AN ROINN OIDEACHAIS M. 27

LEAVING CERTIFICATE EXAMINATION, 1996

MATHEMATICS — ORDINARY LEVEL — PAPER I (300 marks)

THURSDAY, 6 JUNE — MORNING, 9.30 to 12.00

Attempt **SIX QUESTIONS** (50 marks each)

Marks may be lost if all your work is not clearly shown or if you do not indicate where a calculator has been used.

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1. **(a)** Express 250 m as a fraction of 1 km.

(b) A tanker delivered heating oil to a school. Before the delivery the meter reading showed 11 360 litres of oil in the tanker. After the delivery, the meter reading was 7160 litres.

Calculate the cost of the oil delivered if 1 litre of oil cost 20.5p.

When VAT was added to the cost of the oil delivered, the bill to the school amounted to IR£1041.81.
Calculate the rate of VAT added.

(c) **(i)** Calculate the value of

$$\frac{5.1 \times 10^8 + 19 \times 10^7}{1.4 \times 10^{12}}$$

and write your answer as a decimal number.

(ii) Calculate the percentage error if 5 is taken as an approximation for 4.95. Give your answer correct to two places of decimals.

2. **(a)** Solve

$$2x - y = 7$$
$$x + 2y = 6.$$

(b) Write as a power of 2

(i) 16

(ii) $\sqrt{8}$.

Solve for x the equation

$$2^{2x-1} = \left(\frac{16}{\sqrt{8}}\right)^3.$$

(c) Solve

$$\frac{x-1}{x} - \frac{3x}{x-1} = 2, \quad x \neq 0 \text{ and } x \neq 1.$$

OVER →

3. **(a)** Express q in terms of p and t when

$$2(p - 3q) = t.$$

(b) Find the roots of the equation

$$2x^3 - 5x^2 + x + 2 = 0.$$

(c) Let $f(x) = (1 - x)(2 + x), \quad x \in \mathbf{R}$.

Write down the solutions of $f(x) = 0$.

Find the range of values of x for which $f(x) > 0$.

Let $g(x) = f(x) - f(x + 1)$.

Express $g(x)$ in the form $ax + b, \quad a, b \in \mathbf{R}$.

Find the solution set of $g(x) < 0$.

4. **(a)** Let $z = 1 - 4i$, where $i^2 = -1$.
Plot z and $2 + z$ on an Argand diagram.

(b) Let $w = (1 - 3i)(2 + i)$.
Express w in the form $p + qi, \quad p, q \in \mathbf{R}$.

Verify that

$$| w + \overline{w} | = | w - \overline{w} |,$$

where $\overline{w}$ is the complex conjugate of w.

For what value of a is

$$\frac{\overline{w}}{2i} = aw,$$

where $a \in \mathbf{R}$?

(c) Let $z = 2 - i$ be one root of the equation $z^2 + pz + q = 0, \quad p, q \in \mathbf{R}$.
Find the value of p and the value of q.

5. (a) The first two terms of an arithmetic series are given as

$$2 + 8 + \ldots\ldots$$

Find

(i) d, the common difference
(ii) T_{10}, the tenth term
(iii) the value of n such that $T_n = 200$
(iv) S_{16}, the sum to 16 terms.

(b) The n-th term, T_n, of a geometric series is

$$T_n = 3^{n-1}.$$

Find

(i) T_1, the first term
(ii) r, the common ratio
(iii) S_n, the sum to n terms.

Investigate if

$$2S_n - T_n = 2T_n - 1.$$

6. (a) Let $f(x) = 3x + k$, $x \in \mathbf{R}$.
If $f(5) = 0$, find the value of k.

(b) Let $g(x) = x^2 + bx + c$, $x \in \mathbf{R}$.

The solutions of $g(x) = 0$ are symmetrical about the line $x = 1$.
If $x = -3$ is one solution of $g(x) = 0$, find the other solution.

Find the value of b and the value of c.

(c) Let $f(x) = \dfrac{1}{x - 2}$, for $x \in \mathbf{R}$ and $x \neq 2$.

Find the derivative of $f(x)$.

Tangents to $f(x)$ make an angle of 135° with the x axis.
Find the coordinates of the points on the curve of $f(x)$ at which this occurs.

OVER →

7. (a) Differentiate from first principles

$$3x - 7$$

with respect to x.

(b) (i) Find $\frac{dy}{dx}$ when $y = \frac{2x}{4 - x^2}$, for $x \in \mathbf{R}$ and $x \neq \pm 2$.

Show that $\frac{dy}{dx} > 0$.

(ii) Differentiate $\left(x^5 - \frac{1}{x^2}\right)^7$ with respect to x, $x \neq 0$.

(c) A stone is dropped from a height of 80 metres. Its height h metres above the ground after t seconds is given by

$$h = 80 - 10t^2.$$

Find

(i) its speed after t seconds

(ii) its speed after 2.5 seconds

(iii) the time it takes to fall the first 14.4 metres.

8. (a) Find $\frac{ds}{dt}$ when $s = 6t^2 - 3t + 7$.

(b) Let $f(x) = x^3 - 3x^2$, for $x \in \mathbf{R}$.

(i) Find $f'(x)$, the derivative of $f(x)$. Hence, calculate the coordinates of the local maximum and the local minimum of $f(x)$.

(ii) Draw the graph of

$$f(x) = x^3 - 3x^2$$

for $-1 \leq x \leq 3$.

(iii) Use your graph to estimate the values of x for which

$$f(x) + 2 = 0.$$

(iv) Use your graph to estimate the range of values of x for which

$$f'(x) < 0.$$