



**Coimisiún na Scrúduithe Stáit  
State Examinations Commission**

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**LEAVING CERTIFICATE EXAMINATION, 2003**

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**MATHEMATICS — ORDINARY LEVEL**

**PAPER 2 (300 marks)**

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**MONDAY, 9 JUNE — MORNING, 9:30 to 12:00**

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Attempt **FIVE** questions from **Section A** and **ONE** question from **Section B**.  
Each question carries 50 marks.

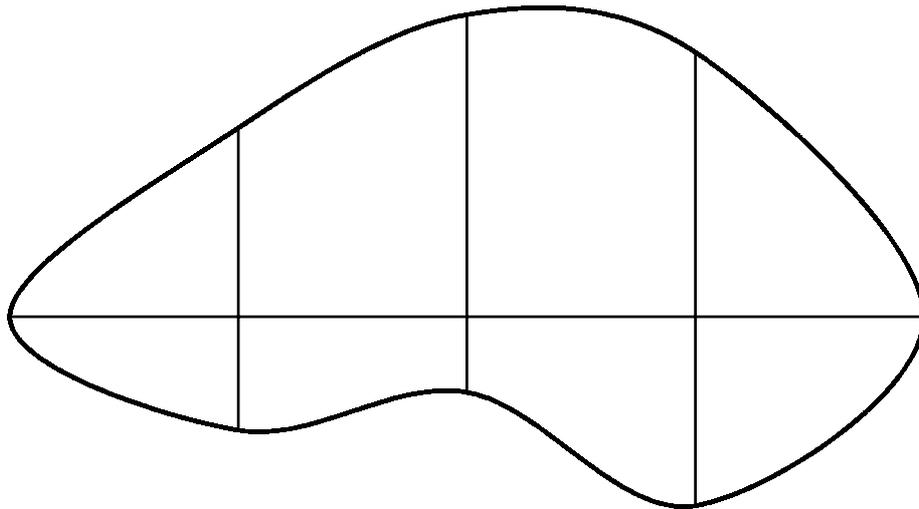
**WARNING: Marks will be lost if all necessary work is not clearly shown.**

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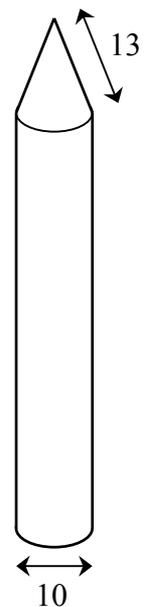
**SECTION A**  
**Attempt FIVE questions from this section.**

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1. (a) A right-angled triangle has sides of length 8 cm, 15 cm and 17 cm. Find its area.
- (b) In order to estimate the area of the irregular shape below, a horizontal line is drawn across the widest part of the shape and three offsets (perpendicular lines) are drawn at equal intervals along this line.

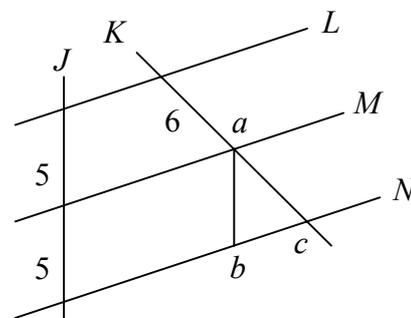


- (i) Measure the horizontal line and the offsets as accurately as you can. Make a rough sketch of the shape in your answerbook and record the measurements on it.
- (ii) Use Simpson's Rule with these measurements to estimate the area of the shape.
- (c) A wax crayon is in the shape of a cylinder of diameter 10 mm, surmounted by a cone of slant height 13 mm.
- (i) Show that the vertical height of the cone is 12 mm.
- (ii) Show that the volume of the cone is  $100\pi \text{ mm}^3$ .
- (iii) Given that the volume of the cylinder is 15 times the volume of the cone, find the volume of the crayon, in  $\text{cm}^3$ , correct to two decimal places.
- (iv) How many complete crayons like this one can be made from 1 kg of wax, given that each  $\text{cm}^3$  of wax weighs 0.75 grammes?



2. (a) Find the distance between the two points (3, 2) and (8, 14).
- (b)  $a(-2, 2)$ ,  $b(4, 6)$  and  $c(0, -4)$  are three points.  
 $p$  is the midpoint of  $[ab]$  and  $q$  is the midpoint of  $[ac]$ .
- (i) Find the co-ordinates of  $p$  and the co-ordinates of  $q$ .
- (ii) Plot  $a$ ,  $b$ ,  $c$ ,  $p$  and  $q$  on a co-ordinate diagram on graph paper.  
 Show the line segments  $[bc]$  and  $[pq]$  on your diagram.
- (iii) Using slopes, or otherwise, prove that  $pq$  is parallel to  $bc$ .
- (c)  $L$  is the line  $3x + 2y + 12 = 0$ .  
 $K$  is the line that passes through the point (7, 3) and is perpendicular to  $L$ .  
 Find the equation of  $K$  and hence find the point of intersection of  $K$  and  $L$ .
3. (a) The circle  $C$  has equation  $x^2 + y^2 = 25$ .
- (i) Verify that the point  $(-4, 3)$  is on the circle  $C$ .
- (ii) Write down the co-ordinates of a point that lies outside  $C$  and give a reason for your answer.
- (b) The line  $x - 2y + 5 = 0$  intersects the circle  $x^2 + y^2 = 10$  at the points  $a$  and  $b$ .
- (i) Find the co-ordinates of  $a$  and the co-ordinates of  $b$ .
- (ii) Draw a co-ordinate diagram on graph paper, showing the line, the circle and the points of intersection.
- (c) The circle  $K$  has equation  $(x + 2)^2 + (y - 3)^2 = 25$ .  
 $p$  and  $q$  are the endpoints of a diameter of  $K$  and  $pq$  is horizontal.
- (i) Find the co-ordinates of  $p$  and the co-ordinates of  $q$ .
- (ii) Hence, or otherwise, write down the equations of the two vertical tangents to  $K$ .
- (iii) Another circle also has these two vertical lines as tangents.  
 The centre of this circle is on the  $x$ -axis.  
 Find the equation of this circle.

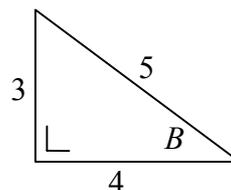
4. (a) In the diagram,  $L$ ,  $M$  and  $N$  are parallel lines. They make intercepts of the indicated lengths on  $J$  and  $K$ .  $ab$  is parallel to  $J$ .



- (i) Write down the length of  $[ab]$ .  
 (ii) Write down the length of  $[ac]$ .

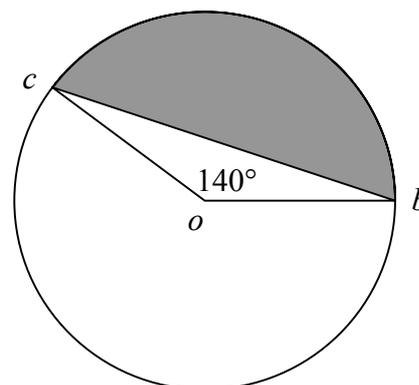
- (b) (i) Prove that the sum of the degree-measures of the angles of a triangle is  $180^\circ$ .  
 (ii) Deduce that the degree-measure of an exterior angle of a triangle is equal to the sum of the degree-measures of the two remote interior angles.
- (c) (i) Construct a triangle  $abc$  in which  $|ab| = 10.5$  cm,  $|bc| = 5$  cm and  $|ac| = 8.5$  cm.  
 (ii) Choose any point  $p$  that is *outside* the triangle and construct the image of  $abc$  under the enlargement of scale factor 0.4 and centre  $p$ .  
 (iii) Given that the area of this image triangle is  $3.36$  cm<sup>2</sup>, calculate the area of the original triangle  $abc$ .

5. (a) The lengths of the sides of a right-angled triangle are shown in the diagram and  $B$  is the angle indicated.



Find the value of  $\sin B \cos B$ , as a fraction.

- (b) A circle has centre  $o$  and radius 7 cm. The two points  $b$  and  $c$  are on the circle and  $|\angle boc| = 140^\circ$ .



- (i) Find the area of the triangle  $obc$ , correct to the nearest cm<sup>2</sup>.  
 (ii) Find the area of the sector  $obc$ , correct to the nearest cm<sup>2</sup>.  
 (iii) Taking the areas correct to the nearest cm<sup>2</sup>, express the area of the shaded region as a fraction of the total area enclosed by the circle. Give your answer as a fraction in its simplest form.

- (c) One side of a triangle has length 8 cm and another has length 3 cm. The angle between these two sides measures  $60^\circ$ .

- (i) Find the length of the third side.  
 (ii) Find the measures of the two remaining angles, correct to the nearest degree.

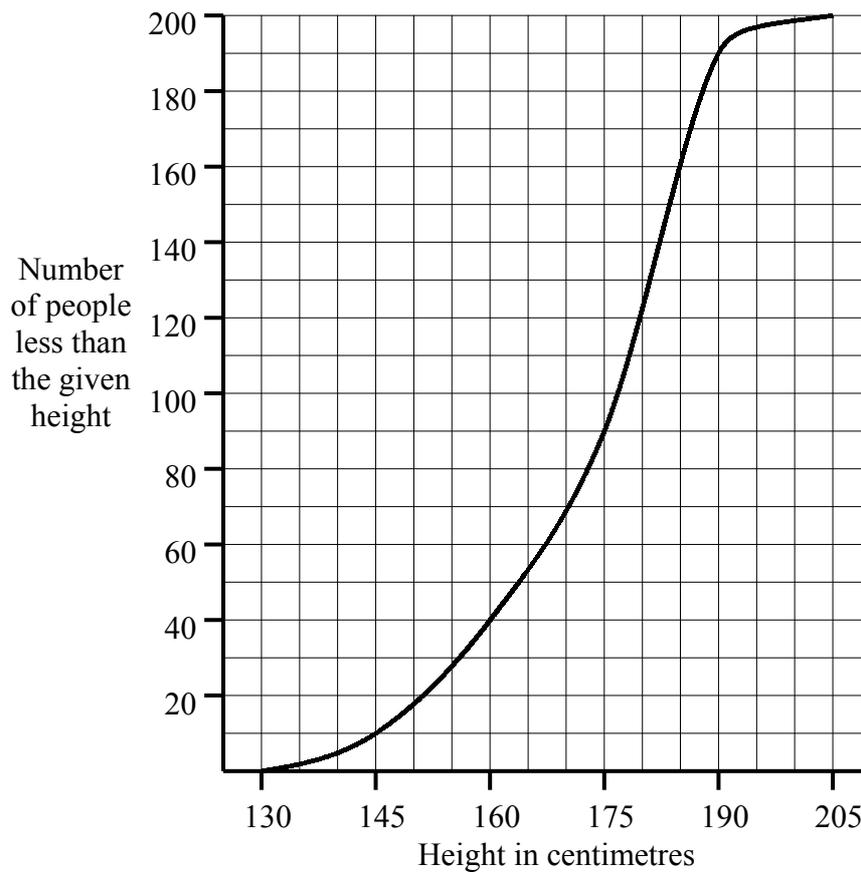
6. (a) I write down today's date as 09062003 and then select one of the digits at random.
- (i) What is the probability that I select the 9?
- (ii) What is the probability that I select an odd digit?
- (b) Two women, Ann and Brid, and two men, Con and Declan, sit in a row for a photograph.
- (i) How many different arrangements of the four people are possible?
- (ii) Write out the four possible arrangements that have the two women in the middle.
- (iii) If the arrangement of the four people is chosen at random from all of the possible arrangements, what is the probability that the two women will be in the middle?
- (c) In a certain school the examination subjects for senior students are grouped as follows:

Compulsory Subjects	Block A	Block B	Block C
Irish English mathematics	French German	biology home economics construction studies accounting	business organisation history physics

As well as taking all three of the compulsory subjects, each student must choose *one* subject from Block A, *two* from Block B and *one* from Block C.

- (i) In choosing two subjects from Block B, how many different selections are possible?
- (ii) In choosing the full range of subjects, how many different selections are possible?
- (iii) One student has already decided to do German and construction studies. How many different selections of the remaining subjects are possible for this student?
- (iv) If the student referred to in part (iii) selects her remaining subjects at random, what is the probability that she will select both biology and physics?

7. (a) The heights of 200 people are recorded to the nearest centimetre. The results are represented by the ogive below.



- (i) Copy the cumulative frequency table below and use the ogive to complete it.

Height	<130	<145	<160	<175	<190	<205
Number of people	0					

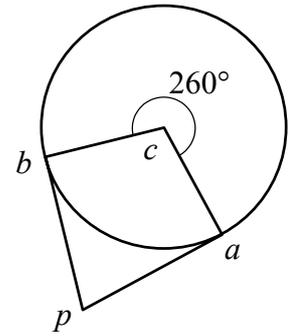
- (ii) Hence, copy and complete the following grouped frequency table:

Height	130 - 144	145 - 159	160 - 174	175 - 189	190 - 204
Number of people					

- (iii) Using your grouped frequency table, and taking mid-interval values, find an estimate of the mean height.
- (iv) Use the ogive to estimate the number of people who are taller than the mean.
- (b) (i) The mean of the following five numbers is 10. Find the standard deviation of the numbers.  
7, 9, 10, 11, 13.
- (ii) The mean of the following five numbers is also 10. Find the standard deviation of these numbers.  
5, 7, 9, 13, 16.
- (iii) What does comparing the two standard deviations tell you about the two sets of numbers?

**SECTION B**  
**Attempt ONE question from this section.**

8. (a) In the diagram, the lines  $pa$  and  $pb$  are tangents to the circle at  $a$  and  $b$  respectively.  $c$  is the centre of the circle.

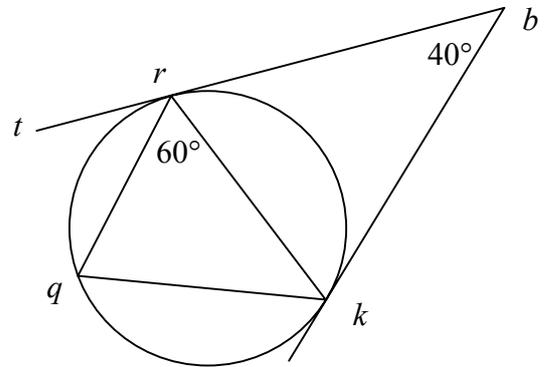


Find

- (i)  $|\angle bca|$   
 (ii)  $|\angle apb|$ .

- (b) Prove that the degree-measure of an angle subtended at the centre of a circle by a chord is equal to twice the degree-measure of an angle subtended by the chord at a point of the arc of the circle which is on the same side of the chordal line as is the centre.

- (c) In the diagram,  $br$  and  $bk$  are tangents to the circle at  $r$  and  $k$  respectively.  $q$  is a point on the circle and  $t$  is a point on  $br$ , as shown.

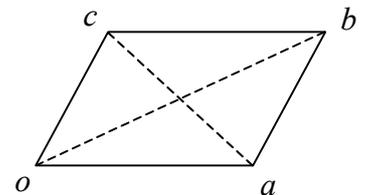


Find

- (i)  $|\angle bkr|$   
 (ii)  $|\angle qrt|$   
 (iii)  $|\angle qkr|$ .

9. (a) The diagram shows a parallelogram  $oabc$ , where  $o$  is the origin.

- (i) Express  $\vec{b}$  in terms of  $\vec{a}$  and  $\vec{c}$ .  
 (ii) Express  $\vec{ac}$  in terms of  $\vec{a}$  and  $\vec{c}$ .



- (b) Let  $\vec{p} = \vec{i} - 3\vec{j}$  and  $\vec{q} = -2\vec{i} + 4\vec{j}$ .  
 (i) Express  $3\vec{q} - 2\vec{p}$  in terms of  $\vec{i}$  and  $\vec{j}$ .  
 (ii) Calculate  $\left| 3\vec{q} - 2\vec{p} \right|$ , correct to one decimal place.  
 (iii) Express  $\vec{qp}$  in terms of  $\vec{i}$  and  $\vec{j}$ .

- (c) Let  $\vec{x} = 8\vec{i} + 6\vec{j}$  and  $\vec{y} = 8\vec{i} - 15\vec{j}$ .

- (i) Find  $\vec{x} \cdot \vec{y}$ , the dot product of  $\vec{x}$  and  $\vec{y}$ .  
 (ii) Hence, find the measure of the angle between  $\vec{x}$  and  $\vec{y}$ , correct to the nearest degree.

10. (a) Expand  $(1 + x)^6$  fully.

(b) (i) Find the sum to infinity of the geometric series:

$$\frac{2}{5} + \frac{2}{50} + \frac{2}{500} + \dots$$

(ii) Use your result from part (i) to express the recurring decimal  $1.444\dots$  as a fraction, (that is, in the form  $\frac{a}{b}$ , where  $a, b \in \mathbf{N}$ ).

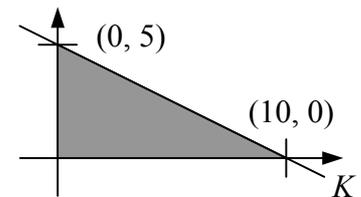
(c) (i) €500 is invested at 4% per annum compound interest. Find the value of the investment after ten years.

(ii) A person invests €500 in an account at the beginning of each year for ten consecutive years. Compound interest is added to the account at the rate of 4% per annum. By treating the values of the ten investments as terms in a geometric series, find the total amount in the account at the end of the tenth year.

11. (a) The line  $K$  cuts the  $x$ -axis at  $(10, 0)$  and the  $y$ -axis at  $(0, 5)$ .

(i) Find the equation of  $K$ .

(ii) Write down the three inequalities that together define the region enclosed by  $K$ , the  $x$ -axis and the  $y$ -axis.



(b) A developer is planning a scheme of holiday homes, consisting of large and small bungalows. Each large bungalow will accommodate 8 people and each small bungalow will accommodate 6 people. The development is not permitted to accommodate more than 216 people. The floor area of each large bungalow is  $200 \text{ m}^2$  and the floor area of each small bungalow is  $100 \text{ m}^2$ . The total floor area of all the bungalows must not exceed  $4000 \text{ m}^2$ .

(i) Taking  $x$  as the number of large bungalows and  $y$  as the number of small bungalows, write down two inequalities in  $x$  and  $y$  and illustrate these on graph paper.

(ii) The expected net annual income from each large bungalow is €14 000 and from each small bungalow is €8000. How many of each type should be built in order to maximise the total expected net annual income?

(iii) The developer decides to build as indicated in part (ii). The cost of building each large bungalow is €110 000 and the cost of building each small bungalow is €85 000. The total cost of the development is equal to the building costs plus €1.58 million. How many years will it take to recoup the total cost of the development?