Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2009

MATHEMATICS – HIGHER LEVEL

PAPER 1 (300 marks)

FRIDAY, 5 JUNE – MORNING, 9.30 to 12.00

Attempt **SIX QUESTIONS** (50 marks each).

WARNING: **Marks will be lost if all necessary work is not clearly shown.**

Answers should include the appropriate units of measurement, where relevant.

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1. **(a)** Find the value of $\frac{x}{y}$ when $\frac{2x+3y}{x+6y}=\frac{4}{5}$.

(b) Let $f(x)=x^2-7x+12$.

(i) Show that if $f(x+1)\neq 0$, then $\frac{f(x)}{f(x+1)}$ simplifies to $\frac{x-4}{x-2}$.

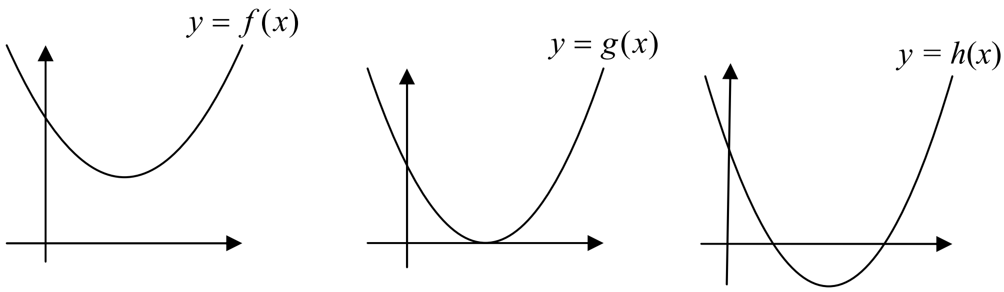
(ii) Find the range of values of x for which $\frac{f(x)}{f(x+1)}>3$.

(c) Given that $x-c+1$ is a factor of $x^2-5x+5cx-6b^2$, express c in terms of b.

2. **(a)** Solve the simultaneous equations

$$x-y+8=0$$
$$x^2+xy+8=0.$$

(b) **(i)** The graphs of three quadratic functions, f, g and h, are shown.



In each case, state the nature of the roots of the function.

(ii) The equation $kx^2+(1-k)x+k=0$ has equal real roots.
Find the possible values of k.

(c) **(i)** One of the roots of $px^2+qx+r=0$ is n times the other root.
Express r in terms of p, q and n.

(ii) One of the roots of $x^2+qx+r=0$ is five times the other.
If q and r are positive integers, determine the set of possible values of q.

3. **(a)** $z_1 = a + bi$ and $z_2 = c + di$, where $i^2 = -1$.

Show that $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$, where $\bar{z}$ is the complex conjugate of z.

(b) Let $A = \frac{1}{2}\begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$.

(i) Express A^3 in the form $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, where $a, b \in \mathbf{Z}$.

(ii) Hence, or otherwise, find A^{17}.

(c) **(i)** Use De Moivre's theorem to prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

(ii) Hence, find $\int \sin^3\theta \, d\theta$.

4. **(a)** Three consecutive terms of an arithmetic series are $4x + 11$, $2x + 11$, and $3x + 17$. Find the value of x.

(b) **(i)** Show that $\dfrac{2}{r^2 - 1} = \dfrac{1}{r - 1} - \dfrac{1}{r + 1}$, where $r \neq \pm 1$.

(ii) Hence, find $\displaystyle\sum_{r=2}^{n} \frac{2}{r^2 - 1}$.

(iii) Hence, evaluate $\displaystyle\sum_{r=2}^{\infty} \frac{2}{r^2 - 1}$.

(c) A finite geometric sequence has first term a and common ratio r. The sequence has $2m + 1$ terms, where $m \in \mathbf{N}$.

(i) Write down the last term, in terms of a, r, and m.

(ii) Write down the middle term, in terms of a, r, and m.

(iii) Show that the product of all of the terms of the sequence is equal to the middle term raised to the power of the number of terms.

5. **(a)** Solve for x: $x - 2 = \sqrt{3x - 2}$.

(b) Prove by induction that, for all positive integers n, 5 is a factor of $n^5 - n$.

(c) Solve the simultaneous equations

$$\log_3 x + \log_3 y = 2$$

$$\log_3 (2y - 3) - 2\log_9 x = 1 \ .$$

6. **(a)** Differentiate $\sin(3x^2 - x)$ with respect to x.

(b) **(i)** Differentiate $\sqrt{x}$ with respect to x, from first principles.

(ii) An object moves in a straight line such that its distance from a fixed point is given by $s = \sqrt{t^2 + 1}$, where s is in metres and t is in seconds.

Find the speed of the object when $t = 5$ seconds.

(c) The equation of a curve is $y = \dfrac{2}{x - 3}$.

(i) Write down the equations of the asymptotes and hence sketch the curve.

(ii) Prove that no two tangents to the curve are perpendicular to each other.

7. **(a)** The equation of a curve is $x^2 - y^2 = 25$. Find $\dfrac{dy}{dx}$ in terms of x and y.

(b) A curve is defined by the parametric equations

$$x = \frac{3t}{t^2 - 2} \text{ and } y = \frac{6}{t^2 - 2}, \text{ where } t \neq \pm\sqrt{2}.$$

(i) Find $\dfrac{dy}{dx}$ in terms of t.

(ii) Find the equation of the tangent to the curve at the point given by $t = 2$.

(c) The function $f(x) = x^3 - 3x^2 + 3x - 4$ has only one real root.

(i) Show that the root lies between 2 and 3.

Anne and Barry are each using the Newton-Raphson method to approximate the root. Anne is starting with 2 as a first approximation and Barry is starting with 3.

(ii) Show that Anne's starting approximation is closer to the root than Barry's. (That is, show that the root is less than 2·5.)

(iii) Show, however, that Barry's next approximation is closer to the root than Anne's.

8. **(a)** Find $\displaystyle\int\left(6x + 3 + \frac{1}{x^2}\right)dx$.

(b) Evaluate **(i)** $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin 3x \sin x \, dx$ **(ii)** $\displaystyle\int_{\ln 3}^{\ln 8} e^x\sqrt{1 + e^x}\, dx$.

(c) Use integration methods to establish the standard formula for the volume of a cone.

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