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M. 30

24518

LEAVING CERTIFICATE EXAMINATION, 1997

MATHEMATICS - HIGHER LEVEL

PAPER 2 (300 marks)

FRIDAY, 13 JUNE – MORNING, 9·30 to 12·00

Attempt **five** questions from Section A and **one** question from Section B.
Each question carries 50 marks.

Marks may be lost if necessary work is not shown or if you do not indicate where a calculator has been used.

SECTION A

1. *(a)* The equation of a circle is

$$(x + 7)(x + 3) + (y - 2)(y + 2) = 0.$$

Find the centre and radius length of the circle.

(b) Prove that the equation of the tangent to the circle $x^2 + y^2 = r^2$ at the point (x_1, y_1) on the circle is

$$xx_1 + yy_1 = r^2.$$

(c) The x axis is a tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$.

Show that

$$g^2 = c.$$

The x axis is a tangent to a circle K at the point (3, 0).

The point $(-1, 4) \in$ K.

Find the equation of K.

OVER→

2. *(a)* $oabc$ is a parallelogram where o is the origin.

If $\vec{a} = 6\vec{i} - 2\vec{j}$ and $\vec{b} = 2\vec{i} - 5\vec{j}$, express $\vec{c}$ in terms of $\vec{i}$ and $\vec{j}$.

(b) $\vec{p} = 2\vec{i} + 3\vec{j}$ and $\vec{p}^{\perp}$ is its related vector $-3\vec{i} + 2\vec{j}$.

Let $\vec{q} = \vec{p}^{\perp} - \vec{p}$ and $\vec{r} = \vec{q} + \vec{q}^{\perp}$.

(i) Express $\vec{q}$ and $\vec{r}$ in terms of $\vec{i}$ and $\vec{j}$.

(ii) Find the measure of the angle between $\vec{q}$ and $\vec{r}$.

(c) $\vec{o}$, $\vec{x}$ and $\vec{y}$ are non-collinear vectors where o is the origin.

(i) Show $\vec{x}.\vec{x} = |\vec{x}|^2$ and $\vec{x}.\vec{y} = \vec{y}.\vec{x}$.

(ii) If $|\vec{x} + \vec{y}| = |\vec{x} - \vec{y}|$, prove that $\vec{x} \perp \vec{y}$.

3. *(a)* A triangle has vertices $(1, -1)$, $(5, 1)$ and $(-\frac{5}{2}, -5)$.
Find the area of the triangle.

(b) K_1 and K_2 are two lines with slopes m_1 and m_2, respectively.
If θ is an angle between K_1 and K_2, prove that

$$\tan\theta = \pm \frac{m_1 - m_2}{1 + m_1 m_2}.$$

(c) f is the transformation $(x, y) \rightarrow (x', y')$ where

$$x' = 4x - y$$
$$y' = 2x + y.$$

For the points $a(0,0)$, $b(-2, -5)$ and $c(4, 9)$, find $f(a)$, $f(b)$ and $f(c)$.

(i) L is the line ac. The image of L under f is the line f(L).
Find the equation of f(L).

(ii) f(M) is the image of the line M under f.
f(M) is perpendicular to f(L) and $f(b) \in f$(M).
Find the equation of the line M.

4. *(a)* Find the value of θ for which

$$\tan\theta = -\frac{1}{\sqrt{3}}$$

where $0° \leq \theta \leq 180°$.

(b) In a triangle pqr, $|\angle pqr| = 30°$, $|qr| = 15$ and $|rp| = 5\sqrt{3}$.

Find two values for $|\angle qpr|$ and sketch the two resulting triangles.

Calculate the ratio of the areas of the two triangles.

(c) Show, using the formula for $\sin(A + B)$, that
$$\sin 2A = 2\sin A\cos A.$$

Using Tables p.9, or otherwise, show that
$$\sin 3A = 3\sin A - 4\sin^3 A.$$

Use the result for $\sin 3A$, or otherwise, to show that
$$\sin 3(A - \frac{\pi}{2}) = 4\cos^3 A - 3\cos A.$$

5. *(a)* Evaluate $\lim_{x \to 0} \frac{\sin 3x}{x}$.

(b) (i) Express $\sin 5x + \sin 3x$ as a product of sine and cosine.

(ii) Find all the solutions of the equation
$$\sin 5x + \sin 3x = 0$$
in the domain $0° \leq x \leq 180°$.

(c) A triangle has sides of length a, b and c with A being the angle opposite the side of length a.

Derive the formula for a^2 in terms of b, c and A.

When $90° < A < 180°$ prove that
$$a^2 > b^2 + c^2.$$

OVER→

6. (a) How many different four digit numbers greater than 5000 can be formed from the digits 2, 4, 5, 8, 9 if each digit can be used only once in any given number ?
How many of these numbers are odd ?

(b) Solve the difference equation

$$u_{n+2} - 4u_{n+1} + u_n = 0, \; n \geq 0$$

where $u_0 = 4$ and $u_1 = 8$.

(c) The following data give the age and gender of twenty five pupils in a class on a given day:

	Boys	Girls
Number of pupils aged sixteen years	5	7
Number of pupils aged seventeen years	7	6

(i) One of the pupils is picked at random. What is the probability that a boy aged sixteen years or a girl aged seventeen years is picked ?

(ii) Each pupil in the class is given his/her examination results. Only three pupils scored full marks. Determine the probability that these three pupils are of the same age and the same gender.

7. *(a)* In how many ways can a group of four people be selected from three men and four women ?
In how many of these groups are there more women than men ?

(b) Two persons look at the letters in the word DISCOVERY.
Independently of one another, each person writes down two of the letters from the word DISCOVERY.

What is the probability that

(i) one person writes down two vowels and the other person two consonants ?

(ii) the two persons write down different letters, that is, they have no letters in common ?

(c) The data in the set $\{1, 2, 5, x, y\}$ have a mean of 5.
Express, in terms of x,

(i) y

(ii) σ, the standard deviation of the data.

If the standard deviation is $\sqrt{\frac{99}{10}}$, find the value of x and the value of y.

SECTION B

Answer ONE question from this section.

8. *(a)* Use the ratio test to show that

$$\sum_{n=1}^{\infty} \frac{x^n}{n!}$$

is convergent for all $x \in \mathbf{R}$.

(b) $f(x) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \ldots$ is the Maclaurin series.

Write the first four terms of the Maclaurin series for

$$f(x) = \sqrt{1 + x}.$$

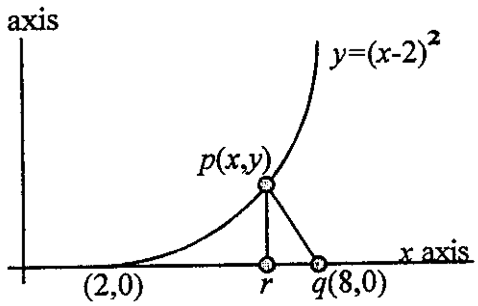
As your expansion converges for $-1 < x < 1$, use it to evaluate $\sqrt{10}$ correct to one place of decimals.

(c) $p(x, y)$ is a point on the curve $y = (x-2)^2$ in the domain $2 < x < 8$.
q is the point $(8, 0)$ and $pr \perp rq$.

Express, in terms of x, the area of the triangle prq.

What value of x maximises the area of triangle prq?

Find the maximum area of triangle prq.



9. *(a)* Four students work separately on a mathematical problem. The probabilities that the four students have of solving the problem are as follows:

$$\frac{3}{4}, \frac{1}{2}, \frac{4}{7}, \frac{2}{3}.$$

Show that the probability that the problem will be solved by at least one of the four students is $\frac{55}{56}$.

(b) The probability of a car having a defective brake light is $\frac{1}{10}$.

In a survey of 150 cars, what is the probability that 20 cars or more have a defective brake light ?

(c) A manufacturer of electronic components employs the following quality control plan. Out of each batch of components, 15 are randomly selected and tested. If 3 or more of the 15 components are found to be defective, the entire batch is rejected.

Find, correct to three decimal places, the probability that a batch will be rejected if $\frac{1}{20}$ th. of its components are defective ?

OVER→

10. *(a)* $\mathbf{Z}, *$ is a group with $x * y = x + y - 2$, where $x, y \in \mathbf{Z}$.

Find

(i) e, the identity element

(ii) x^{-1}, the inverse of x, in terms of x.

(b) Let G = {1, 2, 3, 4, 5, 6}.

(i) Show that G is a group under multiplication modulo 7.
You may assume associativity under multiplication in **N**.

(ii) State the order of each element in G and write down the proper subgroups of G.

(iii) H, $\times$ is a group where H = $\{1, w, w^2\}$ and $w^3 = 1$.
Show that H, $\times$ is isomorphic to a proper subgroup of G.

11. *(a)* Prove that under a similarity transformation the image of any circle is a circle.

(b) Show that the ratios of the lengths of line segments in parallel lines are invariant under every affine transformation.

(c) The tangents from an external point p to a circle S touch the circle at q and r.
Prove that qr is the polar of p.