(u + \frac{1}{u}\right) + ti = 18$$

for real k and real t.

5. **(a)** The first two terms of an arithmetic sequence are 17, 13, . . .

Find **(i)** d, the common difference

(ii) T_7, the seventh term.

(b) The nth term of a geometric sequence is

$$T_n = \frac{2^n}{3^n}.$$

(i) Find the first three terms of the sequence.

(ii) Show that S_5, the sum of the first five terms, is $\frac{422}{243}$.

(c) The first three terms of an arithmetic series are

$$2d + 3d + 4d + \ldots$$

where d is a real number.

(i) Find, in terms of d, an expression for T_{10}, the tenth term.

(ii) Find, in terms of d, an expression for S_{10}, the sum to 10 terms.

(iii) If $S_{10} - T_{10} = 162$, find the value of d and write down the first four terms of the series.

6. **(a)** If $f(x) = 5x - 8$ and $g(x) = 13 - 2x$, find the value of x for which

$$f(x) = g(x).$$

(b) The speed, v, in metres per second of an engine moving along a track is related to time, t, in seconds by

$$v = \frac{1}{3}(2t + 5).$$

(i) Draw the straight line graph of this relation, putting t on the horizontal axis, for $0 \leq t \leq 8$.

(ii) Use your graph to estimate the speed when $t = 2.5$ seconds.

(iii) Use your graph to estimate the time at which the speed reaches 6 metres per second.

(c) $f(x) = (x + k)(x - 2)^2$, where k is a real number.

(i) If $f(3) = 7$, find the value of k.

(ii) Using this value for k, find the coordinates of the local maximum and of the local minimum of $f(x)$.

7. **(a)** Differentiate with respect to x

(i) $x^2 - 3x$

(ii) $\frac{1}{x^2}$.

(b) **(i)** Find $\frac{dy}{dx}$ when $y = \frac{2x}{x^2 + 1}$.

(ii) Find $\frac{dy}{dx}$ when $y = (4 - 3x^2)^7$ and write down the range of values of x for which $\frac{dy}{dx} > 0$.

(c) The volume of water, V, in cm^3, that remains in a leaking tank after t seconds is given by

$$V = 45\,000 - 300t + 0.5t^2.$$

(i) After how many seconds will the tank be empty?

(ii) Find the rate of change of the volume with respect to t when $t = 50$ seconds.

8. Let $f(x) = \frac{1}{x - 1}$, for $x \in \mathbf{R}$ and $x \neq 1$.

(i) Find the value of $f(-2)$, $f(0)$, $f\left(\frac{3}{2}\right)$ and $f(5)$.

(ii) Find $f'(x)$, the derivative of $f(x)$.

(iii) Draw the graph of $f(x)$ for $-2 \leq x \leq 5$.

(iv) Find the equation of the tangent T to the curve at the point $(0, -1)$.

(v) Find the coordinates of the other point on the graph of $f(x)$ at which the tangent to the curve is parallel to T.