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LEAVING CERTIFICATE EXAMINATION, 1997

MATHEMATICS — ORDINARY LEVEL — PAPER 2 (300 marks)

FRIDAY, 13 JUNE — MORNING, 9.30 to 12.00

Attempt **5 Questions** from Section A and **ONE Question** from Section B. Each question carries 50 marks.
Marks may be lost if necessary work is not shown or if you do not indicate
where a calculator has been used.

SECTION A

1. **(a)** Find the slant height, l, of a cone which has
    perpendicular height of 4 cm and base with
    radius of length 3 cm.
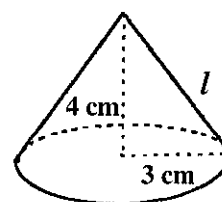
    Write down the curved surface area of the
    cone in terms of π.

    (b) The diagram shows a sketch of a piece of paper $abcd$ with one uneven edge. At equal intervals of h cm
    along $[bc]$, perpendicular measurements of 12 cm, 8 cm, 9 cm, 6 cm, 5 cm, 7 cm and 11 cm are made to
    the top edge.
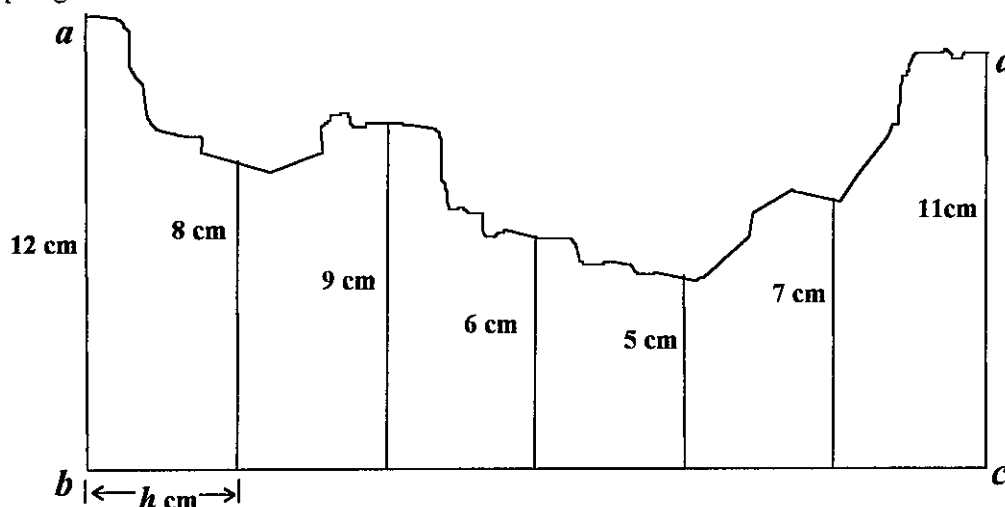
    Using Simpson's Rule the area of the piece of paper is estimated to be 180 cm^2.

    Calculate the value of h.   [See Tables, page 42.]

    (c) Find the volume of a solid sphere which has radius of length 2.1 cm. Give your answer correct to the
    nearest cm^3. Take $\frac{22}{7}$ as an approximation for π.

    This sphere and a solid cube with edge of length 3 cm are completely submerged in water in a cylinder.
    The cylinder has radius of length r cm.

    Both the sphere and the cube are then removed from the cylinder. The water level drops by 4 cm.
    Find r, correct to one place of decimals. [Take $\pi = \frac{22}{7}$.]

OVER→

2. **(a)** Find the distance between the two points $(-5, 1)$ and $(7, -4)$.

(b) L is the line $x - 2y + 2 = 0$.
M is the line $3x + y - 8 = 0$.

Find the co-ordinates of p, the point of intersection of L and M.

L and M cut the x-axis at q and r, respectively.

Find the area of triangle pqr.

(c) K is the line which contains the points $a(0,4)$ and $b(3,0)$.

Find the equation of K.

N is the line which is perpendicular to K and which contains the origin.

Find the equation of N.

Investigate if b is the image of a under the axial symmetry in N.

3. **(a)** The equation of a circle is

$$x^2 + y^2 = 49.$$

Write down

  (i) its radius length

  (ii) the co-ordinates of the points where it intersects the x-axis.

(b) Prove that line $x - 2y + 10 = 0$ is a tangent to the circle whose equation is $x^2 + y^2 = 20$.

(c) C is the circle with centre $(-1, 2)$ and radius 5.
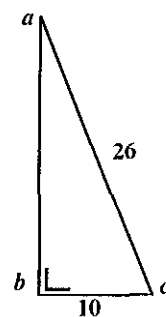
Write down the equation of C.

The circle K has equation

$$(x - 8)^2 + (y - 14)^2 = 100.$$

Prove that the point $p(2, 6)$ is on C and on K.

Show that p lies on the line which joins the centres of the two circles.

OVER→

4. **(a)** Find the area of triangle abc if $|\angle abc| = 90°$,

$|ac| = 26$ and $|bc| = 10$.

(b) Prove that the products of the lengths of the sides of a triangle by the corresponding altitudes are equal.

(c) The triangle odc is the image of the triangle oab under an enlargement, centre o.
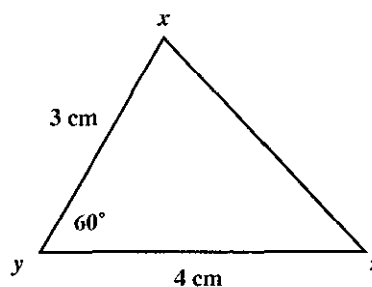$|cd| = 9$ and $|ab| = 15$.

   (i) Find the scale factor of the enlargement.

   (ii) If the area of triangle oab is 87.5 square units, find the area of triangle odc.

   (iii) Write down the area of the region $abcd$.
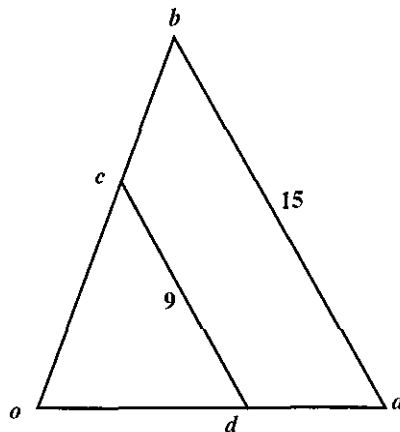
5. **(a)** In the triangle xyz, $|xy| = 3$ cm, $|yz| = 4$ cm and $|\angle xyz| = 60°$.

Use the cosine rule to find $|xz|$, correct to one place of decimals.

[See Tables, page 9.]

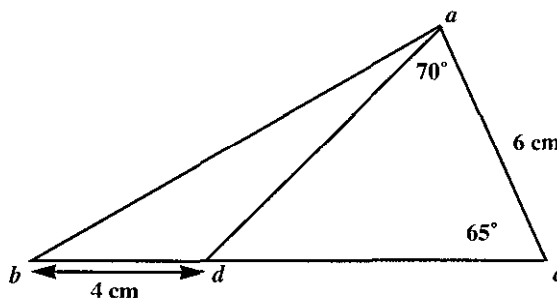**(b)** θ is an acute angle where $\tan \theta = \dfrac{5}{12}$.

Find, as a fraction, the value of

   (i) $\cos \theta$

   (ii) $\sin \theta$

   (iii) $\cos 2\theta$.   [Note: $\cos 2\theta = \cos(\theta + \theta)$.]

(c) abc is a triangle and $d \in [bc]$, as shown.

If $|bd| = 4$ cm, $|ac| = 6$ cm, $|\angle acd| = 65°$ and $|\angle dac| = 70°$, find

   (i) $|dc|$, correct to the nearest cm

   (ii) the area of triangle abc, correct to the nearest cm^2.

6. **(a)** A class of 29 students wins a prize. Two members of the class are chosen to receive the prize. How many different pairs of students can be chosen?

(b) **(i)** In how many different ways can the letters of the word CARPET be arranged?

(ii) How many of these arrangements begin with A?

(iii) In how many of the arrangements do the two vowels come together?

(c) Two people are chosen at random from a large crowd. Each person names the day of the week on which he or she was born. Assuming that each day is equally likely, what is the probability that

(i) both people were born on a Friday

(ii) one person was born on a Tuesday and the other was born on a Thursday

(iii) the two people were born on different days?

7. **(a)** The table shows the distribution of ages of a group of 100 people.

Age (in years)	0–10	10–20	20–30	30–50	50–80
Number of people	10	19	25	30	16

[Note that 10–20 means that 10 is included but 20 is not, etc.]

Taking 5, 15, etc. as mid-interval values, estimate the mean age of the people in the group.

(b) $\{ 2, 5, 6, 4.5, 2.5 \}$

Show that 4 is the mean of this set of numbers.

Then, calculate the standard deviation, correct to one place of decimals.

(c) A new shop opened at 0900 hours. During the first hour of trading, customers were counted as they entered the shop. The following cumulative frequency table shows the number of customers who had entered before the given times:

Time	Before 0910 hours	Before 0920 hours	Before 0930 hours	Before 0940 hours	Before 0950 hours	Before 1000 hours
No. of Customers who had entered	45	69	95	120	144	250

(i) Draw a cumulative frequency curve.

(ii) A photograph was taken of the 100th customer as he or she entered the shop.

Use your curve to estimate the time at which the photograph was taken.

(iii) Use your curve to estimate the number of customers who entered the shop during the 15 minutes immediately after the photograph was taken.

8. (a) In the diagram, *o* is the centre of the circle.
If | *dc* | = 6 cm, | *ce* | = 3 cm and
| *xc* | = 9 cm, find
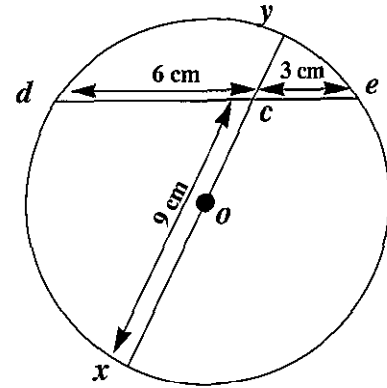
(i) | *cy* |
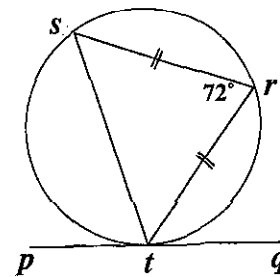
(ii) the length of the radius of the circle.

(b) Prove that an angle between a tangent *ak* and a chord [*ab*] of a circle has degree-measure equal to that of any angle in the alternate segment.

(c) (i) The line *pq* is a tangent to the circle at *t*.

| *rs* | = | *rt* | and | ∠*srt* | = 72°.

Find | ∠*stp* |.

Find | ∠*rtq* |.

(ii) A circle, with centre *o*, has tangents *uv* and *uw* at *v* and *w*, respectively.

kv is parallel to *wu*. | ∠*zvk* | = 82°.
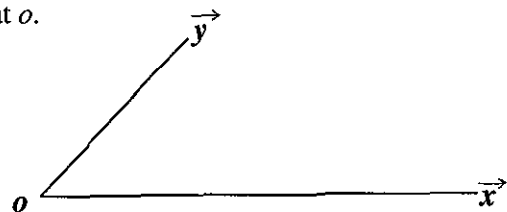
Find | ∠*kwv* |.

Find | ∠*vwu* |.
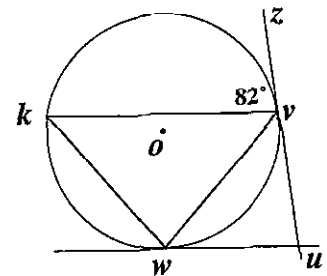
9. (a) The diagram shows $\vec{x}$ and $\vec{y}$ with respect to the origin at *o*.
Copy the diagram and show on it $\vec{r}$ and $\vec{s}$ such that
$$\vec{r} = -\vec{x}$$
$$\vec{s} = 2\vec{y}.$$

(b) Let $\vec{p} = 3\vec{i} + \vec{j}$ and $\vec{q} = 2\vec{i} - 3\vec{j}.$

(i) Express $\vec{p} + \vec{q}$ in terms of $\vec{i}$ and $\vec{j}.$

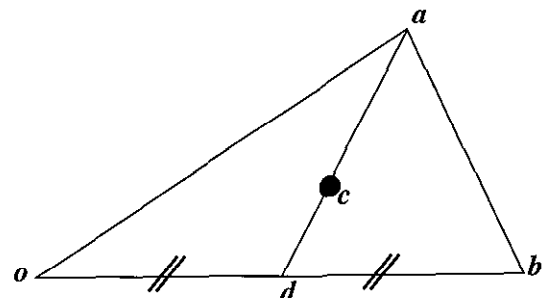(ii) Calculate $| \vec{p} + \vec{q} |.$

(iii) Calculate $\vec{p}.\vec{q}.$

(c) *abo* is a triangle where *o* is the origin.

$| od | = | db |$ and $| cd | = \frac{1}{2} | ac |.$

Express (i) $\vec{d}$ in terms of $\vec{b}$

(ii) $\vec{dc}$ in terms of $\vec{da}$

(iii) $\vec{c}$ in the form $k(\vec{a} + \vec{b})$ where $k \, \varepsilon \, \mathbf{Q}.$

OVER→

10. (a) Find the sum to infinity of the geometric series

$$1 \; + \; \frac{1}{2} \; + \; \frac{1}{4} \; + \; \frac{1}{8} \; + \; \dots\dots\dots$$

(b) Expand $(1 + x)^5$ in ascending powers of x.

By putting $x = 0.01$, use the expansion to show that $(1.01)^5 > 1.05$.

(c) A company invested IR£10 000 in new machinery at the beginning of each year for three consecutive years. The machinery depreciated at the rate of 10% per annum.

Find

(i) the value of the first investment of IR£10 000 at the end of the third year

(ii) the total value of all the investments at the end of the third year.

11. (a) On one diagram, illustrate the set of points (x, y) that satisfy the three inequalities

$$x + y \leq 7$$
$$2x + y \geq 8$$
$$x \geq 0.$$

(b) A factory, which manufactures television sets makes two types of set – a wide screen model and a standard model.

In any week, 500 sets at most can be manufactured.

Each wide screen model costs IR£200 to produce. Each standard model costs IR£150 to produce. Total weekly production costs must not be greater than IR£90 000.

(i) If the factory manufactures x of the wide screen model and y of the standard model, write down two inequalities in x and y and illustrate these on graph paper.

(ii) If the profit on a wide screen model is IR£100 and the profit on a standard model is IR£70, how many of each type of set should be manufactured in order to maximise profit?