Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2015

Mathematics

Paper 2

Higher Level

Monday 8 June Morning 9:30 – 12:00

300 marks

Examination number

Centre stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A — Concepts and Skills — 150 marks

Answer **all six** questions from this section.

Question 1 (25 marks)

An experiment consists of throwing two fair, standard, six-sided dice and noting the sum of the two numbers thrown. If the sum is 9 or greater it is recorded as a "win" (W). If the sum is 8 or less it is recorded as a "loss" (L).

(a) Complete the table below to show all possible outcomes of the experiment.

		Die 2					
		1	2	3	4	5	6
Die 1	1		L				
	2						
	3						
	4						
	5						W
	6						

(b) **(i)** Find the probability of a win on one throw of the two dice.

(ii) Find the probability that each of 3 successive throws of the two dice results in a loss. Give your answer correct to four decimal places.

(c) The experiment is repeated until a total of 3 wins occur. Find the probability that the third win occurs on the tenth throw of the two dice. Give your answer correct to four decimal places.

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Question 2 **(25 marks)**

A survey of 100 shoppers, randomly selected from a large number of Saturday supermarket shoppers, showed that the mean shopping spend was €90·45. The standard deviation of this sample was €20·73.

(a) Find a 95% confidence interval for the mean amount spent in a supermarket on that Saturday.

(b) A supermarket has claimed that the mean amount spent by shoppers on a Saturday is €94. Based on the survey, test the supermarket's claim using a 5% level of significance. Clearly state your null hypothesis, your alternative hypothesis, and your conclusion.

(c) Find the *p*-value of the test you performed in part **(b)** above and explain what this value represents in the context of the question.

p-value:

Explanation:

Question 3 **(25 marks)**

(a) The co-ordinates of two points are $A(4, -1)$ and $B(7, t)$.

The line $l_1 : 3x - 4y - 12 = 0$ is perpendicular to AB. Find the value of t.

(b) Find, in terms of k, the distance between the point $P(10, k)$ and l_1.

(c) $P(10, k)$ is on a bisector of the angles between the lines l_1 and $l_2 : 5x + 12y - 20 = 0$.

(i) Find the possible values of k.

(ii) If $k > 0$, find the distance from P to l_1.

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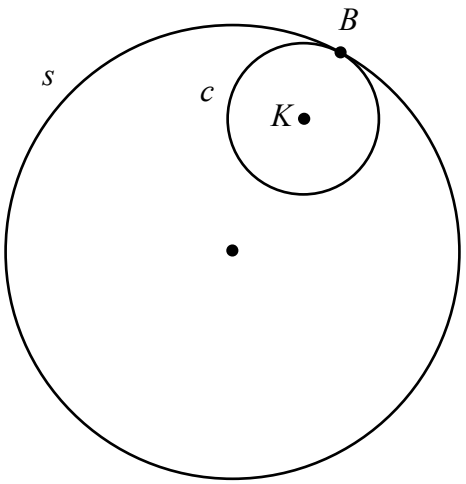
Question 4 **(25 marks)**

Two circles s and c touch internally at B, as shown.



(a) The equation of the circle s is $(x-1)^2+(y+6)^2=360$.

Write down the co-ordinates of the centre of s.

Centre:_____

Write down the radius of s in the form $a\sqrt{10}$, where $a \in \mathbb{N}$.

Radius:_____

(b) **(i)** The point K is the centre of circle c.
The radius of c is one-third the radius of s.
The co-ordinates of B are (7, 12).
Find the co-ordinates of K.

(ii) Find the equation of c.

(c) Find the equation of the common tangent at B.
Give your answer in the form $ax+by+c=0$, where $a, b, c \in \mathbb{Z}$.

Question 5 **(25 marks)**

(a) Prove that $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$.

(b) Find all the values of x for which $\sin(3x) = \dfrac{\sqrt{3}}{2}$, $0 \le x \le 360$, x in degrees.

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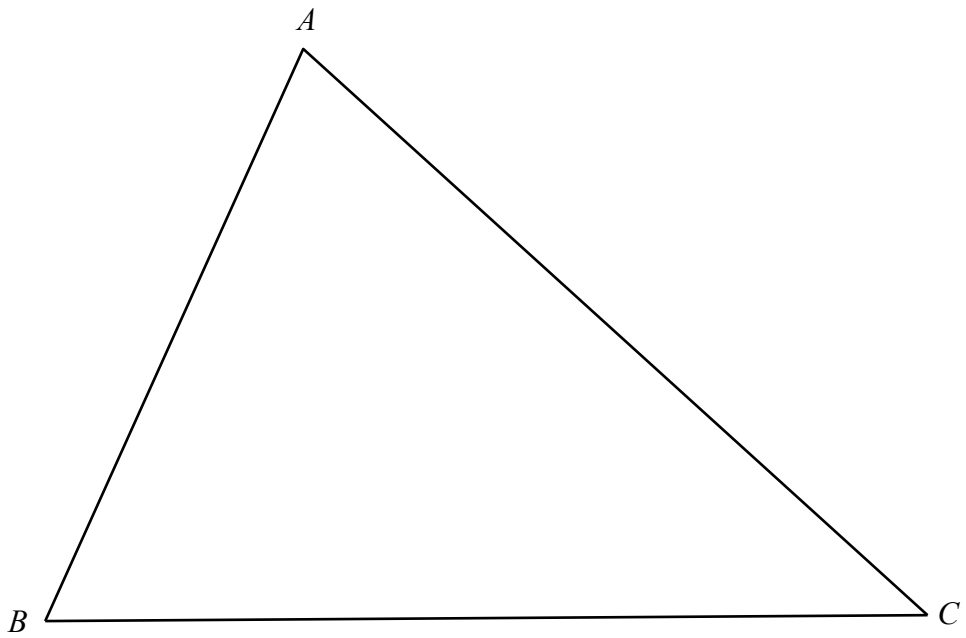
Question 6 **(25 marks)**

(a) Construct the centroid of the triangle *ABC* below. Show all construction lines.
(Where measurement is used, show all relevant measurements and calculations clearly.)



(b) Prove that, if three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal line.

Diagram:

Given:

To Prove:

Construction:

Proof:

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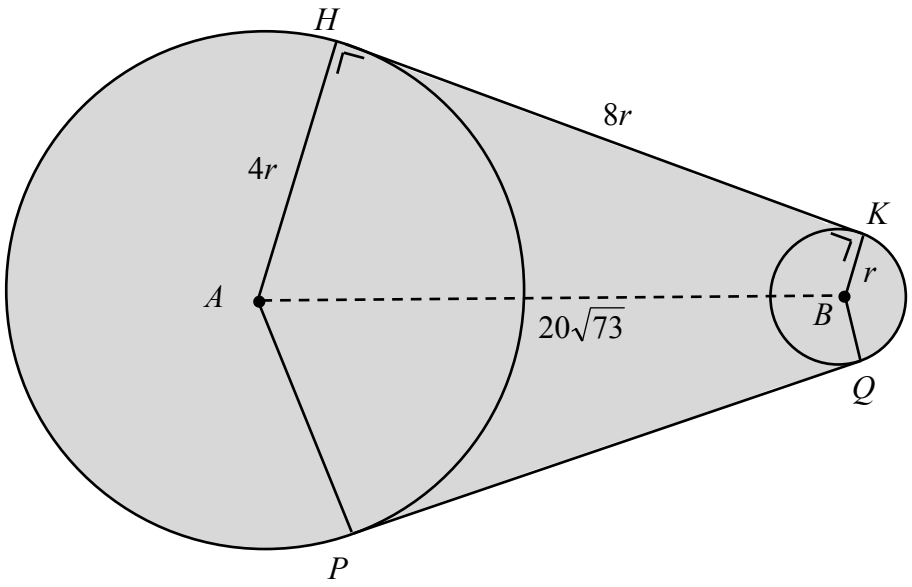
Section B — Contexts and Applications — 150 marks

Answer **all three** questions from this section.

Question 7 (40 marks)

A flat machine part consists of two circular ends attached to a plate, as shown (diagram not to scale).
The sides of the plate, HK and PQ, are tangential to each circle.
The larger circle has centre A and radius $4r$ cm.
The smaller circle has centre B and radius r cm.
The length of $[HK]$ is $8r$ cm and $|AB| = 20\sqrt{73}$ cm.



(a) Find r, the radius of the smaller circle. (Hint: Draw $BT \parallel KH$, $T \in AH$.)

(b) Find the area of the quadrilateral $ABKH$.

(c) **(i)** Find $|\angle HAP|$, in degrees, correct to one decimal place.

(ii) Find the area of the machine part, correct to the nearest cm^2.

Question 8 **(65 marks)**

In basketball, players often have to take free throws. When Michael takes his first free throw in any game, the probability that he is successful is 0·7.
For all subsequent free throws in the game, the probability that he is successful is:

- 0·8 if he has been successful on the previous throw
- 0·6 if he has been unsuccessful on the previous throw.

(a) Find the probability that Michael is successful (S) with all three of his first three free throws in a game.

P(S, S, S) =

(b) Find the probability that Michael is unsuccessful (U) with his first two free throws and successful with the third.

P(U, U, S) =

(c) List all the ways that Michael could be successful with his third free throw in a game and hence find the probability that Michael is successful with his third free throw.

(d) **(i)** Let p_n be the probability that Michael is successful with his n^{th} free throw in the game (and hence $(1-p_n)$ is the probability that Michael is unsuccessful with his n^{th} free throw). Show that $p_{n+1} = 0{\cdot}6 + 0{\cdot}2p_n$.

(ii) Assume that p is Michael's success rate in the long run; that is, for large values of n, we have $p_{n+1} \approx p_n \approx p$.
Using the result from part **(d) (i)** above, or otherwise, show that $p = 0{\cdot}75$.

(e) For all positive integers n, let $a_n = p - p_n$, where $p = 0{\cdot}75$ as above.

(i) Use the ratio $\frac{a_{n+1}}{a_n}$ to show that a_n is a geometric sequence with common ratio $\frac{1}{5}$.

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(ii) Find the smallest value of n for which $p - p_n < 0{\cdot}00001$.

(f) You arrive at a game in which Michael is playing. You know that he has already taken many free throws, but you do not know what pattern of success he has had.

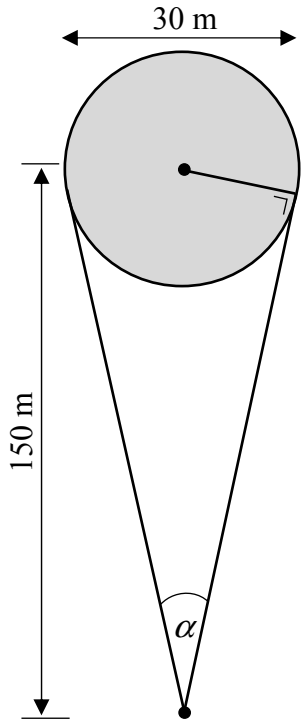
(i) Based on this knowledge, what is your estimate of the probability that Michael will be successful with his next free throw in the game?

Answer: _____

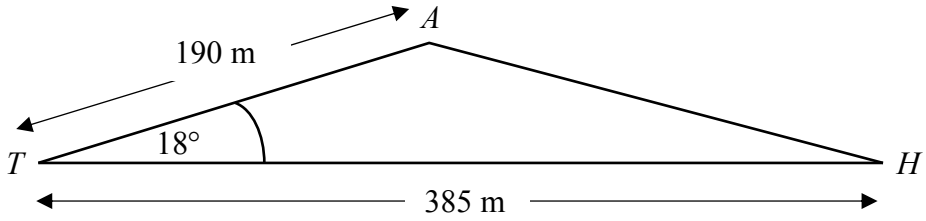
(ii) Why would it **not** be appropriate to consider Michael's subsequent free throws in the game as a sequence of Bernoulli trials?

Question 9 **(45 marks)**

(a) Joan is playing golf. She is 150 m from the centre of a circular green of diameter 30 m. The diagram shows the range of directions in which Joan can hit the ball so that it could land on the green. Find α, the measure of the angle of this range of directions. Give your answer, in degrees, correct to one decimal place.



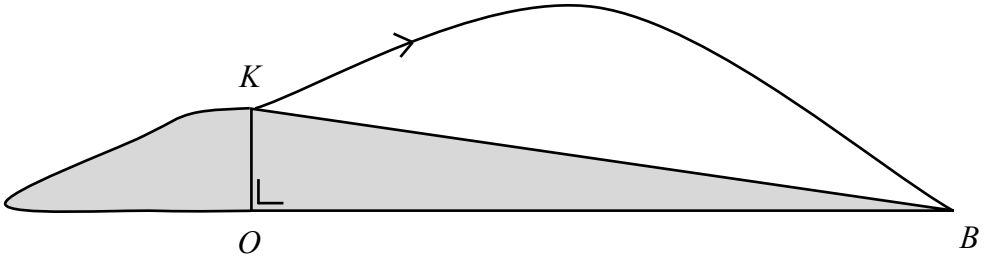
(b) At the next hole, Joan, at T, attempts to hit the ball in the direction of the hole H. Her shot is off target and the ball lands at A, a distance of 190 metres from T, where $|\angle ATH| = 18°$. $|TH|$ is 385 metres. Find $|AH|$, the distance from the ball to the hole, correct to the nearest metre.



(c) At another hole, where the ground is not level, Joan hits the ball from K, as shown. The ball lands at B. The height of the ball, in metres, above the horizontal line OB is given by

$$h = -6t^2 + 22t + 8$$

where t is the time in seconds after the ball is struck and h is the height of the ball.



(i) Find the height of K above OB.

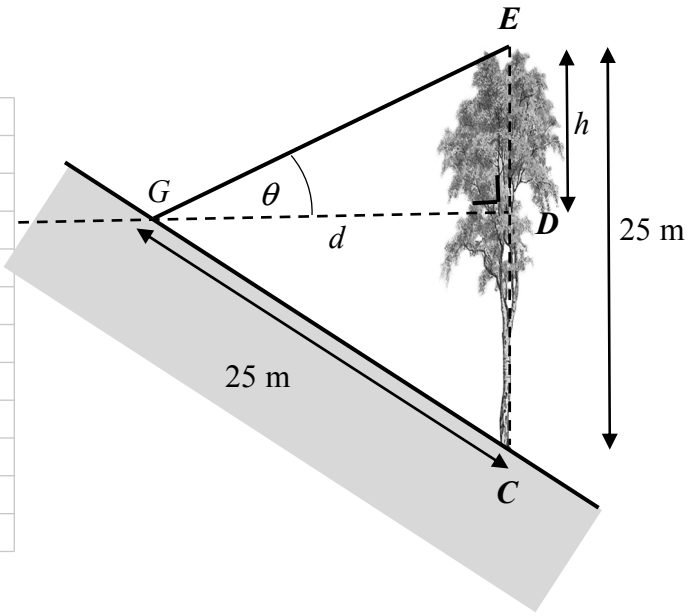
(ii) The horizontal speed of the ball over the straight distance $[OB]$ is a constant 38 m s^{-1}. Find the angle of elevation of K from B, correct to the nearest degree.

(d) At a later hole, Joan's first shot lands at the point G, on ground that is sloping downwards, as shown. A vertical tree, $[CE]$, 25 metres high, stands between G and the hole. The distance, $|GC|$, from the ball to the bottom of the tree is also 25 metres.

The angle of elevation at G to the top of the tree, E, is θ, where $\theta = \tan^{-1}\frac{1}{2}$.

The height of the top of the tree above the horizontal, GD, is h metres and $|GD| = d$ metres.

(i) Write d and $|CD|$ in terms of h.



$d =$

$|CD| =$

(ii) Hence, or otherwise, find h.

You may use this page for extra work.

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