Coimisiún na Scrúduithe Stáit
State Examinations Commission

Junior Certificate 2015

Marking Scheme

Mathematics

Higher Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

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Coimisiún na Scrúduithe Stáit
State Examinations Commission

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Mathematics

Paper 1

Higher Level

Model Solutions – Paper 1

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Instructions

There are 14 questions on this examination paper. Answer **all** questions.

Questions do not necessarily carry equal marks. To help you manage your time during this examination, a maximum time for each question is suggested. If you remain within these times you should have about 10 minutes left to review your work.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. You may ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

You may lose marks if the appropriate units of measurement are not included, where relevant.

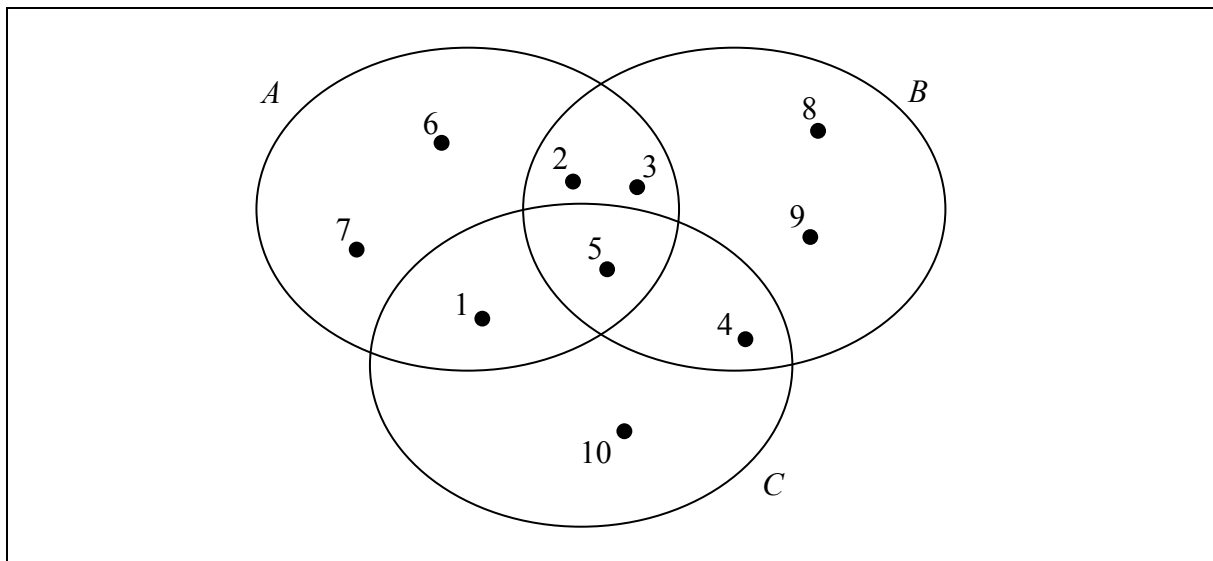**You may lose marks if your answers are not given in simplest form, where relevant.**

Write the make and model of your calculator(s) here:

Question 1   **15 Marks**

The sets A, B, and C are as follows:

$A = \{1, 2, 3, 5, 6, 7\}$ $B = \{2, 3, 4, 5, 8, 9\}$ $C = \{1, 4, 5, 10\}$.

(a) Complete the Venn diagram below.

(b) List the elements of each of the following sets.

$A \cup B =$ $\{1, 2, 3, 5, 6, 7, 4, 8, 9\}$

$A \setminus C =$ $\{2, 3, 6, 7\}$

$A \cup (B \cap C) =$ $\{1, 2, 3, 5, 6, 7, 4\}$

(c) Complete the following identity.

$$A \cup (B \cap C) = (A \cup B) \cap \left(A \cup C \right)$$

Question 2   **15 Marks**

(a)   David weighs 88 kg. The average male triathlete of his height weighs 83 kg.

If David aims to reach this weight, what **percentage decrease** is required?
Give your answer correct to two decimal places.

Decrease $= 88 - 83 = 5$ kg

% Decrease $= \dfrac{5}{88} \times 100 = 5{\cdot}681... = 5{\cdot}68\%$ (2 decimal places)

(b)   Mary's house was worth €200 000.
Mary increased the value of her house by 15% by building a conservatory.
She then increased its value by a further 10% by repaving the driveway.

Find the **total percentage** increase in value.

	€200 000 add 15%	=	€230 000		
	€230 000 add 10%	=	€253 000		
⇒	Total Increase	=	€53 000		
⇒	Percentage Increase	=	$\dfrac{53\,000}{200\,000} \times 100$	=	26·5%

OR

	100% add 15%	=	1·15		
	115% add 10%	=	$1{\cdot}10 \times 1{\cdot}15$	=	1·265
⇒	Percentage Increase	=	26·5%		

OR

10% of 15% = 1·5%		
15% + 10% + 1·5%	=	26·5%

Question 3   **25 Marks**

Eleanor has a **gross** income of €38 500 for the year.
She has an annual tax credit of €3300.
The standard rate cut-off point is €33 800.
The standard rate of income tax is 20% and the higher rate is 40%.

(a)   Find Eleanor's **net** income for the year (i.e. after tax is paid).

	€33 800 at 20%	=	€6760		
	€4 700 at 40%	=	€1880		
$\Rightarrow$	Gross Tax	=	6760 + 1880	=	€8640
	Tax Credit	=	€3300		
$\Rightarrow$	Net Tax	=	8640 – 3300	=	€5340
$\Rightarrow$	Net Income	=	38 500 – 5340		
		=	€33 160.		

Eleanor receives a pay rise. As a result, her **net** income for the year is €34 780.

(b)   Find Eleanor's new **gross** income for the year.

I	Increase in net income	=	34 780 – 33 160		
		=	€1620	= 60% of increase in gross income	
$\Rightarrow$	1% of increase in gross income	=	$\dfrac{1620}{60}$ = €27		
$\Rightarrow$	100% of increase in gross income	=	€27 × 100	= €2700	
$\Rightarrow$	New gross income	=	38 500 + 2700		
		=	€41 200.		

<div align="center">OR</div>

II	New gross income	= 38 500 + x
$\Rightarrow$	New net tax	= 5340 + 0·4x
$\Rightarrow$	New net income	= (38 500 + x) – (5340 + 0·4x)= 33 160 + 0·6x
$\Rightarrow$	33 160 + 0·6x = 34 780	
$\Rightarrow$	0·6x = 1620 so x = $\dfrac{1620}{0\cdot6}$= 2700	
$\Rightarrow$	New gross income	= 38 500 + 2700 = €41 200

	OR		
III	€34780 − €3300	=	€31 480
	Net income at standard rate	=	€33 800 × 0·8 = €27 040
	Net income at higher rate	=	€31 480 − €27 040 = €4440
	60% of Gross income at higher rate	=	€4440
	Gross income at higher rate	=	€4440 ÷ 60 × 100
	Total Gross Income	=	€33 800 + €7400 = €41 200

Question 4   **10 Marks**

Let $f(x) = 3x + 5$, for $x \in \mathbb{R}$.

(a) Find the value of $f(7)$.

$$f(7) = 3(7) + 5 = 26.$$

(b) Write $f(k)$ in terms of k.

$$f(k) = 3k + 5$$

(c) Using your answer to part **(b)**, or otherwise, find the value of k for which $f(k) = k$.

$$3k + 5 = k$$
$$\Rightarrow \quad 2k = -5$$
$$\Rightarrow \quad k = -\frac{5}{2}$$

Question 5   15 Marks

The Kelvin scale is one way of measuring temperature.
To convert a temperature from degrees Fahrenheit (F) to kelvin (K), you:

add 459·67 to F, then multiply your answer by 5 and divide by 9.

(a) Convert 212 degrees Fahrenheit (F) to kelvin (K).

$(212 + 459·67) \times 5 \div 9 \qquad = \qquad 373·15.$

(b) Write an algebraic formula to express K in terms of F.

$$K \quad = \quad \frac{(F + 459·67) \times 5}{9}$$

(c) Hence, or otherwise, convert 400 kelvin (K) to degrees Fahrenheit (F).

$$400 \quad = \frac{(F + 459·67) \times 5}{9}$$

$\Rightarrow \qquad 3600 \quad = (F + 459·67) \times 5$

$\Rightarrow \qquad 720 \quad = F + 459·67$

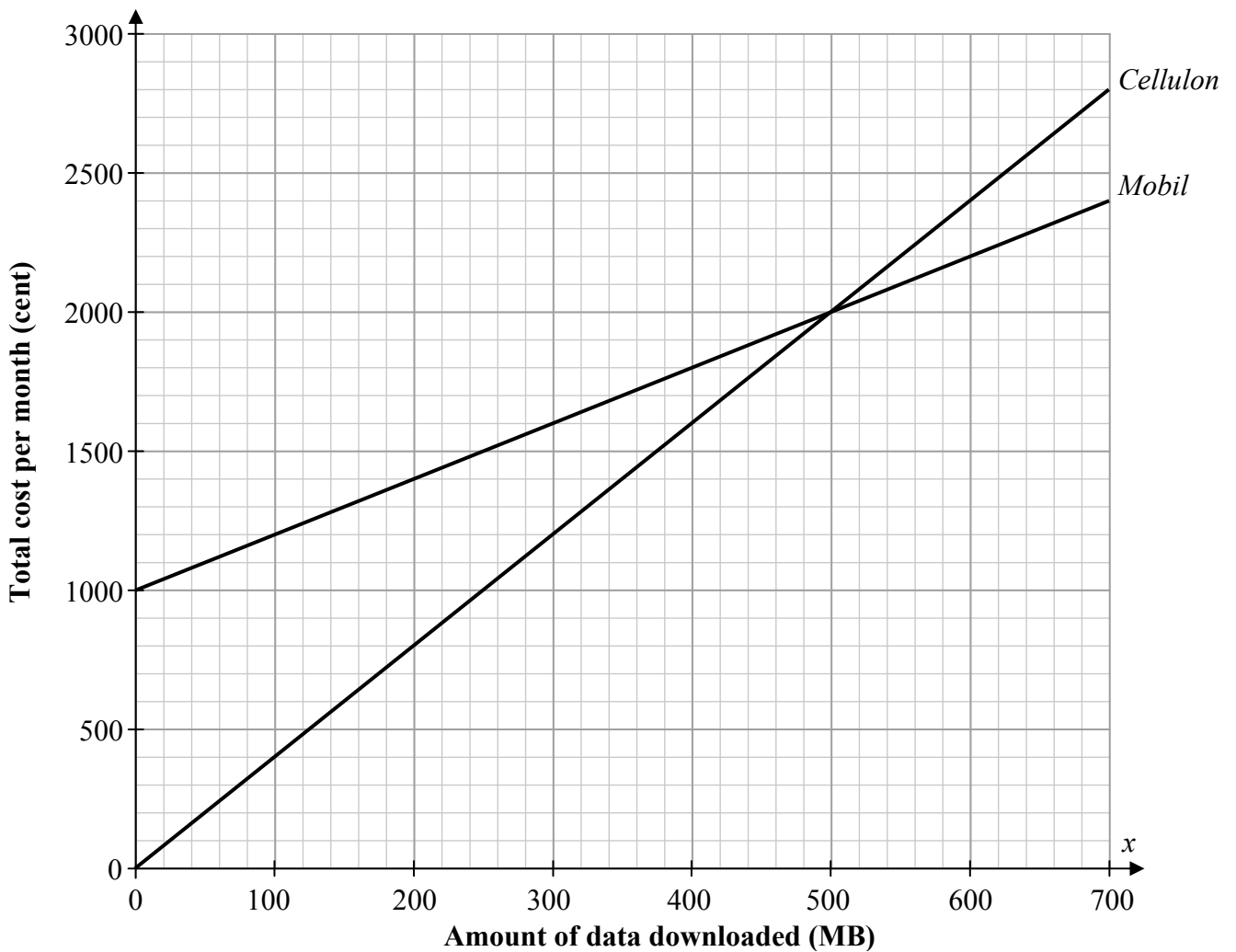$\Rightarrow \qquad F \quad = 720 - 459·67$

$\qquad\qquad\qquad = 260·33.$

Question 6 30 Marks

Two mobile phone companies, *Cellulon* and *Mobil*, offer price plans for mobile internet access.
A formula, in x, for the total cost per month for each company is shown in the table below.
x is the number of MB of data downloaded per month.

Phone company	Total cost per month (cent)
Cellulon	$c(x) = 4x$
Mobil	$m(x) = 1000 + 2x$

(a) **Draw** the graphs of $c(x)$ and $m(x)$ on the co-ordinate grid below to show the total cost per month for each phone company, for $0 \le x \le 700$. **Label** each graph clearly.

$c(0) = 0;$ $c(700) = 2800.$

$m(0) = 1000;$ $m(700) = 2400.$

(b) Which company charges **no** fixed monthly fee?
Justify your answer, with reference to the relevant **formula** or **graph**.

> Answer: *Cellulon*
>
> Justification: $c(0) = 0$, so no cost if no data used. **(formula)**
>
> **OR**
>
> The line goes through (0, 0), so no cost if no data used. **(graph)**

(c) Write down the **point of intersection** of the two graphs.

> (500, 2000)

Fergus wants to buy a mobile phone from one of these two companies, and wants his mobile internet bill to be as low as possible.

(d) **Explain** how your answer to part **(c)** would help Fergus choose between *Cellulon* and *Mobil*.

> If the data is less than 500 MB per month, *Cellulon* is cheaper.
>
> If the data is more than 500 MB per month, *Mobil* is cheaper.

Question 7   **20 Marks**

(a)   Multiply out and simplify $(x + 5)(x^2 - 2x + 6)$.

$$(x + 5)(x^2 - 2x + 6) \quad = \quad x^3 - 2x^2 + 6x + 5x^2 - 10x + 30$$

$$= \quad x^3 + 3x^2 - 4x + 30.$$

OR

	x^2	$- 2x$	$+6$
x	x^3	$- 2x^2$	$+6x$
$+ 5$	$+ 5x^2$	$-10x$	$+30$

$$= \quad x^3 + 3x^2 - 4x + 30$$

(b)   Factorise fully $ac - ad - bd + bc$.

$$ac - ad - bd + bc \quad = \quad a(c - d) + b(c - d)$$

$$= \quad (c - d)(a + b).$$

OR

$$ac + bc - ad - bd \quad = \quad c(a + b) - d(a + b)$$

$$= \quad (c - d)(a + b).$$

(c)   Write the following as a single fraction in its simplest form.

$$\frac{x + 2}{3} - \frac{x - 3}{4}$$

$$\frac{x + 2}{3} - \frac{x - 3}{4} \quad = \quad \frac{4(x + 2) - 3(x - 3)}{12}$$

$$= \quad \frac{4x + 8 - 3x + 9}{12}$$

$$= \quad \frac{x + 17}{12}.$$

Question 8   **15 Marks**

(a)   **Complete** the inequality in n below so that it has the solution set shown.

Inequality	**Solution Set**
$\boxed{2} \ \le n \le \ \boxed{4\cdot7} \ , \ n \in \mathbb{N}.$	

Solution Set number line: points marked at 2, 3, 4 on a line labelled 0 1 2 3 4 5 6.

(b)   **Complete** the inequality in x below so that there is only **one** possible value of x, where $x \in \mathbb{R}$.

$$\boxed{17\cdot3} \ \le x \le \ \boxed{17\cdot3} \ , \ x \in \mathbb{R}.$$

Question 9 **20 Marks**

(a) **(i)** Factorise $x^2 + 7x - 30$.

$$x^2 + 7x - 30 = (x + 10)(x - 3).$$

OR

$$x^2 + 7x - 30 = x^2 + 10x - 3x - 30$$

$$= x(x + 10) - 3(x + 10)$$

$$= (x + 10)(x - 3).$$

(ii) Hence, or otherwise, solve the equation $x^2 + 7x - 30 = 0$.

$$(x + 10)(x - 3) = 0$$

$\Rightarrow$ $x + 10 = 0$ or $x - 3 = 0$

$\Rightarrow$ $x = -10$ or $x = 3$.

OR

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(-30)}}{2(1)}$$

$$= \frac{-7 \pm \sqrt{169}}{2}$$

$$= -10 \quad \text{or} \quad 3.$$

(b) Solve the equation $2x^2 - 7x - 10 = 0$.
Give each answer correct to two decimal places.

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(2)(-10)}}{2 \times 2}$$

$$= \frac{7 \pm \sqrt{129}}{4}$$

$$= 4 \cdot 589... \quad \text{or} \quad -1 \cdot 089...$$

$$= 4 \cdot 59 \quad \text{or} \quad -1 \cdot 09 \quad \text{(2 decimal places)}$$

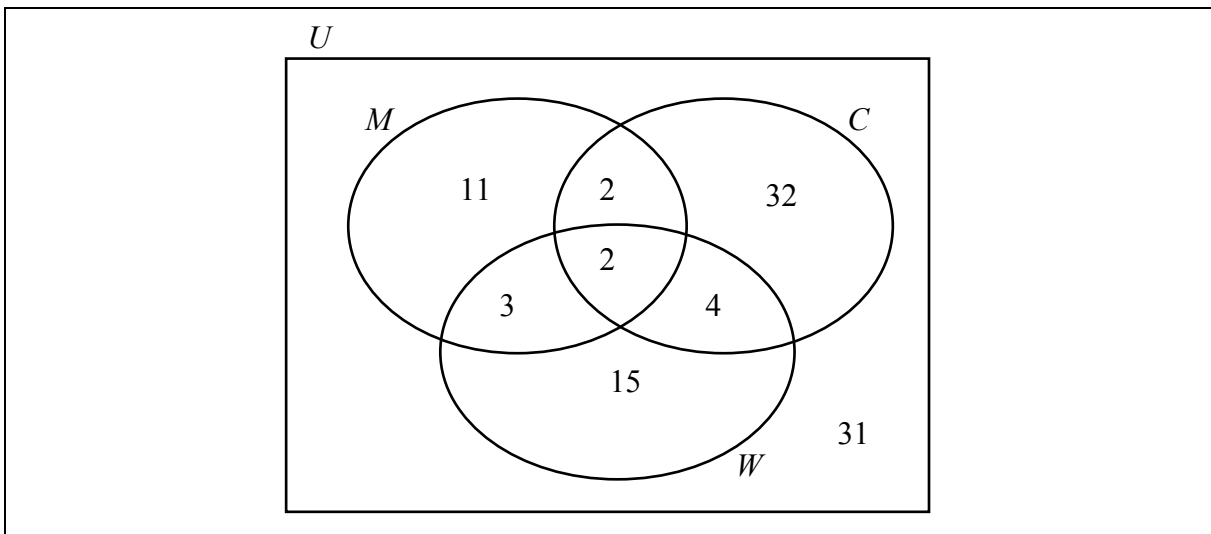**Question 10**   **25 Marks**

A researcher has found old census data about Measles (*M*), Chickenpox (*C*),
and Whooping cough (*W*) among 12-year-old children. In a group of 100 children:

> 31 had **none** of these diseases
>
> 2 had **all three** diseases
>
> 2 had Measles **and** Chickenpox, but **not** Whooping cough
>
> 6 had Whooping cough **and** Chickenpox
>
> 11 had **at least two** diseases
>
> 18 had Measles
>
> 40 had Chickenpox.

(a)   Use this data to **fill in** the Venn diagram.

(b)   Find the **probability** that a child chosen at random from the group had Chickenpox.

$$P(\text{C}) \;=\; \frac{40}{100} \quad \text{or} \quad \frac{2}{5}.$$

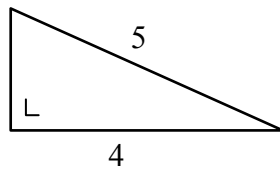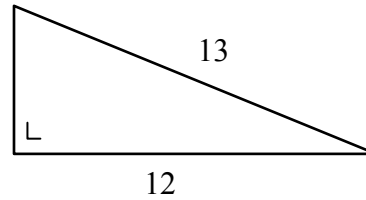The table below shows 3 statements. Each statement is written in English and in set notation.

(c)   Complete the table.

	English	**Set notation**
Statement 1	6 had Whooping cough **and** Chickenpox	$6 = \#(W \cap C)$
Statement 2	36 had Chickenpox but **not** Measles	$36 = \#(C \setminus M)$
Statement 3	2 had Measles **and** Chickenpox but **not** Whooping cough	$2 = \#[\,(M \cap C) \setminus W\,]$ or $2 = \#[\,M \cap (C \setminus W)\,]$

Question 11   **40 Marks**

Two right-angled triangles are shown below.

(a) Find the height of each triangle.
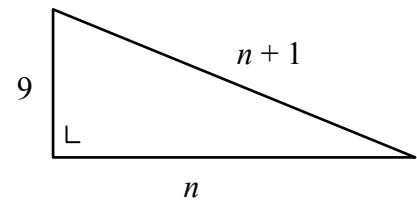Write each answer in the box below the appropriate diagram.

Height = 3   Height = 5

$$x^2 + 4^2 = 5^2 \Rightarrow \quad x^2 + 16 = 25$$
$$\Rightarrow \quad x^2 = 9$$
$$\Rightarrow \quad x = 3.$$

$$y^2 + 12^2 = 13^2 \Rightarrow y^2 + 144 = 169$$
$$\Rightarrow y^2 = 25$$
$$\Rightarrow y = 5.$$

The triangles above are the first two triangles (with sides of integer lengths) where the hypotenuse is 1 unit longer than the base.

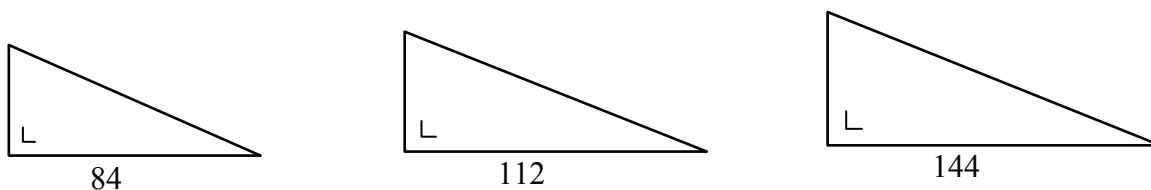**(b)** Another such triangle is shown on the right.
It has a height of 9 units.

Use the Theorem of Pythagoras to find the value of n,
the length of the base of this triangle.

$$9^2 + n^2 = (n+1)^2$$
$$\Rightarrow \quad 81 + n^2 = n^2 + 2n + 1$$
$$\Rightarrow \quad 2n = 80$$
$$\Rightarrow \quad n = 40.$$

These triangles can be put in a sequence of increasing size.
The lengths of the bases of the triangles in this sequence follow a **quadratic** pattern.
Three consecutive triangles in this sequence are shown below.

(c) Use this information to find the length of the base of the next triangle in the sequence.

	$112 - 84$		$144 - 112$
1st difference	$= 28$		$= 32$

2nd difference $\quad\quad\quad\quad \dfrac{32 - 28}{= 4}$

$\Rightarrow$ next 1st difference $\quad = 32 + 4 \quad\quad = 36.$

$\Rightarrow$ next base $\quad\quad\quad = 144 + 36 = 180.$

The length of the hypotenuse, h, of triangle x in this sequence is given by the function below, where b and c are integers.

$$h(x) = 2x^2 + bx + c$$

Also, $h(1) = 5$ and $h(2) = 13$.

(d) **(i)** Use this information to write two equations in b and c.

Equation 1:	Equation 2:
$h(1) \quad = 2(1)^2 + b(1) + c = 5$	$h(2) \quad = 2(2)^2 + b(2) + c = 13$
$\Rightarrow \quad\quad 2 + b + c = 5$	$\Rightarrow \quad\quad 8 + 2b + c = 13$
$\Rightarrow \quad\quad b + c = 3$	$\Rightarrow \quad\quad 2b + c = 5$

(ii) Solve these simultaneous equations to find the value of b and the value of c.

Equation 1 $\times (-1)$: $\quad -b - c = -3$

Equation 2 : $\quad\quad \underline{2b + c = 5 \quad\quad}$.

$\Rightarrow \quad\quad\quad\quad b \quad = 2 \quad\quad\quad \Rightarrow \quad\quad c = 1.$

OR

Equation 1 $\Rightarrow \quad\quad c = 3 - b$

Equation 2 $\Rightarrow \quad\quad 2b + 3 - b = 5$

$\Rightarrow \quad\quad\quad\quad b = 2 \quad\quad\quad\quad \Rightarrow \quad\quad c = 1.$

Question 12   **20 Marks**

(a)   **(i)**   Factorise $n^2 - 1$.

$$n^2 - 1 = (n - 1)(n + 1)$$

Hence, or otherwise, answer the following question.
    (ii)   The **product** of two **consecutive odd** positive numbers is 399. Find the two numbers.

	$(n - 1)(n + 1)$	$= 399$
$\Rightarrow$	$n^2 - 1$	$= 399$
$\Rightarrow$	n^2	$= 400$ $\Rightarrow$ $n = 20$
$\Rightarrow$	Two numbers are 19 and 21.	

OR

	$\sqrt{399} = 19\cdot97...$	
$\Rightarrow$	Two numbers are 19 and 21.	

(b)   Divide $x^3 + 5x^2 - 29x - 105$ by $x + 3$.

$$
\begin{array}{r}
x^2 + 2x - 35 \\
x+3 \overline{)\, x^3 + 5x^2 - 29x - 105} \\
\underline{x^3 + 3x^2} \\
2x^2 - 29x - 105 \\
\underline{2x^2 + 6x} \\
-35x - 105 \\
\underline{-35x - 105} \\
0
\end{array}
$$

Answer: $x^2 + 2x - 35$

OR

	x^2	$2x$	-35
x	x^3	$2x^2$	$-35x$
3	$3x^2$	$6x$	-105

Answer: $x^2 + 2x - 35$
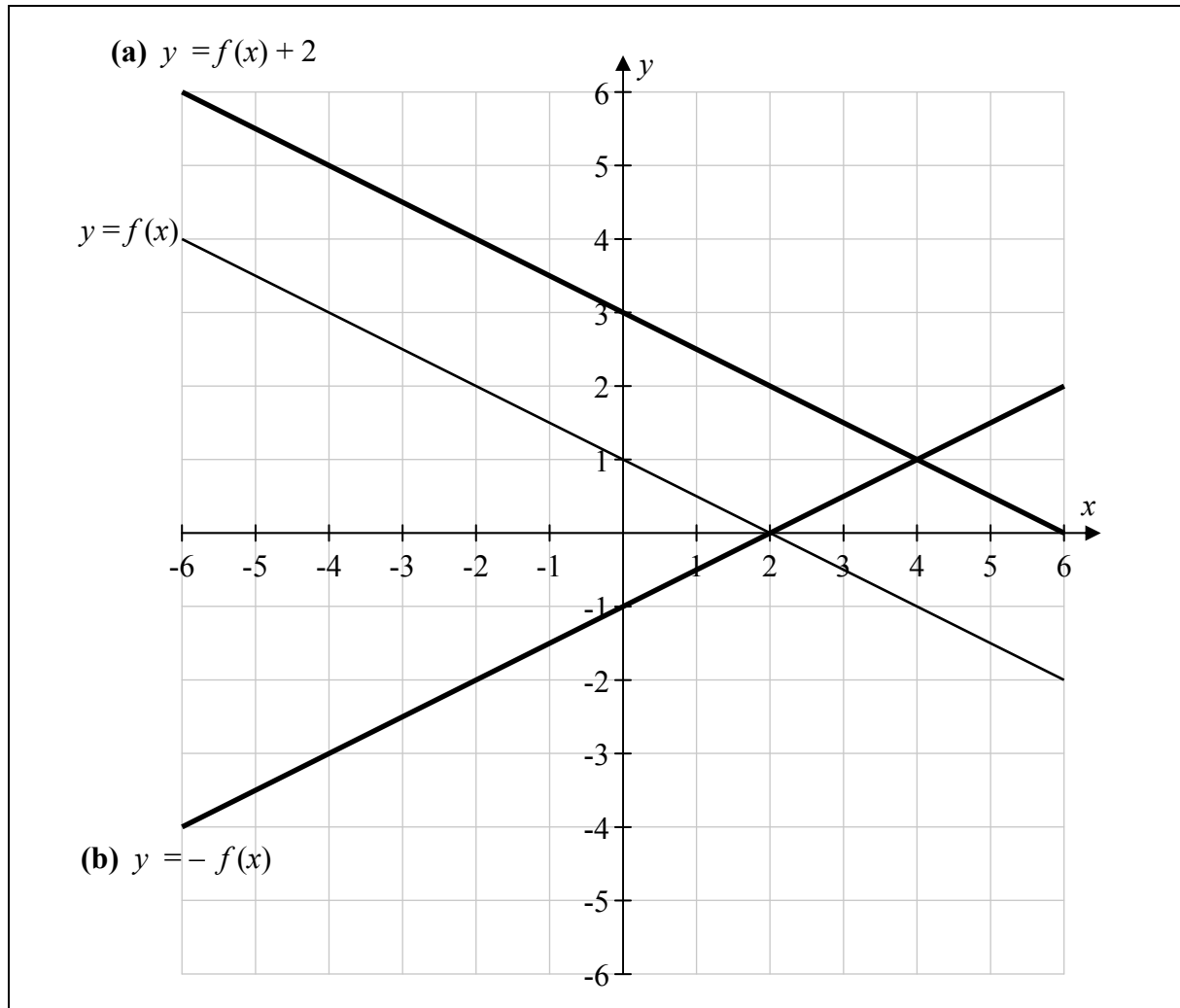
Question 13   20 Marks

The graph of the linear function $y = f(x)$ is drawn on the co-ordinate grid below.

Using the same axes, draw the graph of each of the following functions, where $-6 \le x \le 6$, $x \in \mathbb{R}$. Label each graph clearly.
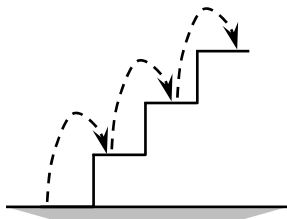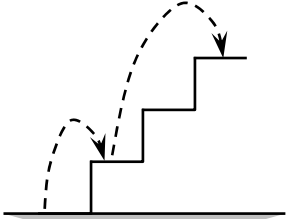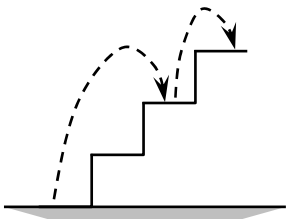
(a) $y = f(x) + 2$

(b) $y = -f(x)$

(a) $y = f(x) + 2$

$y = f(x)$

(b) $y = -f(x)$

Question 14　30 Marks

A boxer runs up stairs as part of her training.
She can go up 1 step or 2 steps with each stride, as shown.

Up 1 step　　　　　　　　　Up 2 steps

The boxer wants to count how many different ways she can reach the nth step.
She calls this T_n, the nth Taylor number.

For example, she has **3** different ways to reach the 3rd step, as shown in the tables below.
So $T_3 = 3$.

3rd step: way 1	3rd step: way 2	3rd step: way 3
Up 1 step, then 1 step, then 1 step	Up 1 step, then 2 steps	Up 2 steps, then 1 step
1 + 1 + 1	1 + 2	2 + 1

(a) Find the value of T_1 and T_2.

$T_1 = 1$　[way]	$T_2 = 2$　[ways]
[1 step]	[1 step + 1 step　or　2 steps]

(b) **List** all the different ways that she can reach the 4th step;　one way is already done for you.
Hence **write down** the value of T_4.

Different ways to reach the 4th step:　**1 + 1 + 1 + 1**

1 + 1 + 2

1 + 2 + 1

2 + 1 + 1

2 + 2　　[steps]

$\Rightarrow$　T₄ = 5.　[ways]

Some of the ways to reach the *n*th step start by going up **1 step**; others start by going up **2 steps**.

(c) **(i)** **List** the different ways that she can reach the 5th step, if she starts by going up **1 step**.

> 1 + 1 + 1 + 1 + 1
>
> 1 + 1 + 1 + 2
>
> 1 + 1 + 2 + 1
>
> 1 + 2 + 1 + 1
>
> 1 + 2 + 2   [steps]
>
> [5 ways]

(ii) **List** the different ways that she can reach the 5th step, if she starts by going up **2 steps**.

> 2 + 1 + 1 + 1
>
> 2 + 1 + 2
>
> 2 + 2 + 1   [steps]
>
> [3 ways]

(d) **Explain** why $T_{100} = T_{99} + T_{98}$.

> To get to the 100th step, you must start by going up either 1 step or 2 steps.
>
> If you start by going up 1 step, there are T_{99} ways to finish.
>
> If you start by going up 2 steps, there are T_{98} ways to finish.

Marking Scheme – Paper 1

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect), scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D
No of categories	2	3	4	5
5-mark scale		0, 2, 5	0, 2, 3, 5	
10-mark scale		0, 4, 10	0, 4, 7, 10	0, 4, 6, 8, 10
15-mark scale			0, 6, 11, 15	0, 6, 9, 12, 15
20-mark scale			0, 6, 12, 20	0, 5, 10, 15, 20

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)
- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)
- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (mid partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work, or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may be awarded. Thus, for example, in Scale 10C, 9 marks may be awarded.

No marks may be awarded other than those on the appropriate scale, and *Full Credit –1*.

In general, accept a candidate's work in one part of a question for use in subsequent parts of the question, unless this oversimplifies the work involved.

Summary of mark allocations and scales to be applied

Question 1 (15)

(a)   5C

(b)&(c) 10D

Question 2 (15)

(a)   5B

(b)   10C

Question 3 (25)

(a)   10D

(b)   15C

Question 4 (10)

(a)&(b) 5C

(c)   5B

Question 5 (15)

(a)   5B

(b)   5B

(c)   5B

Question 6 (30)

(a)   15C

(b)   5B

(c)   5B

(d)   5B

Question 7 (20)

(a)   5C

(b)   10C

(c)   5C

Question 8 (15)

(a)&(b) 15C

Question 9 (20)

(a)(i)   5B

(a)(ii)   5B

(b)   10D

Question 10 (25)

(a)   15C

(b)&(c) 10C

Question 11 (40)

(a)   5B

(b)   10C

(c)   5C

(d)(i)&(ii) 20D

Question 12 (20)

(a)(i)&(ii) 10C

(b)   10C

Question 13 (20)

(a)&(b) 20C

Question 14 (30)

(a)   5C

(b)   5C

(c)(i)&(ii) 10C

(d)   10B

Detailed marking notes

Question 1 (15 Marks)

(a)   **Scale 5C (0, 2, 3, 5)**

Low Partial Credit
- Two elements correctly placed

High Partial Credit
- 4 parts of Venn diagram filled in correctly

(b)&(c) Scale 10D (0, 4, 6, 8, 10)

Low Partial Credit
- One element correct in one part of **(b)**

Mid Partial Credit
- Two parts of **(b)** correct *or* **(c)** correct

High Partial Credit
- Three parts of **(b)** correct
- **(c)** correct *and* one part of **(b)** correct

Full Credit
- Accept $(A \cup B) \cap (A \cup (B \cap C))$
- Accept elements listed in **(c)**

Question 2 (15 Marks)

(a)   **Scale 5B (0, 2, 5)**

Partial Credit
- Some work of merit *e.g.* $\frac{83}{88}$, $88 - 83$, 5, 100, 94·32.

Full Credit − 1
- 5·681, 5·6, 5·7 without work

(b)   **Scale 10C (0, 4, 7, 10)**

No Credit
- 25% or 15% + 10% and stops

Low Partial Credit
- €30 000 or €230 000
- Shows some understanding of percentages *e.g.* $\frac{15}{100}$ or ·1
- 10% of 15% = 1·5%
- 250 000 and stops

High Partial Credit
- €53 000 or €23 000 or €253 000
- Significant work of merit with error(s) in the final step
- 15 +10 + 1·5 and stops
- 10% of €200 000 leading to final answer of 25% with work shown
- 0·265% or 0·265 or 126·5

Question 3 (25 Marks)

(a)   **Scale 10D (0, 4, 6, 8, 10)**

Low Partial Credit
- Uses a relevant percentage correctly
- Some work of merit

Mid Partial Credit
- Relevant percentage of a relevant number *e.g.* €6760 and/or €1880
- Finds Gross Tax €8640
- Uses a relevant percentage correctly and does one other step correctly

High Partial Credit
- Substantial work of merit (mishandles / omits one step, otherwise fully correct)
- Finds Net Tax €5340
- Omission/error involving tax credit *e.g.* €29 860
- Swaps percentages *i.e.* €33800 at 40% and €4700 at 20 %

Full Credit − 1
- Unit omitted (once per question)

(b) **Scale 15C (0, 6, 11, 15)**

Low Partial Credit
- €1620 **(I)** €31 480 **(III)**
- Some relevant work of merit *e.g.* indicates division by 60 or ·6
- Trial and error fully worked out with wrong answer

High Partial Credit
- €27 or €2700 **(I) (II)** or €74 or €7400 **(III)**
- Correct answer with no supporting work **or** correct answer with invalid supporting work

Full Credit − 1
- Unit omitted (once per question)

Full Credit
- Trial and error fully worked out with correct answer

Question 4 (10 Marks)

(a)&(b) Scale 5C (0, 2, 3, 5)

No Credit
- 36 and/or 8k without work
- $f(x) = 7$

Low Partial Credit
- Any correct substitution *e.g.* 3(7), 3k, 21

High Partial Credit
- **(a)** or **(b)** correct
- 3(7) + 5 **and** 3k

(c) **Scale 5B (0, 2, 5)**

Partial Credit
- Some work of merit *e.g.* $3k + 5 = k$
- Evaluates LHS, $k \neq -2·5$
- Makes incorrect equation from answer in **(b)**, finished correctly (*e.g.* sets answer in **(b)** = 0, instead of = k)

Question 5 (15 Marks)

(a)   **Scale 5B (0, 2, 5)**

Partial Credit
- Some work of merit *e.g.* adds (671·67) or multiplies by 5 or divides by 9

Full Credit − 1
- Rounding error, once per question

Full Credit
- Correct answer without work

(b)   **Scale 5B (0, 2, 5)**

Partial Credit
- One correct operation with correct number *i.e.* × 5 **or** ÷ 9 **or** + 459·67

Full Credit − 1
- Letter other than *F* (or *K*) used, otherwise correct
- *F* in terms of *K* correctly
- No brackets, otherwise correct

(c)   **Scale 5B (0, 2, 5)**

Oversimplification is worth *Partial Credit* at most

No Credit
- *F* = 400 and stops

Partial Credit
- Some work of merit
- 400 = Candidate's answer in **(b)**
- 400 × 9 or 400 ÷ 5
- Oversimplification

Full Credit − 1
- Rounding error, once per question

Full Credit
- Correct answer without work

Question 6 (30 Marks)

(a)   **Scale 15C (0, 6, 11, 15)**

Tolerance = ± 40 on *x*-axis, reasonably joined

Low Partial Credit
- 1 point correctly plotted
- Some valid work *e.g.* substitution

High Partial Credit
- 1 line correct
- 2 correct lines not filling the domain

Full Credit − 1
- No label(s) or incorrect labels

Full Credit
- Two correct lines with one correct label

(b)   **Scale 5B (0, 2, 5)**

Partial Credit
- Company correct only, no reason or incorrect reason
- Valid reason only, no company

(c)   **Scale 5B (0, 2, 5)**

Partial Credit
- Co-ordinates reversed
- Zeros missing *e.g.* (5, 20)
- One co-ordinate correct
- Point of intersection significantly marked on graph, but not stated in **(c)**
- Shows understanding of point of intersection
- Outside tolerance of ± 40 on *x*-axis
- Based on incorrect graphs in **(a)** with no point of intersection, candidate acknowledges this

Full Credit − 1
- (500, 200)

Full Credit
- Accept point of intersection as "500MB at 2000 cent"

(d)   **Scale 5B (0, 2, 5)**

Partial Credit
- Answer of some merit

Question 7 (20 Marks)

(a)   **Scale 5C (0, 2, 3, 5)**

Low Partial Credit
- Any correct multiplication *i.e.* one correct term (sign, number and *x* term)
- $x(x^2 - 2x + 6) + 5(x^2 - 2x + 6)$ or sets up grid correctly

High Partial Credit
- 4 terms correctly multiplied (including signs)

(b)   **Scale 10C (0, 4, 7, 10)**

Low Partial Credit
- One common factor
- Grouping indicated

High Partial Credit
- 1 correct factorisation (outside and inside bracket)
- 2 factorisations with sign errors

(c)   **Scale 5C (0, 2, 3, 5)**

Low Partial Credit
- Some relevant work of merit, e.g. 3(4), 12, 4(*x* + 2), 3(*x* − 3)

High Partial Credit
- $\dfrac{4(x+2)-3(x-3)}{12}$ or similar
- Multiplies by 12 instead of putting over 12, finishes correctly (i.e. $x + 17$)

Question 8 (15 Marks)

(a)&(b) Scale 15C (0, 6, 11, 15)

Three answers to check: each box in **(a)** counts as a separate answer, while two boxes together in **(b)** count as one answer.

No Credit
- Uses x in **(b)**

Low Partial Credit
- One value correct in **(a)**
- Correct values in **(a)** reversed
- $1 \le x \le 5$ in **(a)**

High Partial Credit
- **(a)** or **(b)** correct

Full Credit
- Accept same letter, other than *x,* in part **(b)**

Question 9 (20 Marks)

(a)(i)   Scale 5B (0, 2, 5)

Partial Credit
- Any correct factorisation of x^2 or 30, or one factor of each *e.g.* x and 6, or 2 and 15
- Two digits to total 7
- Fully substituted relevant formula

(a)(ii)   Scale 5B (0, 2, 5)

You may accept work presented in **(a)(i)**

Partial Credit
- Answer **(a)(i)** = 0
- Some work of merit *e.g.* formula, correct factor, correct substitution *etc.*
- Solves one linear equation, award *Partial Credit* at most

(b)   Scale 10D (0, 4, 6, 8, 10)

Low Partial Credit
- Identifies *a*, *b* or *c*
- Fully correct formula

Mid Partial Credit
- Fully correct substitution into formula
- 2 steps correct (from 4 steps involved: (1) correct formula or correctly identifies *a*, *b*, and *c*; (2) fully correct substitution; (3) works out to surd form; (4) finishes correctly to two decimal solutions).

High Partial Credit
- 3 correct steps from 4 steps involved
- One correct solution

Question 10 (25 Marks)

(a) **Scale 15C (0, 6, 11, 15)**

Low Partial Credit
* Two correct entries

High Partial Credit
* 4 correct entries (Check work, entries are dependent on each other)

(b)&(c) **Scale 10C (0, 4, 7, 10)**

Three answers to check: one in **(b)** and two in **(c)**

Low Partial Credit
* One correct
* Some work of merit in **(b)**

High Partial Credit
* Two correct

Full Credit − 1
* Omission of # in **(c)**

Question 11 (40 Marks)

(a)   **Scale 5B (0, 2, 5)**

No Credit
- Measures given sides

Partial Credit
- One correct
- Squares two numbers in one triangle
- $a^2 + b^2 = c^2$ or similar
- Some relevant trigonometry filled in *e.g.* Cos α $= \frac{4}{5}$ *or* Sin β $= \frac{4}{5}$ *or* 36·86°
  or 22·62° unfinished

(b)   **Scale 10C (0, 4, 7, 10)**

Low Partial Credit
- Squares or indicates squaring of two terms
- $a^2 + b^2 = c^2$
- T + E: One substitution (incorrect value of n) fully worked out

High Partial Credit
- $81 + n^2 = n^2 + 2n + 1$ or equivalent *i.e.* $n + 1$ must be squared correctly
- Incorrect hypotenuse fully finished correctly (5·84..., or −6·84..., or −41)
- $n = 40$ with no supporting work or $n = 40$ with invalid work

Full Credit
- Accept fully verified substitution of correct answer *i.e.* $9^2 + \mathbf{40}^2 = (\mathbf{40} + 1)^2$
  ⇒ 1681 = 1681

(c)   **Scale 5C (0, 2, 3, 5)**

Low Partial Credit
- Relevant subtraction
- Mentions *difference*
- Finds height of one triangle (13, 15 or 17), or one co-efficient of quadratic formula, or one equation involving coefficients of quadratic formula

High Partial Credit
- Find second difference (4) or next first difference (36)
- Finds height of next triangle (19) or finds all 3 coefficients of quadratic formula

Full Credit
Accept correct answer no work

(d)   **Scale 20D (0, 5, 10, 15, 20)**

Solution by substitution must be fully verified

Low Partial Credit
- Work of merit in one part

Mid Partial Credit
- Work of merit in both parts *or* one part correct

High Partial Credit
- One part correct *and* work of merit in the other

Question 12 (20 Marks)

(a)   **Scale 10C (0, 4, 7, 10)**

No Credit
- (   ) (   )

Low Partial Credit
- Some work of merit for **(i)** *e.g.* $n - 1$ or $n + 1$ or factors of n^2, or states "*difference of two squares*"
- Some work of merit for **(ii)** *e.g.* product of two odd consecutive numbers (e.g. 5×3), or factorises 399 (*e.g.* 3×133), or prime factors (3, 7, 19), or finds square root of 399, or relevant term = 399.

High Partial Credit
- **(i)** or **(ii)** correct

(b)   **Scale 10C (0, 4, 7, 10)**

Low Partial Credit
- Sets up division
- Any relevant step

High Partial Credit
- Substantial work *e.g.* x^2 on top to $2x^2 - 29x$ stage
- 2 correct terms in answer

Question 13 (20 Marks)

(a)&(b) Scale 20C (0, 6, 12, 20)

Tolerance = ± 2 mm

Low Partial Credit
- One correct point of $y = f(x) + 2$ *or* $y = -f(x)$ written or plotted
- Line parallel to either solution
- Correct value of the given function, *e.g.* $f(0) = 1$

High Partial Credit
- One correct line (full domain)

Full Credit − 1
- No label(s)

Question 14 (30 Marks)

(a)   **Scale 5C (0, 2, 3, 5)**

Low Partial Credit
- Some work of merit *e.g.* indication of understanding on diagram or 1 + 1 [steps]

High Partial Credit
- One correct answer

(b)   **Scale 5C (0, 2, 3, 5)**

"+" not required for a successful list
Lists can be given as diagrams

No Credit
- $T_4 = 4$ with no work shown

Low Partial Credit
- Any one of list correct
- No correct way listed, but T_4 value consistent with work
- $T_4 = 5$, no list

High Partial Credit
- Any two correct listings

Full Credit −1
- List correct with incorrect conclusion or no conclusion *e.g.* states "$T_4 = 4$" instead of "$T_4 = 5$", with correct list

(c)   **Scale 10C (0, 4, 7, 10)**

Low Partial Credit
- Any one way correct

High Partial Credit
- One correct in each part *i.e.* one correct way starting with 1 **and** one correct way starting with 2
- **(i)** or **(ii)** fully correct

(d)   **Scale 10B (0, 4, 10)**

Partial Credit
- Identifies pattern *e.g.* $T_1 + T_2 = T_3$
- Links T_{99} to one step *or* Links T_{98} to two steps
- Mentions Fibonacci

Full Credit
- Clearly links T_{99} to one step **and** links T_{98} to two steps

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Junior Certificate Examination 2015

Mathematics

Paper 2

Higher Level

Model Solutions – Paper 2

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Instructions

There are 14 questions on this examination paper. Answer **all** questions.

Questions do not necessarily carry equal marks. To help you manage your time during this examination, a maximum time for each question is suggested. If you remain within these times you should have about 10 minutes left to review your work.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

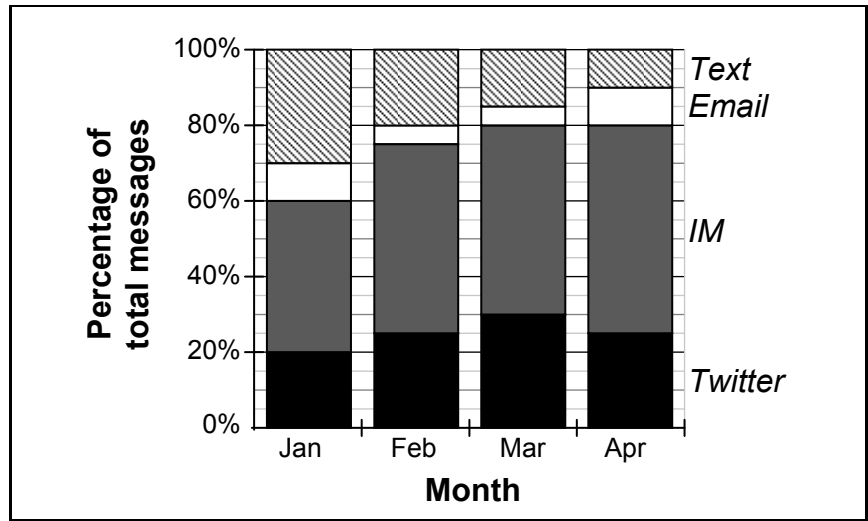Write the make and model of your calculator(s) here:

Question 1 15 Marks

The students in a class recorded how many messages they sent using different forms of messaging (*Text*, *Email*, *IM*, and *Twitter*) over four months.

Diagram 1 shows the percentage of messages sent using each form of messaging in each of the four months.

(a) Using **Diagram 1**, complete the table below to show the percentage of messages sent using *Email* in each of the four months.
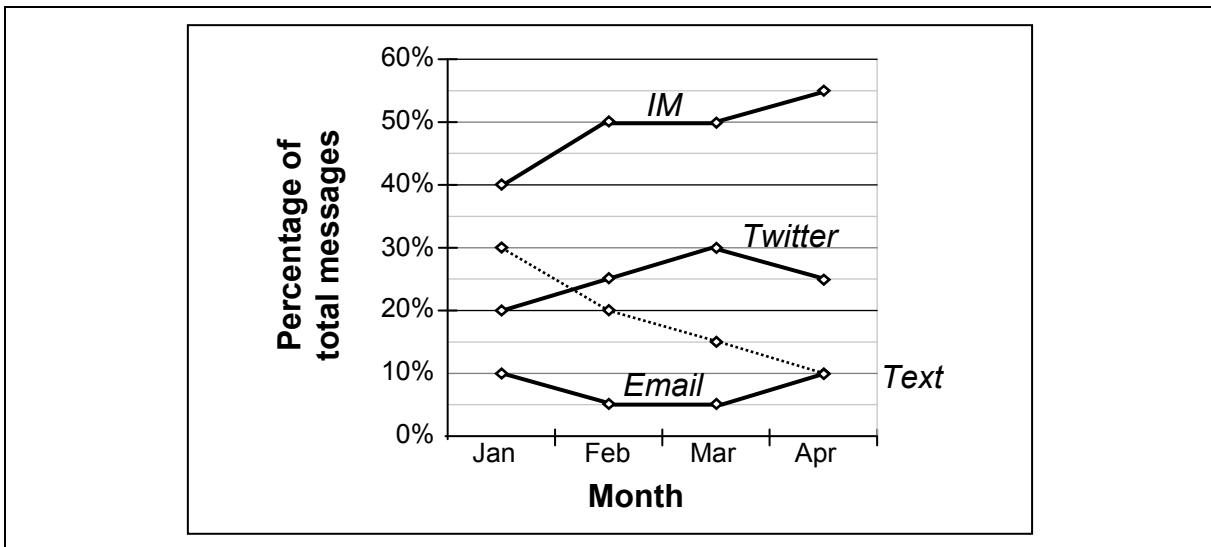

Month	Jan	Feb	Mar	Apr
Percentage of messages sent by *Email*	10%	**5%**	**5%**	**10%**

Diagram 2 shows the trend graph for *Text* over the four months.

(b) **Complete Diagram 2** to show the trend graphs for *Email*, *IM*, and *Twitter* over the four months, using the data in **Diagram 1**.

Label each trend graph clearly.

Question 2   **20 Marks**

Below is a menu from a restaurant.
A 3-course dinner is made up of one *Starter*, one *Main Course*, and one *Dessert*.

Starter	*Main Course*	*Dessert*
• Soup • Garlic Bread • Onion Rings • Chowder	• Pizza • Spaghetti • Steak • Lamb • Salmon	• Cheesecake • Chocolate Cake • Ice-cream

(a) Calculate the number of different 3-course dinners that can be ordered from this menu.

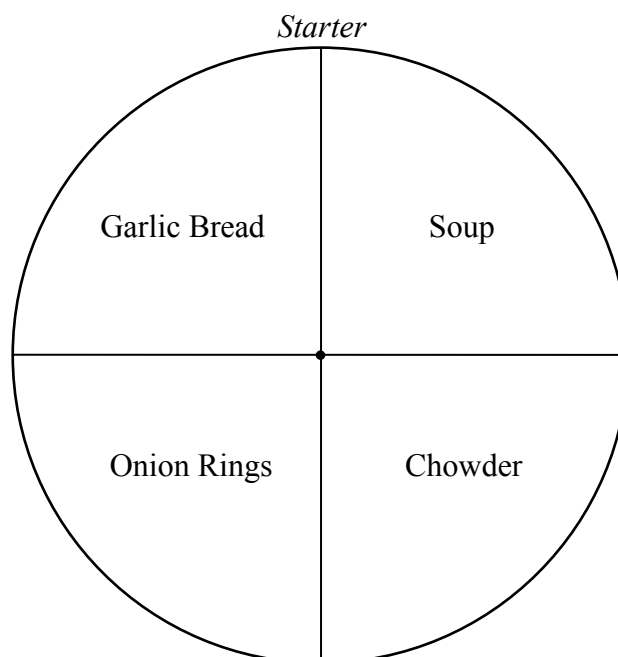Number of different 3-course dinners = $4 \times 5 \times 3 = 60$.

In the restaurant there is a circular poster for each course, where the options are represented by sectors of equal area.

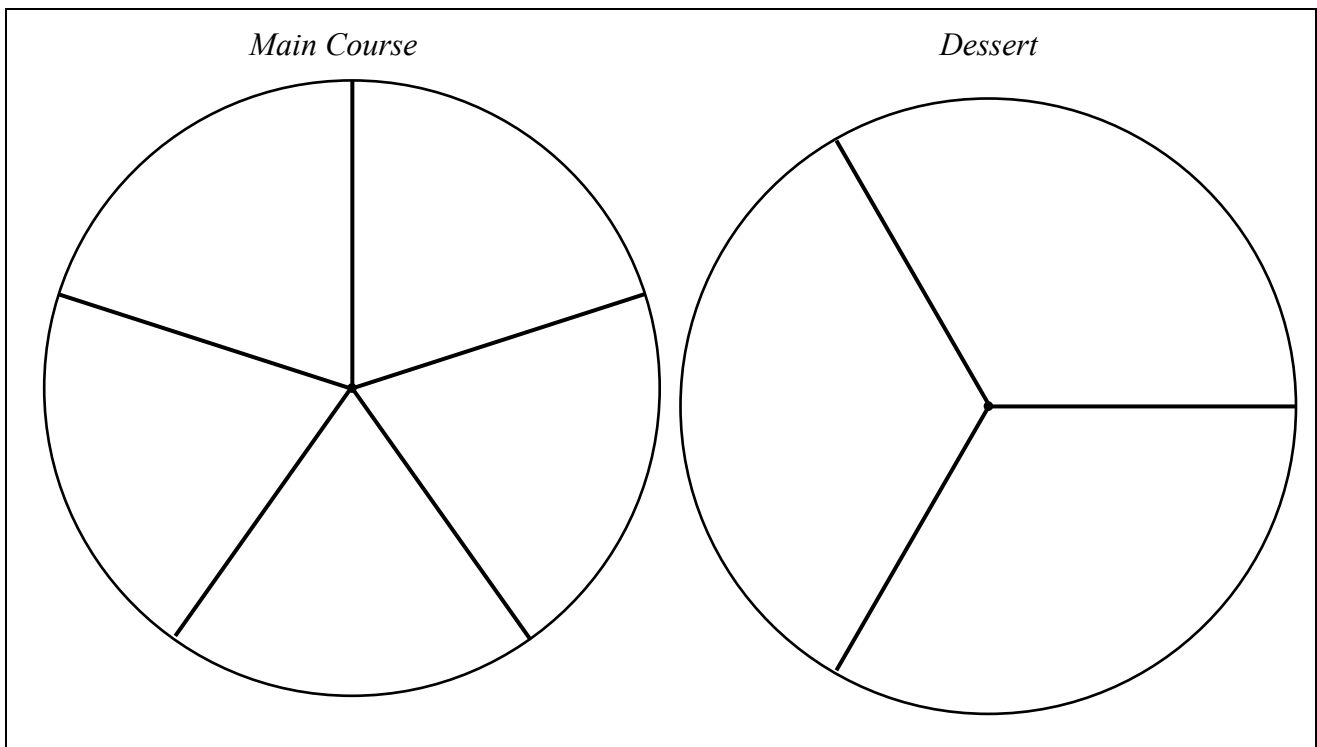The poster for the *Starter* is shown in the diagram on the right.

(b) **Calculate** the angle of each sector in the *Main Course* poster and each sector in the *Dessert* poster.

    Construct these sectors in the diagrams on the next page.

Starter

Garlic Bread   Soup

Onion Rings   Chowder

Main Course Angle = $360 \div 5 = 72°$.

Dessert Angle = $360 \div 3 = 120°$.

Main Course   *Dessert*

(c) The owner of the restaurant wants to increase the number of different 3-course dinners that can be ordered.

She will add **either** one *Starter*, **or** one *Main Course*, **or** one *Dessert* to the menu.

Which should she add to make the number of different 3-course dinners that can be ordered as **large** as possible? Justify your answer fully.

Answer:	Dessert	
Justification:	Extra Starter ⇒	$5 \times 5 \times 3 = 75$ dinners
	Extra Main Course ⇒	$4 \times 6 \times 3 = 72$ dinners
	Extra Dessert ⇒	$4 \times 5 \times 4 = 80$ dinners

Question 3   **20 Marks**

Eithne is going to survey post-primary Geography teachers in Ireland.

(a) Some of the questions in the survey are shown in the table below.
Put a tick (✓) in the correct box to show what type of data each question would give.

Question	Numerical Continuous	Numerical Discrete	Categorical Nominal	Categorical Ordinal
How many Geography classes do you teach each week?		✓		
How much do you like teaching Geography? A lot ☐ A little ☐ Not at all ☐				✓
What subjects (other than Geography) do you teach?			✓	

Eithne is going to send her survey to some of the post-primary schools in Ireland.

(b) Describe how Eithne could select a **Simple Random Sample** from all the post-primary schools in Ireland.

> Get a list of all of the post-primary schools in Ireland.
> [Step 1]
>
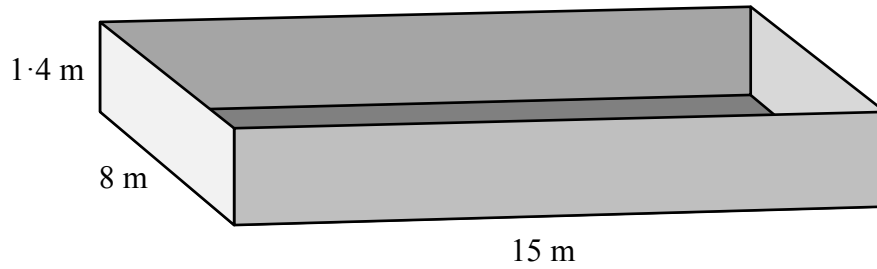> Randomly select a number of them, e.g. using random number generator.
> [Step 2]

Eithne is considering sending her survey by email.

(c) State **one advantage** and **one disadvantage** of using email to collect data.

Advantage: Quick / Convenient / Cheap / *etc*.	Disadvantage: Not everyone has email / May go to Spam / Faulty computer / *etc*.

Question 4   **30 Marks**

A swimming pool is 15 m long, 8 m wide, and 1·4 m deep, as shown in the diagram.

Harry says: "The area of the bottom of the swimming pool is $8 \times 15 = 120 \text{ cm}^2$."

(a) Explain what is **wrong** with Harry's answer.

> The unit is cm^2 – it should be m^2.

Harry will use $20 \text{ cm} \times 20 \text{ cm}$ tiles to cover the **inside** of the pool.

(b) Find the **minimum** number of tiles that Harry will need.

Area of bottom $= 8 \times 15$	$= 120 \text{ m}^2$	or	$1{,}200{,}000 \text{ cm}^2$
Area of front & back sides $= 2 \times 15 \times 1·4$	$= 42 \text{ m}^2$	or	$420{,}000 \text{ cm}^2$
Area of left & right sides $= 2 \times 8 \times 1·4$	$= 22·4 \text{ m}^2$	or	$224{,}000 \text{ cm}^2$
Total Area	$= 184·4 \text{ m}^2$	or	$1{,}844{,}000 \text{ cm}^2$
Total Area of 1 tile $0·2 \times 0·2$	$= 0·04 \text{ m}^2$	or	400 cm^2

Number of tiles $= 184·4 \div 0·04$ or $1{,}844{,}000 \div 400 = 4610$

The surface of the water in the swimming pool is 10 cm below the top of the pool.

(c) Find the volume of water in the swimming pool.
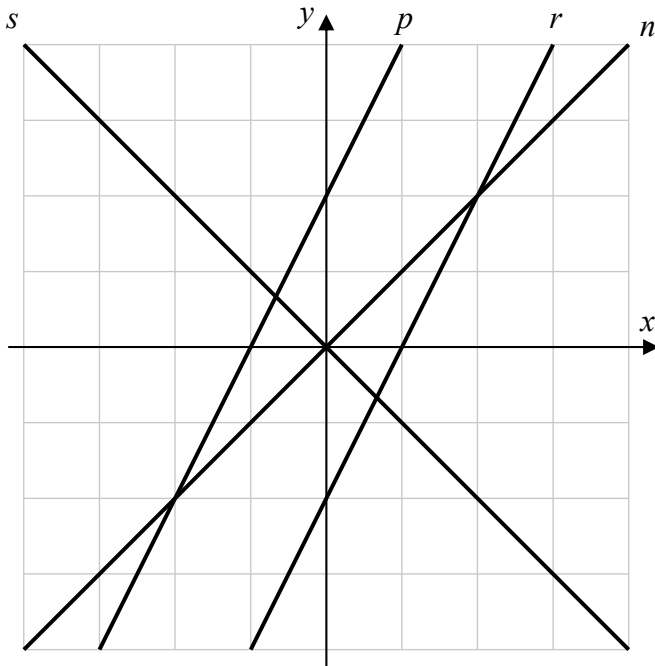
> Volume $= 15 \times 8 \times 1·3 = 156 \text{ m}^3$
>
> **OR**
>
> Volume $= 1500 \times 800 \times 130 = 156{,}000{,}000 \text{ cm}^3$

Question 5   25 Marks

The co-ordinate diagram below shows the lines *n*, *p*, *r*, and *s*.
The table shows the equation of each line.

Equation	Line
$y = 2x - 4$	**r**
$y = x$	**n**
$y = -x$	**s**
$y = 2x + 4$	**p**

(a) Write the letters *n*, *p*, *r*, and *s* into the table to match each line to its equation.

> [See table above]

Complete the following sentences. Write one of the letters *n*, *p*, *r*, or *s* in each box.

(b) You can use a **translation** to map the line ┃ *p* or *r* ┃ onto the line ┃ *r* or *p* ┃ .

(c) You can use an **axial symmetry** in the *y*-axis to map the line ┃ *n* or *s* ┃ onto the line ┃ *s* or *n* ┃ .

(d) The line ┃ *s* or *n* ┃ is mapped onto itself under **central symmetry** in the point (0, 0).

Question 6   **20 Marks**

The equation of the line l is $5 + y - 2x = 0$.

(a) Find the co-ordinates of the points where l cuts the axes.

l cuts the x-axis at $(2\cdot5, 0)$	l cuts the y-axis at $(0, -5)$

(b) Find the **slope** of the line l.

$y = 2x - 5$ $\Rightarrow$ slope $= 2$
OR
$m = \dfrac{-5-0}{0-2\cdot5} = 2$
OR
slope $= \dfrac{\text{rise}}{\text{run}}$ $= \dfrac{5}{2\cdot5}$ $= 2$

The line j goes through the point $(11, 6)$ and is **perpendicular** to the line l.

(c) **(i)** Write down the **slope** of the line j.

Slope of $j = -\dfrac{1}{2}$.

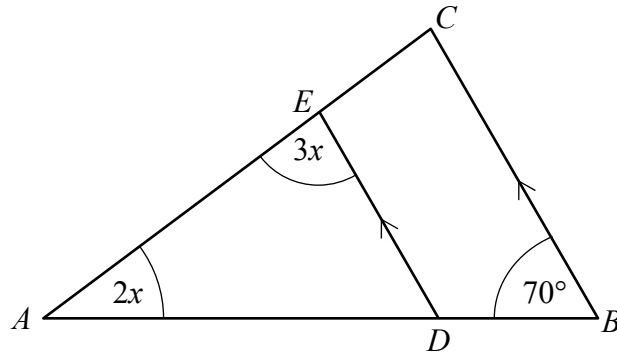**(ii)** Find the **equation** of the line j.

Equation of j: $\quad y - 6 = -\dfrac{1}{2}(x - 11)$ $\quad\Rightarrow\quad x + 2y - 23 = 0.$

Question 7 15 Marks

The diagram shows the triangle *ABC*.
DE is parallel to *BC*.
The sizes of some of the angles are shown.

(a) Find the value of *x*.

$$2x + 3x + 70° \qquad = \qquad 180°$$

$$\Rightarrow \quad 5x \qquad\qquad = \qquad 110°$$
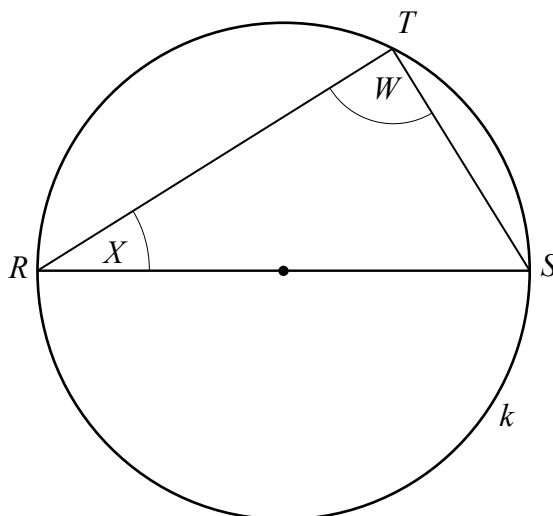
$$\Rightarrow \quad x \qquad\qquad = \qquad 22°$$

(b) Given that $|AE| = 100$, $|AC| = 130$, and $|DE| = 74$, find the value of $|BC|$.

$$\frac{|BC|}{130} \qquad = \qquad \frac{74}{100}$$

$$\Rightarrow \qquad 100\,|BC| \qquad = 9620$$

$$\Rightarrow \qquad |BC| \quad = 96{\cdot}2$$

Question 8 15 Marks

The diagram shows the triangle *RST* inscribed in the circle *k*.
The line segment [*RS*] is a **diameter** of the circle.

Gavin says: "The size of the angle *W* **must** be 90°."

(a) State one result on your course (a theorem or a corollary) that shows that Gavin is correct.

The angle at the centre of a circle is twice the angle at the circumference standing on the same arc [Theorem 19].
OR
Each angle in a semi-circle is a right angle [Corollary 3].

|*ST*| = 10 and |*RS*| = 30.

(b) Using this information, and trigonometry, find the size of ∠*X*.
Give your answer in degrees, correct to one decimal place.

$$\sin X \qquad = \frac{10}{30}$$

$$\Rightarrow \qquad X \qquad = \sin^{-1}\left(\frac{10}{30}\right)$$

$$= 19 \cdot 47...$$

$$= 19 \cdot 5° \text{ (1 decimal place)}$$

Question 9 30 Marks

A class of 25 students was surveyed to find out how many *WhatsApp* messages they each sent
in a particular week. The results are shown in the table below.

Number of messages	0 – 30	30 – 50	50 – 70	70 – 100	100 – 160
Number of students	1	2	10	7	5

Note: 30 – 50 means at least 30 but less than 50, etc.

(a) A student is picked at random from the class.
Find the probability that this student sent 50 or more messages.

> 50 or more messages: $\quad 10 + 7 + 5 = 22$
>
> Probability of $\geq 50 \quad = \dfrac{22}{25}$ or $0{\cdot}88$.

(b) A student is picked at random from those who sent 50 or more messages.
Find the probability that this student sent 50 – 70 messages.

> 50 or more messages: $\quad 10 + 7 + 5 = 22$
>
> Probability of 50 – 70 $\quad = \dfrac{10}{22}$ or $\dfrac{5}{11}$ or $0{\cdot}4545\ldots$

(c) Using mid-interval values, estimate the **mean** number of messages sent per student.

> $$\text{Mean} \quad = \quad \frac{(1)(15)+(2)(40)+(10)(60)+(7)(85)+(5)(130)}{1+2+10+7+5}$$
>
> $$= \quad \frac{15+80+600+595+650}{25}$$
>
> $$= \quad \frac{1940}{25} \ = \ \frac{388}{5} \ \text{or } 77{\cdot}6.$$

The students also found the **total** number of *WhatsApp* messages they sent in this particular week.

(d) Use the data in the table to find the **smallest** value that this total could be.

> $(0 \times 1) + (30 \times 2) + (50 \times 10) \ + (70 \times 7) + (100 \times 5)$
>
> $= \quad 1550.$

Question 10   **15 Marks**

There are 10 students in a class. All 10 of them sat a test.
The table below shows the **mean** mark, the **median** mark, and the **range** of the marks on the test.

	Results on the test	Answers to part **(b)**
Mean mark	25·1	**27·1**
Median mark	24	**26**
Range of the marks	14	**14**

32 was the **highest mark** got by a student on the test.

(a) Use the range to find the **lowest mark** got by a student on the test.

> Lowest mark   = 32 − 14
>
>   = 18.

An external examiner suggested that 2 be added onto each student's mark.

(b) Find what the **mean**, the **median**, and the **range** would be in this case.
Fill your answers into the table above.

> [See table above]

Bob says: "Whenever the median of a list of numbers is 24, then at least one of the numbers in the list **must be 24**."
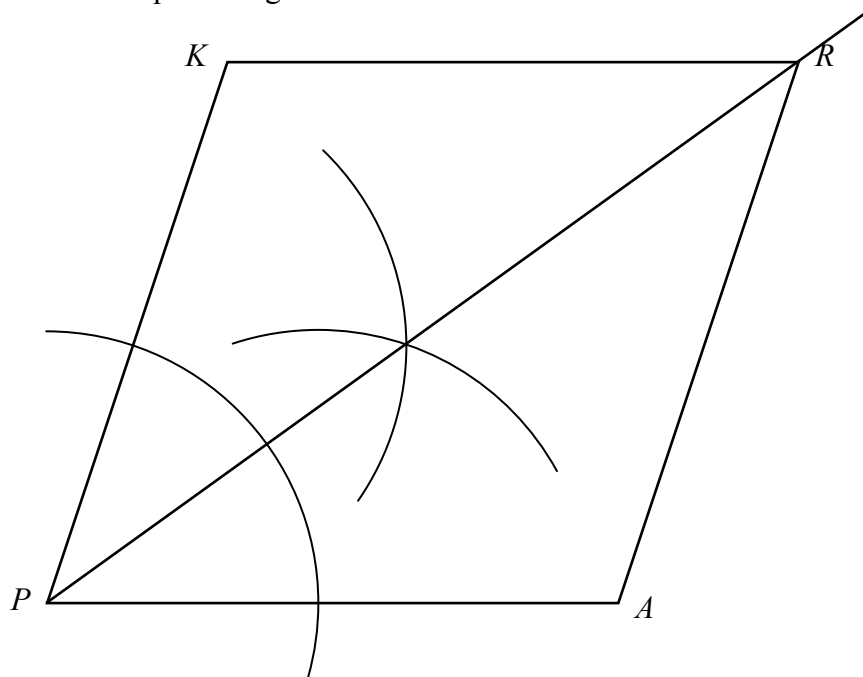
(c) Give an example to show that Bob is **not** correct.

> List   = 7, 23, 25, 96.
>
> Median   = 24.
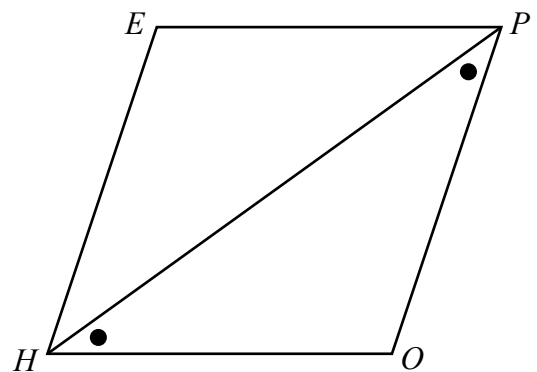
The diagram below shows the parallelogram *PARK*.

(a) **Construct** the **bisector** of ∠*KPA* on the diagram above, using only a compass and straight edge. Show your construction lines clearly.

[See diagram above]

The diagram on the right shows the parallelogram *HOPE*.
|∠*PHO*| = |∠*OPH*|, as shown.

(b) **Prove** that all four sides of the parallelogram are equal in length.

Give a reason for each of the statements that you make in your proof.

|*HO*| = |*EP*| & |*HE*| = |*OP*| ....Opposite sides of parallelogram   [Step 1]

|*HO*| = |*OP*| ....Isosceles triangle *PHO*   [Step 2]

⇒ |*HE*| = |*OP*| = |*HO*| = |*EP*| [Step 3]

$|\angle PHO| = |\angle HPE|$ ....Alternate angles

$|\angle OPH| = |\angle EHP|$ ....Alternate angles

$|HP| = |HP|$ ....Common side

$\therefore \Delta HOP \equiv \Delta PEH$ ........................A.S.A

$\therefore |OP| = |HE|$ and $|HO| = |EP|$ ....   Corresponding sides   [Step 1]

$|HO| = |OP|$ ....Isosceles triangle *PHO*   [Step 2]

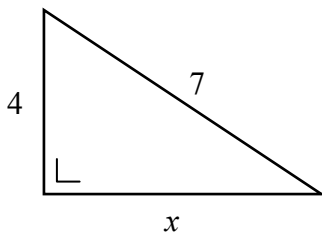$\Rightarrow$ $|HE| = |OP| = |HO| = |EP|$ [Step 3]

Question 12 {20 Marks}

(a)   The triangle *PQR* has sides of length 8, 11, and *y*.
Write down **one** value of *y* for which $\triangle PQR$ is an **isosceles** triangle.

$y =$ | 8 **or** 11 |

(b)   The triangle *STU* has sides of length 4, 7, and *x*.
Find the **two** values of *x* for which $\triangle STU$ is a **right-angled** triangle.
Give each answer in surd form.

$4^2 + x^2 = 7^2$
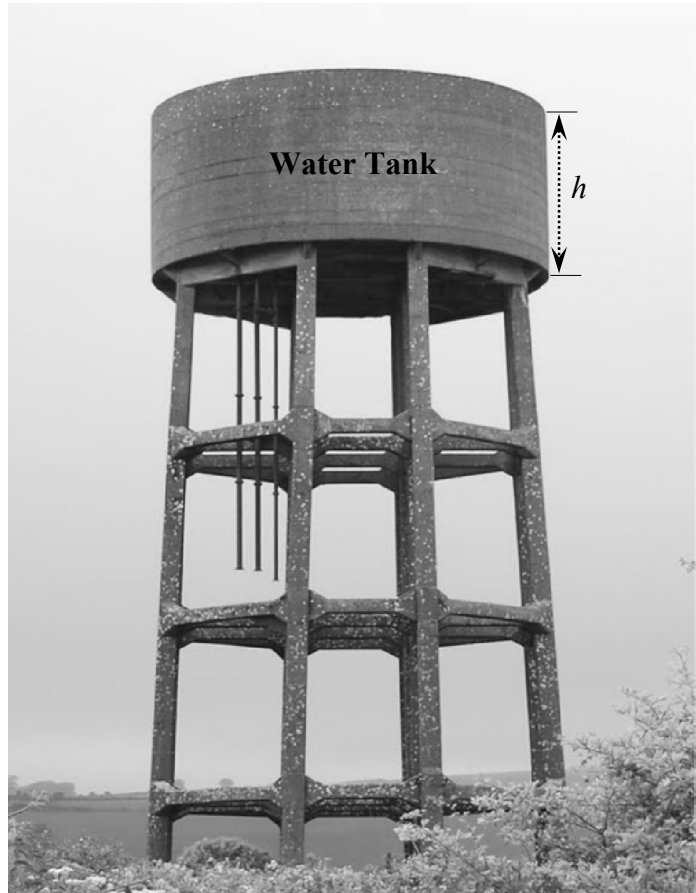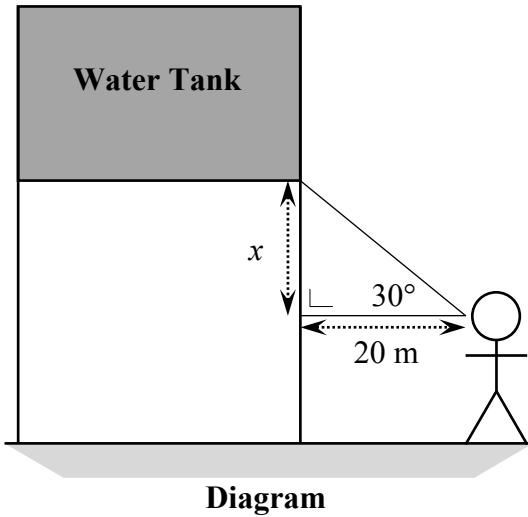
$\Rightarrow x = \sqrt{33}$

$4^2 + 7^2 = x^2$

$\Rightarrow x = \sqrt{65}$

Miriam is trying to find the volume of the water tank shown in the photograph on the right.

She takes some measurements and draws a diagram. Part of her diagram is shown below.

Water Tank

x

30°

20 m

Diagram

(a) Using the diagram, find the value of x. Give your answer in metres, correct to two decimal places.

Source: www.watertowersofireland.com. Altered.

$$\frac{x}{20} = \tan 30° \qquad \Rightarrow \qquad x = 11{\cdot}547 = 11{\cdot}55 \text{ m (2 decimal places)}$$

(b) The angle of elevation to the bottom of the water tank is 30°, as shown in the diagram. The angle of elevation to the top of the water tank is 38°. Find the distance marked h on the photograph. Give your answer correct to one decimal place.

h

x

38°

30°

20 m

$$\frac{x+h}{20} = \tan 38°$$

$$\Rightarrow \qquad x + h = 20 \tan 38° = 15{\cdot}626$$

$$\Rightarrow \qquad h = 15{\cdot}63 - 11{\cdot}55$$

$$= 4{\cdot}08 \qquad = 4{\cdot}1 \text{ m (1 decimal place)}$$

(c) Hugh is also trying to find the volume of the water tank.
He estimates that the height, h, is 4·5 m.

By taking **measurements** from the photograph and performing **calculations**, use Hugh's value of h to estimate the volume of the water tank as accurately as you can.

Give your answer correct to the nearest m^3.
State clearly what shape you are taking the water tank to be.

Shape of water tank:

   Cylinder   [Step 1]

Measurements from photograph (label each measurement):

   Diameter   = 5·2 cm   [⇒   Radius = 2·6 cm]

   Height   = 2·2 cm   [Step 2]

Calculations:

   $$\frac{\text{Actual radius}}{2 \cdot 6} = \frac{4 \cdot 5}{2 \cdot 2} \quad \Rightarrow \quad \text{Actual radius} = 5 \cdot 318... \text{ m} \qquad \text{[Step 3]}$$

   Volume of cylinder $= \pi \times r^2 \times h$ [Step 4]

   $\qquad\qquad\qquad = \pi \times (5 \cdot 318)^2 \times 4 \cdot 5$

   $\qquad\qquad\qquad = 399 \cdot 81...$ [Step 5]

Volume of water tank, in m^3:

   400 m^3 (nearest m^3)

Question 14   **20 Marks**

A small sphere has a radius of 1·5 cm.

(a) Find the **volume** of the small sphere. Give your answer in cm^3, in terms of π.

> Volume of small sphere $= \dfrac{4}{3} \times \pi \times r^3 \qquad = \dfrac{9}{2} \pi$ cm^3

The volume of a large sphere is three times the volume of the small sphere.

(b) Find the **radius** of the large sphere.

Give your answer in cm, in the form $\dfrac{a \sqrt[3]{a}}{b}$, where $a, b \in \mathbb{N}$.

> Radius of large sphere $= R$
>
> Volume of large sphere $= \dfrac{4}{3} \times \pi \times R^3 = 3 \times \left(\dfrac{9}{2} \pi \right) = \dfrac{27}{2} \pi$
>
> $\Rightarrow \quad R^3 = \dfrac{27 \times 3}{2 \times 4} = \dfrac{81}{8}$
>
> $\Rightarrow \quad R = \sqrt[3]{\dfrac{81}{8}} = \dfrac{3\sqrt[3]{3}}{2}$ cm

Marking Scheme – Paper 2

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect), scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5-mark scale	0, 5	0, 2, 5	0, 2, 3, 5		
10-mark scale		0, 5, 10	0, 4, 7, 10	0, 3, 5, 8, 10	
15-mark scale			0, 5, 10, 15	0, 4, 8, 12, 15	
20-mark scale					0, 4, 8, 12, 16, 20

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)
- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)
- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

E-scales (six categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response just below half-right (low mid partial credit)
- response just above half-right (high mid partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work, or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may be awarded. Thus, for example, in Scale 10C, 9 marks may be awarded.

No marks may be awarded other than those on the appropriate scale, and *Full Credit –1*.

In general, accept a candidate's work in one part of a question for use in subsequent parts of the question, unless this oversimplifies the work involved.

Summary of mark allocations and scales to be applied

Question 1 (15)

(a)   5B

(b)   10C

Question 2 (20)

(a)   5B

(b)   5C

(c)   10B

Question 3 (20)

(a)   10C

(b)   5B

(c)   5B

Question 4 (30)

(a)   5A

(b)   15C

(c)   10C

Question 5 (25)

(a)   15C

(b)–(d) 10C

Question 6 (20)

(a)   5C

(b)   5B

(c)(i)&(ii) 10C

Question 7 (15)

(a)   5B

(b)   10C

Question 8 (15)

(a)   5B

(b)   10C

Question 9 (30)

(a)&(b) 10C

(c)   10C

(d)   10B

Question 10 (15)

(a)&(b) 10D

(c)   5B

Question 11 (20)

(a)   10B

(b)   10C

Question 12 (20)

(a)   5A

(b)   15C

Question 13 (35)

(a)   5C

(b)   10C

(c)   20E

Question 14 (20)

(a)   5B

(b)   15D

Detailed marking notes

Question 1 (15 Marks)

(a)   **Scale 5B (0, 2, 5)**

Partial Credit
- One percentage correct

(b)   **Scale 10C (0, 4, 7, 10)**

Low Partial Credit
- Correct table for *IM* or *Twitter*
- One graph correctly drawn

High Partial Credit
- Correct tables for *IM* and *Twitter*
- Two graphs correctly drawn
- No labels or incorrect labels, and points not joined

Full Credit −1
- No labels, or incorrect labels, or points not joined

Full Credit
- Accept correct points joined with reasonable curves

Question 2 (20 Marks)

(a)   **Scale 5B (0, 2, 5)**

Partial Credit
- 4×5 or 4×3 or 5×3
- $4 \times 5 \times 3$
- Mention of the Fundamental Principle of Counting
- Attempt at a tree diagram or a listing of possible dinners

(b)   **Scale 5C (0, 2, 3, 5)**

Low Partial Credit
- Mention of 360°
- 72° or 120°
- $\dfrac{1}{5}$ or $\dfrac{1}{3}$
- One diagram correct but angle not calculated

High Partial Credit
- One angle and relevant diagram correct
- Both angles correct
- Both diagrams correct but angles not calculated

Full Credit −1
- No units (degree symbol), or diagrams reversed

Full Credit
- Allow a tolerance of $\pm 2°$

(c)   **Scale 10B (0, 5, 10)**

Partial Credit
- Correct answer, no justification
- Use of extra starter or extra main course or extra dessert

Question 3 (20 Marks)

(a)   **Scale 10C (0, 4, 7, 10)**

Low Partial Credit
- One correct

High Partial Credit
- Two correct

(b)   **Scale 5B (0, 2, 5)**

Partial Credit
- One step correct

(c)   **Scale 5B (0, 2, 5)**

Partial Credit
- One advantage
- One disadvantage

Full Credit –1
- Answers reversed

Question 4 (30 Marks)

(a)   **Scale 5A (0, 5)**

(b)   **Scale 15C (0, 5, 10, 15)**

Low Partial Credit
- Area of one relevant side correct
- Number of tiles along one side correct – 7 or 40 or 75
- Area of one tile correct

High Partial Credit
- Total area correct
- Number of tiles for two non-congruent sides correct
- Error(s) in conversion of units but finished correctly
- Correct answer, no work shown

(c)   **Scale 10C (0, 4, 7, 10)**

Low Partial Credit
- Height = 1·3 or 130
- Correct formula: $V = l \times w \times h$

High Partial Credit
- Volume = $15 \times 8 \times 1·3$
- Volume = $15 \times 8 \times 1·4 = 168$
- Error in conversion of units but finished correctly

Full Credit –1
- No units or incorrect units

Full Credit
- Accept work based on candidates measurements from part (b)

Question 5 (25 Marks)

(a)   **Scale 15C (0, 5, 10, 15)**

Low Partial Credit

- One line correct
- Correct formula: $y = mx + c$
- r and p swapped and n and s swapped

High Partial Credit

- Three lines correct
- r and p swapped or n and s swapped, and the other pair correct

(b)–(d) Scale 10C (0, 4, 7, 10)

No Credit

- x or y used

Low Partial Credit

- One correct sentence

High Partial Credit

- Two correct sentences

Full Credit

- Accept the same letter (n, p, r, or s) in both boxes in part (b)

Question 6 (20 Marks)

(a) **Scale 5C (0, 2, 3, 5)**

Low Partial Credit
- Indication of $x = 0$ or $y = 0$
- $x = 2 \cdot 5$ or $y = -5$
- $y = mx + c$ or $y = 2x - 5$

High Partial Credit
- $(2 \cdot 5, 0)$ or $(0, -5)$
- $x = 2 \cdot 5$ and $y = -5$
- Points in incorrect boxes

(b) **Scale 5B (0, 2, 5)**

Partial Credit
- Correct formula: $y = mx + c$ stated, or $y = mx + c$ used
- Indication of $-\dfrac{a}{b}$
- $y = 2x - 5$
- Correct formula: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
- Indication that the slope $= \dfrac{\text{rise}}{\text{run}}$
- Points correctly plotted

Full Credit –1
- $\dfrac{5}{2 \cdot 5}$

(c) **Scale 10C (0, 4, 7, 10)**

Low Partial Credit
- Slope correct in part (i)
- Reference to the property of perpendicular lines – e.g. invert and change the sign, or the product of the slopes is −1
- Indication that "perpendicular" implies that the lines make an angle of 90°
- Correct formula: $y - y_1 = m(x - x_1)$ or $y = mx + c$ in part (ii)
- Some correct substitution into formula $y - y_1 = m(x - x_1)$ or $y = mx + c$ in part (ii)

High Partial Credit
- Slope $= -\dfrac{1}{2}$ and some correct substitution of the point into $y - y_1 = m(x - x_1)$ or $y = mx + c$

Full Credit
- Accept $y - 6 = -\dfrac{1}{2}(x - 11)$

Question 7 (15 Marks)

(a)   **Scale 5B (0, 2, 5)**

Partial Credit

- Indication that corresponding angles are equal - e.g. $|\angle ACB| = 3x$ or $|\angle ADE| = 70^o$
- Indication that the sum of the angles in a triangle is 180°

Full Credit –1
-   No units (degree symbol)

(b)   **Scale 10C (0, 4, 7, 10)**

Low Partial Credit

- Indication that corresponding sides are in proportion
- One correct relevant ratio
- Corresponding sides identified
- Scale factor identified

High Partial Credit

- $\dfrac{|BC|}{130} = \dfrac{74}{100}$ or equivalent
- $|BC| = 96\cdot 2$ with no work shown

Question 8 (15 Marks)

(a)   **Scale 5B (0, 2, 5)**

Partial Credit

-   Indication that the angle at the centre of a circle is 180°

Full Credit –1
- Converse of Corollary 3, e.g. accept "in a right-angled triangle, the hypotenuse is the diameter of a circle" for *Full Credit –1*

(b)   **Scale 10C (0, 4, 7, 10)**

Low Partial Credit
- Any correct trigonometric ratio
- Some use of Pythagoras' theorem

High Partial Credit

- $\sin X = \dfrac{10}{30}$ or $X = \sin^{-1}\left(\dfrac{10}{30}\right)$ or similar
- $|\angle RST| = 70\cdot 5^o$
- Correct answer, no work shown

Full Credit –1
- Answer not rounded or incorrectly rounded, or early rounding, or calculator in incorrect mode

Question 9 (30 Marks)

(a)&(b) Scale 10C (0, 4, 7, 10)

Low Partial Credit
- 50 or more messages = 22
- Fraction with denominator 25 in part (a) or 22 in part (b)
- 50 - 70 messages = 10 in part (b)
- $P = \dfrac{\#\,\text{successes}}{\#\,\text{total}}$ or similar
- 22:25 in (a), or 10:22 (or 5:11) in (b)

High Partial Credit
- One part correct
- 22:25 in (a), and 10:22 (or 5:11) in (b)

Full Credit −1
- 0·9 in (a) and/or 0·5 in (b)

Full Credit
- Accept 0·45 in (b)

(c)   Scale 10C (0, 4, 7, 10)

Low Partial Credit
- Indication of division by 25
- One correct mid-interval value

High Partial Credit
- Consistent incorrect mid-interval values
- $(1)(15)+(2)(40)+(10)(60)+(7)(85)+(5)(130)$
- 1940

Full Credit −1
- $\dfrac{1940}{25}$, or one incorrect mid-interval value

(d)   Scale 10B (0, 5, 10)

Partial Credit
- Indication of the lower value of an interval × number of students
- Lower value of each interval listed (and 160 not listed)

Full Credit −1
- Largest value found (answer = 2330)

Question 10 (15 Marks)

(a)&(b) Scale 10D (0, 3, 5, 8, 10)

Low Partial Credit
- One value correct

Mid Partial Credit
- Two values correct

High Partial Credit
- Three values correct

Full Credit
- Accept values not filled into table

(c)   Scale 5B (0, 2, 5)

Partial Credit
- Shows correct understanding of the median of a list

Full Credit
- A list of numbers not containing 24, with 24 as the median – 24 does not have to be identified as the median

Question 11 (20 Marks)

(a)   Scale 10B (0, 5, 10)

Partial Credit
- Construction lines not shown
- Indication that *PARK* is a rhombus and line *PR* drawn

Full Credit –1
- Wrong angle correctly bisected

Full Credit
- Allow a tolerance of ± 2°

(b)   Scale 10C (0, 4, 7, 10)

Low Partial Credit
- One step correct
- Indication of isosceles triangle
- Mention of congruent triangles
- Indication that the opposite sides of a parallelogram are equal in length

High Partial Credit
- Two steps correct with one valid reason
- Three steps correct with no reasons or incorrect reasons

Full Credit
- Must have reasons for Steps 1 and 2, and Step 3 for full credit

Question 12 (20 Marks)

(a)   **Scale 5A (0, 5)**

(b)   **Scale 15C (0, 5, 10, 15)**

Low Partial Credit
- Indication of Pythagoras' Theorem
- One right-angled triangle drawn with sides correctly indicated
- One answer in decimal form, with or without work

High Partial Credit
- $7^2 = 4^2 + x^2$ or $x^2 = 4^2 + 7^2$
- One answer correct in surd form
- Both answers in decimal form, with or without work

Question 13 (35 Marks)

(a)   **Scale 5C (0, 2, 3, 5)**

Low Partial Credit
- Any correct trigonometric ratio
- $\tan 30° = \dfrac{20}{x}$ and finished correctly
- Some correct use of Pythagoras' theorem

High Partial Credit
- $\tan 30° = \dfrac{x}{20}$ or $x = 20\tan 30°$ or similar

Full Credit –1
- Answer not rounded or incorrectly rounded

(b) **Scale 10C (0, 4, 7, 10)**

Low Partial Credit
- Any correct trigonometric ratio
- Indication of $x + h$
- Correct triangle with 38° or h indicated
- Some correct use of Pythagoras' theorem
- 38° at the top and $\tan 30° = \dfrac{20}{x + h}$

High Partial Credit
- $\tan 38° = \dfrac{x + h}{20}$ or $x + h = 20\tan 38°$ or similar
- 38° at the top and finished correctly (consistent with diagram)

Full Credit –1
- Answer not rounded or incorrectly rounded, or no units or incorrect units

(c) **Scale 20E (0, 4, 8, 12, 16, 20)**

Low Partial Credit
- One step correct

Low-Mid Partial Credit
- Two steps correct

High-Mid Partial Credit
- Three steps correct

High Partial Credit
- Four steps correct

Full Credit –1
- Answer not rounded or incorrectly rounded, or answer in part (b) used

Full Credit
- Allow a tolerance of $\pm 0\cdot 2$ cm

Question 14 (20 Marks)

(a)   **Scale 5B (0, 2, 5)**

Partial Credit

- Correct formula: $V = \dfrac{4}{3}\pi r^3$

- Volume $= \dfrac{4}{3} \times \pi \times 1 \cdot 5^3$

Full Credit –1

- Answer not in terms of π

(b)   **Scale 15D (0, 4, 8, 12, 15)**

Low Partial Credit

- $3\left(\dfrac{9}{2}\pi\right)$ or similar

- Correct formula: $V = \dfrac{4}{3}\pi r^3$

Mid Partial Credit

- $\dfrac{4}{3}\pi r^3 = 3\left(\dfrac{9}{2}\pi\right)$ or $\dfrac{27}{2}\pi$

- $3\left(\dfrac{4}{3}\pi r^3\right) = \dfrac{9}{2}\pi$ and r found (in any form)

High Partial Credit

- $r^3 = \dfrac{27 \times 3}{2 \times 4}$ or $\dfrac{81}{8}$

- Answer in incorrect form, e.g. $2\cdot 16$

Bonus marks for answering through Irish

Bonus marks are applied separately to each paper, as follows:

If the mark achieved is 225 or less, the bonus is 5% of the mark obtained, rounded **down**.
For instance, 198 marks $\times$ 5% $= 9 \cdot 9 \Rightarrow$ bonus $= 9$ marks.

If the mark achieved is above 225, the following table applies:

Bunmharc (Mark achieved)	Marc Bónais (Bonus mark)	Bunmharc (Mark achieved)	Marc Bónais (Bonus mark)
226	11	261 – 266	5
227 – 233	10	267 – 273	4
234 – 240	9	274 – 280	3
241 – 246	8	281 – 286	2
247 – 253	7	287 – 293	1
254 – 260	6	294 – 300	0