AN ROINN OIDEACHAIS AGUS EOLAÍOCHTA | DEPARTMENT OF EDUCATION AND SCIENCE

*Scéim Mharcála* — *Scrúdú an Teastais Shóisearaigh, 2002*

*Matamaitic* — *Ardleibhéal*

*Marking Scheme* — *Junior Certificate Examination, 2002*

*Mathematics* — *Higher Level*

# MARKING SCHEME 2002

# JUNIOR CERTIFICATE EXAMINATION

# MATHEMATICS

# HIGHER LEVEL

# PAPER 1

## GENERAL GUIDELINES FOR EXAMINERS

**1.** Penalties of three types are applied to candidates' work, as follows:

- Blunders - mathematical errors / sign errors / omissions (–3 )
- Slips - numerical errors (–1)
- Misreadings (provided task is not oversimplified) (–1)

Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled as B1, B2, B3, ….. S1, S2, S3, ….. M1, M2, …. etc.

**2.** When awarding attempt marks, e.g. Att(3), it is essential to note that

- any correct relevant step in a part of a question merits, *at least*, the attempt mark for that part
- if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
- a mark between zero and the attempt mark is not awarded.

**3.** Worthless work must be awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2, …. etc.

**4.** The *same* error in the *same* section of a question is penalised *once* only.

**5.** Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

**6.** Particular cases, verifications and answers derived from diagrams (unless requested) qualify for the attempt mark only.

**7.** The phrase "and stops" means that no more work is shown by the candidate.

---

# QUESTION 1

| Each Part | 10 marks | Att 3 |
|---|---|---|

**(i)** **10 marks** **Att 3**

An estate agent's fee for selling a house is €1350.
This fee is 3% of the selling price of the house. Calculate the selling price.

$3\% = 1350 \quad \Rightarrow \quad 1\% = 450 \quad \Rightarrow \quad 100\% = 45\,000$

**or** $100\% = \frac{1350}{3} \times 100 = 45\,000.$

* Accept correct answer and no work.

Blunders (– 3)

| | | | |
|---|---|---|---|
| B1 | Error with decimal point, e.g. 1% = £4500. | | |
| B2 | Mathematical error, e.g. 3 % taken as some incorrect fraction. | | |
| B3 | Fails to get 100% - stops at 1% | ➔ | €450 |
| B4 | Multiplies 450 by some incorrect % | | |
| B5 | Gets 3 % of 1350 and multiplies by 100 | ➔ | €4050 |
| | But **note** if stops at €40.50 also incurs B3 | ➔ | 4 marks |
| B6 | Get 103 % of 1350 | ➔ | €1390.50 |
| B7 | Takes selling price as 97 %, i.e. multiplies 450 by 97 | ➔ | €43 650 |
| | But **note** 1350 x 97 = 130 950 incurs 2 blunders | ➔ | 4 marks |
| B8 | Takes selling price as 103 %, i.e. multiplies 450 by 103 | ➔ | €46 350 |
| B9 | Takes 1350 as 103 % | ➔ | €1310.68 |

Slips (– 1)

S1 Numerical errors to a max of 3.

Misreadings (– 1)

M1 Reads as €1530 or similar.

Attempts (3 marks)

A1 Selling price = 100 % and stops.

A2 $3\% = \frac{3}{100}$ and stops.

Worthless (0)

W1 Incorrect answer with no work, but accept €450 with no work as B3 ➔ 7 marks.

| |
|---|
| A person travelled at an average speed of 72 km/hr for 4 hours and 20 minutes. How far did the person travel? |

| |
|---|
| Distance $= 72 \times 4\frac{1}{3} = 312$ km or 312 000 m |

* No penalty if units omitted.

Blunders (– 3)

B1 Error with decimal point, e.g. 4 – 20 = 4.2 or 4.3, but accept 4.33.

B2 Mathematical error, e.g. $4\,{}^{1}\!/_{3} = \frac{14}{3}$.

B3 Incorrect (relevant) formula e.g. $\text{Distance} = \frac{\text{Speed}}{\text{Time}}$ or $\text{Distance} = \frac{\text{Time}}{\text{Speed}}$.

B4 No multiplication e.g. stops at $72 \times 4\frac{1}{3}$ but **note:** stops at $72 \div 4\frac{1}{3}$ or $4\frac{1}{3} \div 72$ incurs 2 blunders ➔ 4 marks; **note** also A4 below.

B5 Each error in conversion – if done, e.g. 72000 x 260 = 18 720 000 or 72 x 260 = 18 720, but **note** $\frac{72000 \times 260}{60} = 312\,000$ is correct.

Slips (– 1)

S1 Numerical errors to a max of 3.

Attempts (3 marks)

A1 Some correct conversion relevant to values given.

A2 Distance = Speed x Time and no work.

A3 Correct answer without work.

A4 Stops at 72 x 4hrs 20mins.

A5 Multiplies 72 by 4 to get 288 – oversimplification.

Worthless (0)

W1 Incorrect answer with no work.

W2 Stops at $\frac{72}{4-20}$ *or* $\frac{4-20}{72}$.

A box is in the shape of a cube of side 7 cm.
Find the volume of the largest sphere which will fit exactly in the box.
Take $\pi = \frac{22}{7}$.

Radius of sphere = 3.5 cm.
Volume of sphere = $\frac{4}{3} \times \frac{22}{7} \times 3.5^3 = \frac{539}{3} = 179.67$ or $179\frac{2}{3}$ $cm^3$

* Accept a misreading of formula for volume of sphere as $\frac{4}{8}\pi r^3$.

* Accept 180, 179.7, 179.66, 179.6, but **note** S3 below.

Blunders (– 3)
B1 Mathematical error e.g. $3.5^3 = 10.5$
B2 Incorrect substitution into correct formula
B3 Uses 7 as radius of sphere
B4 Misplaced decimal

Slips (– 1)
S1 Numerical errors to a max of 3.
S2 Uses some other approximation for π.
S3 Gives answer as 179.

Attempts (3 marks)
A1 Correct answer and no work
A2 Correct volume of box i.e. 343, but **note** W3 below
A3 Some correct substitution into correct formula for volume of sphere and stops
A4 Radius of sphere = 3.5 and stops

Worthless (0)
W1 Incorrect answer and no work
W2 Volume of sphere = $\frac{4}{3}\pi r^3$ and stops
W3 Volume of box = $7^3$ and stops

**Part (iv)** **10 marks** **Att 3**

Evaluate $\sqrt{\dfrac{1}{0.25}} + (0.6)^2$.

$\sqrt{\dfrac{1}{0.25}} + (0.6)^2 = 2 + 0.36 = 2.36$

Blunders (– 3)

B1 Error with decimal point e.g. $(0{\cdot}6)^2 = 3{\cdot}6$ or $\sqrt{\dfrac{1}{0.25}} = 20$

B2 Mathematical error in reading the tables e.g. wrong page

B3 Assumes $\sqrt{\ }$ sign extends over the $(0{\cdot}6)^2$ ➔ 2·088

B4 Error in calculating reciprocal

B5 Error in getting square root or no root

B6 Error in squaring or failure to square

B7 Failure to perform addition i.e. stops at $2 + 0{\cdot}36$

B8 Mishandles common denominator, e.g. $\dfrac{1}{0.5} + 0.36 = \dfrac{1.36}{0.5}$.

Slips (–1)

S1 Numerical errors to a max of 3.

S2 Slip in reading tables. e.g. reads adjacent row or column.

Attempts (3marks)

A1 Evaluates $\dfrac{1}{0\cdot25}$ as 4 and stops.

A2 Evaluates $(0{\cdot}6)^2$ as 0·36 or similar and stops.

A3 Evaluates $\sqrt{0.25}$ as 0.5 and stops.

Worthless (0)

W1 Mishandles the reciprocal and stops.

W2 Mishandles the square e.g. evaluates $(0{\cdot}6)^2$ as 3·6 and stops.

W3 Mishandles the square root and stops.

**Part (v)** **10 marks** **Att 3**

| If $\frac{3}{a} = \frac{4}{b} - \frac{1}{c}$, express $c$ in terms of $a$ and $b$. |
|---|

| | |
|---|---|
| **1** | $\frac{3}{a} = \frac{4}{b} - \frac{1}{c} \Rightarrow \frac{1}{c} = \frac{4}{b} - \frac{3}{a} = \frac{4a-3b}{ab} \Rightarrow c = \frac{ab}{4a-3b}$ or $c = \frac{-ab}{3b-4a}$ |
| **2** | $\frac{3}{a} = \frac{4}{b} - \frac{1}{c} \Rightarrow \frac{3abc}{a} = \frac{4abc}{b} - \frac{abc}{c} \Rightarrow 3bc - 4ac = -ab \Rightarrow c = \frac{-ab}{3b-4a}$ |

* Accept $c = \frac{1}{\frac{4}{b} - \frac{3}{a}}$ or similar correct expression for $c$.

Blunders (– 3)

B1 Each different transposition error or error with sign.

B2 Mishandles or omits common denominator.

B3 Error in distributive law, e.g. $c(4a - 3b) = 4ac - 3b$.

B4 Stops at $\frac{1}{c} = \frac{4a-3b}{ab}$, but stops at $\frac{1}{c} = \frac{4}{b} - \frac{3}{a}$ gives 2 Blunders ➔ 4 marks.

B5 Correctly finds $a$ or $b$ in terms of the other two variables.

B6 Each term incorrect when multiplying across, cf. method **2** above.

Attempts (3marks)

A1 Some correct relevant work, e.g. one correct transposition or cross multiplication.

A2 Stops at $-\frac{1}{c} = \frac{3}{a} - \frac{4}{b}$.

A3 Correct answer without work.

Worthless (0)

W1 Incorrect answer and no work.

W2 Inverts to get $\frac{a}{3} = \frac{b}{4} - c$ etc.

**Part (vi)** **10 marks** **Att 3**

Find the value of $n$ for which $\frac{4}{2^{n+1}} = 32$.

$$\frac{4}{2^{n+1}} = 32 \Rightarrow \frac{2^2}{2^{n+1}} = 2^5 \Rightarrow 2^{1-n} = 2^5 \Rightarrow 1-n = 5 \Rightarrow n = -4.$$

* Accept $n = -4$ verified correctly.

B1 Each different error in laws of indices.

B2 Each different transposition error.

B3 Each sign error.

B4 Stops at $1 - n = 5$.

B5 Stops at $2^{1-n} = 2^5$ but **note** also incurs B4 ➔ 4 marks.

B6 Mathematical errors, e.g. $2^{n+1} = \frac{32}{4}$ and continues.

Attempts (3 marks)

A1 States $4 = 2^2$ or $32 = 2^5$ and stops.

A2 Stops at $\frac{2^2}{2^{n+1}} = 2^5$.

A3 Correct answer without work.

A4 Correct substitution of $n = -4$ into $\frac{4}{2^{n+1}}$.

A5 Some correct cross multiplication or correct use of indices.

A6 Some incorrect cancelling, e.g. $\frac{2}{n+1} = 5$.

Worthless (0)

W1 Incorrect answer and no work.

**Part (vii)** **10 marks** **Att 3**

| If $\log_3 p = 5$, calculate the value of $p$. |
|---|

| $\log_3 p = 5 \Rightarrow p = 3^5 = 243.$ |
|---|

* Accept correct answer and no work.

Blunders (– 3)

B1 Stops at $p = 3^5$, or $3^5 = 15$ or similar, but **note** A3 below.

B2 Gets $p = 5^3 = 125$ but **note** also incurs B1 if stops at $p = 5^3$ ➔ 4 marks.

B3 Gets $p = \sqrt[3]{5}$ or $\sqrt[5]{3}$, but must evaluate else, also incurs B1.

Attempts (3 marks)

A1 Indicates some knowledge of Logs / Indices.

A2 List relevant rule i.e. $\log_b a = c \Rightarrow a = b^c$.

A3 $\log_3 p = 5 \Rightarrow p = 3 \times 5 = 15$, or 15 without work.

Worthless (0)

W1 Indicates no knowledge of Logs / Indices e.g. $p = 5/3$ or similar.

**Part (viii)** **10 marks** **Att 3**

| |
|---|
| If $x * y = x^2 + 2y + 3$, find the two values of $a$ for which $a * a = 6$. |

| |
|---|
| $a * a = a^2 + 2a + 3 = 6 \Rightarrow a^2 + 2a - 3 = 0 \Rightarrow (a-1)(a+3) = 0 \Rightarrow a = 1, \quad a = -3.$ |

* Accept $a = 1$ **and** $a = -3$ verified correctly.

Blunders (– 3)

B1 Error in substitution but **note** A4 below.

B2 Each different transposition error or no transposition.

B3 Each different mathematical error.

B4 Correct factors but roots not stated or incorrect roots.

B5 Incorrect factors each time and continues, but **note** $(a + 1)(a - 3)$ in only **one** Blunder.

B6 Solves $a^2 + 2a + 3 = 0$.

B7 "Factorises" $x^2 + 2y - 3 = 0$, to get $x = 1$ and $x = -3$.

Slips (– 1)

S1 Numerical errors to a max of 3.

Attempts (3 marks)

A1 Correct answers without work or verification.

A2 Some correct substitution of $a$.

A3 One correct value verified.

A4 Work not leading to a quadratic merits attempt at most.

A5 $a^2 + 2a - 3 = 0$ and stops.

Worthless (0)

W1 Treats $*$ as multiplication.

W2 Incorrect value of $a$ substituted, e.g. $a = 6$.

**Part (ix)** **10 marks** **Att 3**

| |
|---|
| Express $\sqrt{72} - \sqrt{8}$ in the form $k\sqrt{2}$ where $k \in \mathbf{N}$. |

| |
|---|
| $\sqrt{72} - \sqrt{8} = \sqrt{36 \times 2} - \sqrt{4 \times 2} = 6\sqrt{2} - 2\sqrt{2} = 4\sqrt{2}$ |

* Accept correct answer and no work or states $k = 4$ and no work.

* Accept $\sqrt{72} - \sqrt{8} = 8.485 - 2.828 = 5.657 = \sqrt{32} = 4\sqrt{2}$

Blunders (– 3)

B1 States $\sqrt{72} = a\sqrt{2}$, but $a \neq 6$.

B2 States $\sqrt{8} = b\sqrt{2}$, but $b \neq 2$.

B3 Gets $\sqrt{32}$ and stops.

B4 Gets $\sqrt{9 \times 8} - \sqrt{8} = 3\sqrt{8} - \sqrt{8} = 2\sqrt{8}$ and stops.

B5 $\sqrt{32} = 2\sqrt{4}$

B6 No subtraction e.g. stops at $6\sqrt{2} - 2\sqrt{2}$.

B7 Each error in handling the square root, e.g. $8(\sqrt{9} - \sqrt{1})$ and continues.

Slips (– 1)

S1 Numerical to a max of 3.

Misreadings (– 1)

M1 Reads as $\sqrt{72} + \sqrt{8}$.

Attempts (3 marks)

A1 Shows some knowledge of handling surds.

A2 Any factorisation which contains a perfect square.

A3 $\sqrt{72} - \sqrt{8} = 8.485 - 2.828 = 5.657$ and stops.

Worthless (0)

W1 $\sqrt{72} - \sqrt{8} = \sqrt{64}$ even if finishes.

**Part (x)** **10 marks** **Att 3**

| |
|---|
| Solve the equation $x^2 - x - 6 = 0$. <br> Hence, or otherwise, solve the inequality $x^2 - x - 6 \le 0, \quad x \in \mathbf{R}$. |

| |
|---|
| $x^2 - x - 6 = 0 \Rightarrow (x+2)(x-3) = 0 \Rightarrow x = -2, x = 3$ <br> $x^2 - x - 6 \le 0$ for $-2 \le x \le 3$. |

* Accept correct indication on graph or number line.

Blunders (– 3)

B1 Incorrect factors each time and continues, but **note** $(x - 2)(x + 3)$ in only **one** Blunder.
B2 One solution where there should be two.
B3 Correct solution of quadratic and stops.
B4 Stops at $(x+2)(x-3) = 0$ but **note** also incurs B3
B5 Gives solution for $x^2 - x - 6 \ge 0$, if not a misreading.
B6 Gives solution for $x \in \mathbf{Z}$.

Slips (– 1)

S1 Omits equal sign in the inequality, i.e. $-2 < x < 3$.
S2 States that $x$ is between – 2 and + 3.

Attempts (3 marks)

A1 Some quadratic curve drawn.
A2 Incorrect factors and stops.
A3 Correct test on one or more values.

Worthless (0)

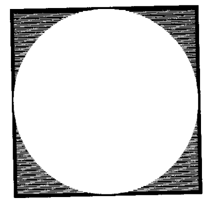
W1 Number line drawn with or without points indicated.

# QUESTION 2

| | | |
|---|---|---|
| **Part (a)** | **10 marks** | **Att 4** |
| **Part (b)** | **20 marks** | **Att 7** |
| **Part (c)** | **20 marks** | **Att 6** |

**Part (a)** **10 marks** **Att (2, 2)**

A circle fits exactly inside a square of area 49 $cm^2$ as shown.



Calculate

**(i)** the length of a side of the square

**(ii)** the area of the shaded region

Take $\pi = \frac{22}{7}$.

**(i)** **5 marks** **Att 2**

$l^2 = 49 \Rightarrow l = 7$ cm

* Accept correct answer and no work.

Blunders (– 3)

B1 Incorrect square root of 49.

B2 Takes side as 49 ÷2.

Misreadings (– 1)

M1 Reads area of circle as 49 and continues. Mark **(i)** and **(ii)** slips and blunders and treat any rounding off of $r$ as in question 1 (iii).

Attempts (2 marks)

A1 Assumes 49 is perimeter.

Worthless (0)

W1 Area equals length x breath.

**(ii)** **5 marks** **Att 2**

Area circle $= \pi r^2 = \frac{22}{7}\left(\frac{7}{2}\right)^2 = 38.5$

Shaded area $= 49 - 38.5 = 10.5$

* Accept candidates answer from **(i)** above.

Blunders (– 3)

B1 Fails to divide by 2 to get the radius.

B2 Mathematical error, e.g. $3.5^2 = 7$.

Slips (–1)

S1 Fails to subtract to get area of the shaded region.

S2 Uses some other approximation for $\pi$.

Attempts (2 marks)

A1 Correct answer without work.

A2 Some correct substitution into correct formula for area of circle.

A3 Stops at $\frac{22}{7} \times 12.25$.

A4 Indicates that $r$ is half of the answer to **a(i)** above.

| **Part (b)** | **20 marks** | **Att (2, 2, 3)** |
|---|---|---|

A cone and a sphere each has radius 2 cm.
The curved surface area of the cone equals the surface area of the sphere.
Find the slant height of the cone.

| **Setting up equality** | **5 marks** | **Att 2** |
|---|---|---|

$\pi r l = 4\pi r^2$

Blunders (– 3)

B1 Equates the curved surface area of the cone to the curved surface area of a cylinder.

B2 Equates the curved surface area of the cone to the area of a circle.

B3 Equates the curved surface area of the cone to the surface area of a hemisphere, e.g. $2\pi r^2$ or $3\pi r^2$.

B4 Uses total surface area of a cone.

Attempts (2 marks)

A1 Equates the curved surface area of the cone to some volume.

A2 Equates volume of cone to volume of sphere.
**Note:** Can continue to get 5 + 10 marks.

Worthless (0)

W1 Must have equality with $l$ else zero, unless A2 above.

**Correct substitution** **5 marks** **Att 2**

| $\pi r l = 4\pi r^2 \quad \Rightarrow \quad \pi 2 l = 4\pi(2)^2$ |
|---|

* Do not penalise incorrect formula if already penalised above.

* Accept reasonable approximation for π if used.

Blunders (– 3)

B1 Each incorrect (inconsistent) substitution into correct formula.

**Solving** **10 marks** **Att 3**

| $\pi 2 l = 16\pi \quad \Rightarrow \quad l = 8$ |
|---|

Blunders (– 3)

B1 Each transposition error.

B2 Misplaced decimal, e.g. when using a value for π.

B3 Stops at $l = \dfrac{16\pi}{2\pi}$ or similar.

Slips (–1)

S1 Stops at $\dfrac{16}{2}$, but if previous errors result in $\dfrac{17}{2}$, then no need to simplify.

Attempts (3 marks)

A1 Earlier errors result in a value for $l$ in terms of $h$.

A2 Gets value for "h" and stops, c.f. A2 in "Setting up equality" above.

Worthless (0)

W1 No use of $l$.

**Part (c)** **20 marks** **Att (3 , 3)**

Water flows through a cylindrical pipe of internal diameter 1 cm at a speed of 2 cm per second.

**(i)** Verify that the rate of flow is $\frac{11}{7}$ cm$^3$ per second, taking $\pi = \frac{22}{7}$.

**(ii)** The water from the pipe flows into an empty hemispherical bowl. It takes 36 seconds to fill the bowl. Calculate the internal radius of the bowl.

**(i)** **10 marks** **Att 3**

Volume per second $= \pi r^2 h = \frac{22}{7} \times \left(\frac{1}{2}\right)^2 \times 2 = \frac{11}{7}$

Blunders (– 3)

B1 Uses formula for volume of cone.

B2 Mathematical error e.g. $\left(\frac{1}{2}\right)^2 = 1$.

B3 Each incorrect (inconsistent) substitution into correct formula.

B4 Takes $r = 1$ i.e. fails to divide diameter by 2.

B5 Misplaced decimal, e.g. when using a value for π.

Slips (–1)

S1 Uses some other approximation for π.

Attempts (3 marks)

A1 Gets curved surface area of cylinder.

A2 Some correct substitution into a relevant volume formula.

A3 Cylinder drawn showing correct radius and/or height.

A4 States $\frac{22}{7} \times \frac{1}{2} = \frac{11}{7}$, i.e. recognises that $r = ½$.

A5 $\text{Speed} = \frac{\text{Distance}}{\text{Time}}$ or equivalent.

Worthless (0)

W1 Volume of cylinder $= \pi r^2 h$ and stops.

W2 Substitution into formula for sphere or circle.

**(ii)** **10 marks** **Att 3**

Volume of bowl = $\frac{2}{3} \times \frac{22}{7} \times r^3 = \frac{11}{7} \times 36$

$r^3 = \frac{11 \times 36 \times 3}{2 \times 22} = 9 \times 3 = 27 \Rightarrow r = 3$

* Accept candidate's answer from **(c)** **(i)** above.

* Accept $\frac{2}{8} \pi r^3$ as volume of hemisphere.

Blunders (– 3)

B1 Uses formula for volume of sphere.

B2 Assumes volume of bowl $= \frac{11}{7}$.

B3 Each incorrect (inconsistent) substitution into correct formula.

B4 Each incorrect transposition.

B5 Fails to get cube root, e.g. $r^3 = 27$ and stops or $r = \sqrt[3]{27}$ and stops.

B6 Misplaced decimal, e.g. when using a value for $\pi$.

Slips (–1)

S1 Uses some other approximation for $\pi$, but do not penalise again if same approximation used as in **c(i)**.

Attempts (3 marks)

A1 Gets volume of hemisphere of radius 36.

A2 Sets up some equation using volume of hemisphere.

A3 $\frac{11}{7} \times 36$ with or without further work.

A4 Correct formula for volume of hemisphere.

# QUESTION 3

| | | |
|---|---|---|
| **Part (a)** | **15 marks** | **Att 6** |
| **Part (b)** | **15 marks** | **Att 5** |
| **Part (c)** | **20 marks** | **Att 7** |

**Part (a)** **15 marks** **Att (2, 2, 2)**

Factorise fully each of the following:

**(i)** $x^2 - 7x + 12$

**(ii)** $4x^2 - 25y^2$

**(iii)** $27x^3 + y^3$.

**(i)** **5 marks** **Att 2**

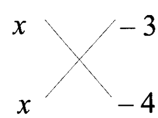
$x^2 - 7x + 12 = (x-3)(x-4)$

* Accept other correct methods, e.g. using formula, big 'X', guide number etc. and mark slips and blunders.

Blunders (– 3)

B1 Incorrect factors.

B2 Leaves answer as:




B3 Stops at $x(x-3) - 4(x-3)$.

B4 Errors in use of quadratic formula.

B5 Uses formula to get $x = 3$ and $x = 4$ but fails to form factors.

Attempts (2 marks)

A1 Any correct factors of $x^2$ and / or 12.

A2 Some correct substitution into correct quadratic formula.

A3 Indicates correct guide (key) number and stops.

A4 Writes down correct quadratic formula and stops.

**(ii)** **5 marks** **Att 2**

| |
|---|
| $4x^2 - 25y^2 = (2x-5y)(2x+5y)$ |

Blunders (– 3)

B1 Errors in sign.

B2 Stops at $2x(2x+5y) - 5y(2x+5y)$.

Attempts (2 marks)

A1 Any correct factors of $4x^2$ and / or $25y^2$.

A2 $(4x-25y)(4x+25y)$.

A3 Indicates some knowledge of the difference of two squares.

**(iii)** **5 marks** **Att 2**

| |
|---|
| $27x^3 + y^3 = (3x+y)(9x^2 - 3xy + y^2)$ |

* Accept $(3x+y)[(3x)^2 - (3x)y + y^2]$
* Apply slips and blunders if candidate divides $27x^3 + y^3$ by $3x+y$.

Blunders (– 3)

B1 Each error in sign.

B2 Treats as $(27x)^3 + y^3$ to get $(27x + y)(729x^2 - 27xy + y^2)$

B3 Each incorrect term, if it is not a slip or if it is not the same error being repeated.

Misreadings (– 1)

M1 Reads as $27x^3 - y^3$.

Attempts (2 marks)

A1 Some correct relevant work, e.g. $(3x)^3$.

A2 Treats as difference of two squares, i.e. $(\sqrt{27}\,x - y)(\sqrt{27}\,x + y)$.

A3 Correct formula for sum / difference of two cubes and stops.

A4 Indicates some knowledge of the sum / difference of two cubes.

A5 Expands $(3x + y)^3$.

A6 Gets $(3x+y)$ and stops.

A7 Any correct factors of $27x^3$ and / or $y^3$.

Worthless (0)

W1 $(27x^3 - y^3)(27x^3 + y^3)$.

**Part (b)** **15 marks** **Att 5**

Simplify

$$(2x^3 + 5x^2 - 14x + 3) \div (2x - 3).$$

$$\begin{array}{r} x^2 + 4x - 1 \\ 2x-3 \overline{)\, 2x^3 + 5x^2 - 14x + 3} \\ \underline{2x^3 - 3x^2} \\ 8x^2 - 14x \\ \underline{8x^2 - 12x} \\ -2x + 3 \\ \underline{-2x + 3} \end{array}$$

or $(2x-3)(x^2 + ax - 1)$

$\Rightarrow$ $2x^3 + 2ax^2 - 2x - 3x^2 - 3ax + 3$

$\Rightarrow$ $2a - 3 = 5$

$\Rightarrow$ $2a = 8$

$\Rightarrow$ $a = 4$

$\Rightarrow$ $x^2 + 4x - 1$ other factor.

* Accept correct answer by factorisation.

Blunders (– 3)

B1 Each error in index and sign.

B2 Incorrect or omitted middle term when factorising – apply 2 blunders → 9 marks

Attempts (5 marks)

A1 Some division to get $x^2$ and/or – 1.

A2 Sets up division correctly.

A3 Multiplies rather than divides – must have at least one correct term.

**Part (c)** **20 marks** **Att (5, 2)**

**(i)** Solve, correct to one decimal place, the equation

$$x^2 - 4x + 2 = 0.$$

**(ii)** Use your answers to part **(i)** to find, correct to one decimal place, the two values of $k$ for which

$$(k-5)^2 - 4(k-5) + 2 = 0.$$

**(i)** **15 marks** **Att 5**

$$x = \frac{4 \pm \sqrt{16 - 4(1)(2)}}{2} = \frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm 2.828}{2} = \frac{6.828}{2} \text{ or } \frac{1.172}{2}$$

$x = 3.414 = 3.4$ or $x = 0.586 = 0.6$

Blunders (– 3)

B1 Each error in formula, e.g. + b $\pm\sqrt{}$ etc

B2 Each different incorrect substitution into formula,
But **note** b = 2 ; c = – 4 → 1 Blunder.

B3 Mathematical error in sign, e.g. $-4(1)(2) = 8$.

B4 Mathematical error in squaring, e.g. $4^2 = 8$ or similar.

B5 Mathematical error in tables (wrong page).

B6 Ignores a minus in square root, e.g. $\sqrt{-8}$ taken as $\sqrt{8}$.

B7 One solution where there should be two.

B8 Misplaced decimal.

B9 Gets $\frac{4\pm\sqrt{8}}{2}$ or $2\pm\sqrt{2}$ and stops, incurs **2 blunders** and S1 below.➔ 8 marks.

Slips (–1)

S1 Failure to round off or rounds off incorrectly, once or twice.

S2 Numerical to max of 3.

Attempts (5 marks)

A1 Incorrect relevant formula with some correct substitution.

A2 Correct formula and stops.

A3 Some effort at completing the square.

Worthless (0)

W1 Some attempt at factorising.

**(ii)** **5 marks** **Att 2**

| |
|---|
| Let $x = k - 5$ |
| Then: $k-5 = 3.4 \Rightarrow k = 8.4$ or $k - 5 = 0.6 \Rightarrow k = 5.6$ |

* Accept candidates' values for $x$ from c(i) above.

* Accept quadratic in $k$ multiplied out and mark blunders and slips as in c(i) above.

Blunders (– 3)

B1 Each different error in transposition.

B2 Deals with only one value of $k$, but if only one value of $x$ in c(i) then no further penalty.

B3 No final transposition.

Slips (–1)

S1 Failure to round off or rounds off incorrectly, once or twice.

Attempts (2 marks)

A1 Correct removal of either bracket and stops.

A2 $x = k - 5$ and stops.

# QUESTION 4

| | | |
|---|---|---|
| **Part (i)** | **10 marks** | **Att 3** |
| **Part (ii)** | **10 marks** | **Att 3** |
| **Part (iii)** | **10 marks** | **Hit/Miss** |
| **Part (iv)** | **10 marks** | **Att 3** |
| **Part (v)** | **10 marks** | **Att 3** |

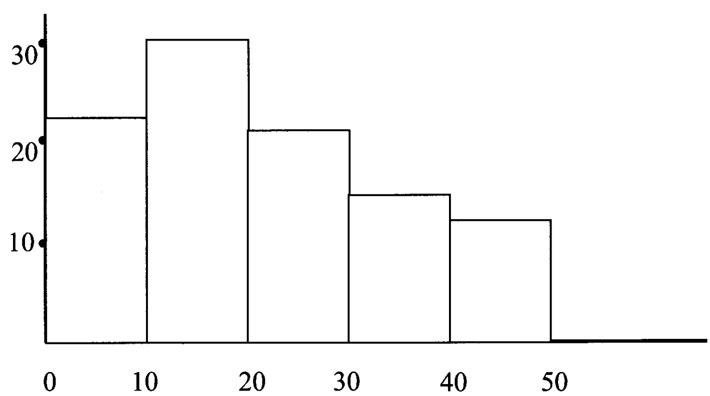
**Part (i)** **10 marks** **Att 3**

The amounts of money spent by 100 customers in a shop are recorded in the following grouped frequency table:

| Amount spent (€) | 0 - 10 | 10 - 20 | 20 - 30 | 30 - 40 | 40 - 50 |
|---|---|---|---|---|---|
| Number of customers | 22 | 30 | 21 | 15 | 12 |

(Note: 0 - 10 means €0 or more, but less than €10, etc.)

Draw a histogram to illustrate this information.




* Accept heights in correct ratio.

Blunders (– 3)

B1 Scale not uniform on horizontal axis.
B2 Bars with correct width and height but separated, i.e. bar chart, but **note** A3 below.
B3 Number of customers on horizontal axis.

Attempts (3 marks)

A1 Axes scaled or partly scaled and stops.
A2 Draws one bar and stops.
A3 Draws bar chart, but spaces bars out of proportion.
A4 Frequency polygon or curve.

Worthless (0)
W1 Pie Chart.

**Part (ii)** **10 marks** **Att 3**

| Use the mid-interval values to calculate the mean amount of money spent per customer. |
|---|

Mean =

$$\frac{5\times22+15\times30+25\times21+35\times15+45\times12}{100}=\frac{110+450+525+525+540}{100}=\frac{2150}{100}=21.5$$

Blunders (– 3)

B1 Incorrect mid-intervals. (Penalise each different blunder).

B2 Multiplies by wrong frequency. (Penalise each different blunder).

B3 $\Sigma f = 5 \Rightarrow \frac{2150}{5} = 430$.

B4 Uses upper limits, i.e. (10 x 22) + (20 x 30) + etc.

B5 Uses $\Sigma x$ instead of $\Sigma f \Rightarrow \frac{2150}{125} = 17.2$.

B6 Gets $\frac{2150}{100}$ and stops.

B7 Mathematical / decimal errors.

Slips (–1)

S1 Numerical to max of 3.

Attempts (3 marks)

A1 Uses interval size instead of mid interval value, i.e. (10 x 22) + (10 x 30) + etc.
A2 Correct answer without work.
A3 Correct formula for mean and stops.
A4 Some correct relevant work, e.g. 5 x 22.
A5 Ignores frequency, i.e. $\frac{5+15+25+35+45}{100}$ or $\frac{\Sigma x}{5}$ or similar & stops, even if continues.

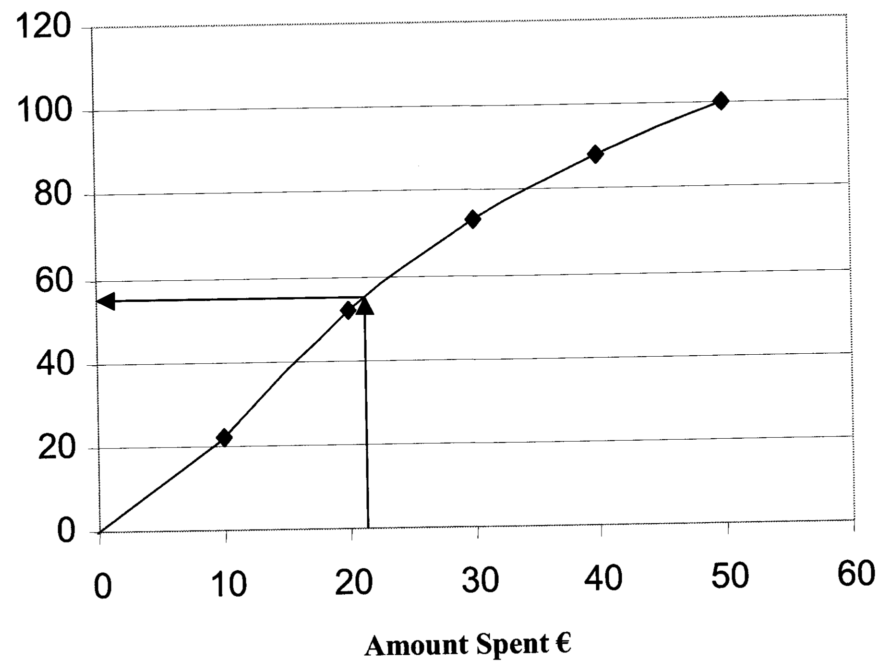
**Part (iii)** **10 marks** **5 x 2 marks**

| Copy and complete the cumulative frequency table: |
|---|

| Amount spent (€) | < 10 | < 20 | < 30 | < 40 | < 50 |
|---|---|---|---|---|---|
| Number of customers | 22 | 52 | 73 | 88 | 100 |

* **Note** Each value filled in correctly gets **2 marks.** Errors do **not** carry forward.

On graph paper, draw the ogive (cumulative frequency curve).




* Use candidate's values from his/her cumulative frequency table.

Blunders (– 3)
B1 Scale not uniform ( 1 x –3 each axis).
B2 Each point omitted or plotted incorrectly (if not consistent or slip).
B3 Points not joined or not a smooth curve.
B4 Number of customers on the horizontal axis.

Slips (–1)
S1 Slip in plotting points (to max 3), allow tolerance of 1 box using graph paper and if not on graph paper then tolerance of ± 2mm.

Attempts (3 marks)
A1 Axes scaled or partly scaled and stops.
A2 Frequency polygon /curve.
A3 Cumulative frequency histogram.
A4 Couples named, e.g. (10,22) and stops.
A5 Uses the frequency distribution table given.

Worthless (0)
W1 Pie chart, bar chart or histogram.

**Part (v)** **10 marks** **Att 3**

| |
|---|
| Use your graph to estimate the number of customers who spent the median amount or more, but less than the mean amount. |

| |
|---|
| Median $\Rightarrow$ 50 customers  Mean $\Rightarrow$ 56 customers  $\Rightarrow$ 6 customers |

* Accept answer consistent with candidate's curve (within tolerance of ± 2).
* Accept calculation using candidate's mean.

Blunders (– 3)
B1 Mean amount rather than number of customers used.
B2 Median amount rather than number of customers used, but **note** apply 1 blunder only if both B1 and B2 occur.
B3 Subtracts values from different axes..
B4 Reads up and across from €25 to get "median".

Slips (–1)
S1 Written value just outside tolerance.

Attempts (3 marks)
A1 A line drawn from correct axis to graph and stops.
A2 Finds Interquartile Range.
A3 Line or lines drawn correctly but value not indicated or written down.
A4 Finds median amount.
A5 No subtraction done or indicated.

# QUESTION 5

| | | |
|---|---|---|
| **Part (a)** | **35 marks** | **Att 13** |
| **Part (b)** | **15 marks** | **Att 6** |

**Part (a)** **35 marks** **Att (7, 2, 2, 2)**

Using the same axes and scales, draw the graphs of

$$f : x \to 2x^2 - 2x - 3$$

$$g : x \to 2 - 3x$$

in the domain $-2 \le x \le 3, \quad x \in \mathbf{R}$.

Use your graphs to estimate

**(i)** the minimum value of $f(x)$

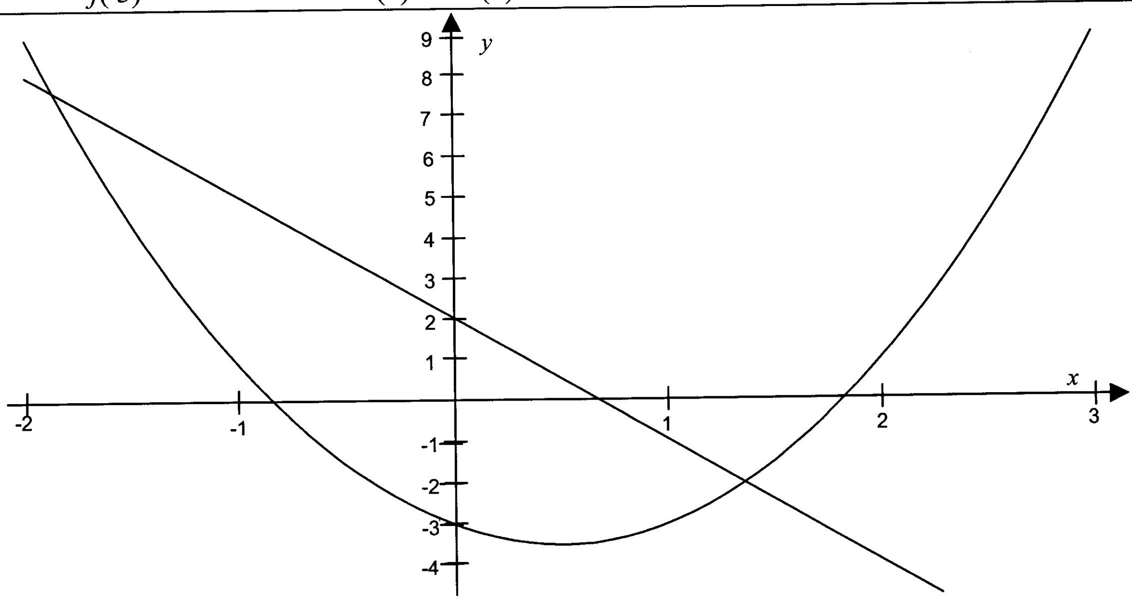
**(ii)** the values of $x$ for which $f(x) = g(x)$.

**(a) Quadratic Graph** **20 marks** **Att 7**

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|
| $2x^2$ | 8 | 2 | 0 | 2 | 8 | 18 |
| $-2x$ | 4 | 2 | 0 | $-2$ | $-4$ | $-6$ |
| $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ |
| $f(x)$ | 9 | 1 | $-3$ | $-3$ | 1 | 9 |

**or**

| | | | | |
|---|---|---|---|---|
| $f(-2)$ | = | $2(-2)^2 - 2(-2) - 3$ | = | 9 |
| $f(-1)$ | = | $2(-1)^2 - 2(-1) - 3$ | = | 1 |
| $f(0)$ | = | $2(0)^2 - 2(0) - 3$ | = | $-3$ |
| $f(1)$ | = | $2(1)^2 - 2(1) - 3$ | = | $-3$ |
| $f(2)$ | = | $2(2)^2 - 2(2) - 3$ | = | 1 |
| $f(3)$ | = | $2(3)^2 - 2(3) - 3$ | = | 9 |



Values for quadratic graph

Blunders (– 3)
B1 Each incorrect $f(x)$ without work.
B2 $x$ row added in, i.e. top row.
B3 Omits $-2x$ or $-3$ row (– 3 for each omitted).
B4 Treating the domain as $-2 < x < 3$, can incur 2 Blunders if both omitted.
B5 Each different blunder which yields an incorrect row (full or part), e.g. $(2x)^2$ for $2x^2$.
B6 Avoids square for some (not all) values.
B7 Mathematical errors in tots, e.g. $-5 + 2 = 3$.
B8 –3 row treated as $3x$ or $-3x$.

Slips (–1)
S1 Numerical slips to a max. of 3.

Attempts (7 marks)
A1 Omits $2x^2$ or does not treat as $x^2$ (Treats as linear expression).
A2 Correct or partly correct table / values but no graph drawn.

**(a) Linear Graph** **5 marks** **Att 2**

| | | | | |
|---|---|---|---|---|
| $g(-2)$ | = | $2-3(-2)$ | = | 8 |
| $g(3)$ | = | $2-3(3)$ | = | $-7$ |

* Table not necessary – Accept any two correct values (may be on graph).
* Do not penalise same error if already penalised on quadratic graph table.

Values for linear graph

Blunders (– 3)
B1 $g(x) = 2 + 3x$ and continues correctly (oversimplifies).
B2 +2 row treated as $2x$.

Attempts (2 marks)
A1 One value only calculated, but no graph drawn.

Plotting the quadratic and linear graphs
* Accept candidate's values from the table.
* Accept correct graph without work (20 marks + 5 marks).

Blunders (– 3)
B1 Points not joined to form a reasonable graph.
B2 $(x, y)$ plotted as $(y, x)$, but apply once only.
B3 + and – sides confused, e.g. (– 2 , 9) plotted as (2 , 9)
B4 Scale not reasonably uniform. 1 x (–3) each axis.
B5 Each different blunder in plotting points from candidate's table / values.
B6 Each point omitted, if graph does not go reasonably close to where point should be.
B7 Points joined with straight lines – applies to the quadratic only.

Attempts (7 marks)
A1 Scaled axis drawn.

**(a) (i)** **5 marks** **Att 2**

| Minimum $f(x) = -3.5$ |
|---|

* Accept answer consistent with candidate's curve (within tolerance of $\pm 0.4$).

Blunders (– 3)
B1 $x$ value of minimum only.

Slips (–1)
S1 Written value just outside tolerance.
S2 Gives coordinates of the minimum rather than the $y$ value.

Attempts (2 marks)
A1 Point indicated on graph only.

Worthless (0)
W1 Answer inconsistent with candidate's graph.

**(a) (ii)** **5 marks** **Att 2**

| $x = 1.4$ or $x = -1.9$ |
|---|

* Accept answer consistent with candidate's graph (within tolerance $\pm 0.2$).

Blunders (– 3)
B1 One correct value only.
B2 Correct indication on graph, but no values given.
B3 Gives answer as $-1.9 \leq x \leq 1.4$ or similar.

Slips (–1)
S1 Written values just outside tolerance.
S2 Gives coordinates of points of intersection rather than $x$ values.

Attempts (2 marks)
A1 $2x^2 - 2x - 3 = 2 - 3x$, even if completes correctly.
A2 Some indication on graph, but no values given.

**Part (b)** **15 marks** **Att (2, 2, 2)**

$h : x \rightarrow 3x + p$ and $k : x \rightarrow 4x^2 - p$ are two functions defined on **R**, where $p \in$ **Z**.

**(i)** If $h(2) = 4$, find the value of $p$.

**(ii)** Hence, find $(h \circ k)(-1)$.

**(iii)** Find the two values of $x$ for which $h(x) + k(x) = 0$.

**(i)** **5 marks** **Att 2**

| $h(x) = 3x + p \Rightarrow h(2) = 3(2) + p = 4 \Rightarrow p = -2$ |
|---|

* Accept correct answer and no work.

Blunders (– 3)

B1 Transposition errors.

B2 Substitutes in wrong value and continues.

Misreadings (– 1)

M1 $k(2) = 4$ and finds $p$.

Attempts (2 marks)

A1 Some effort at substitution and stops.

Worthless (0)

W1 Incorrect "$p$" and no work.

**(ii)** **5 marks** **Att 2**

| $k(-1) = 4-(-2) = 6; \quad (h \circ k)(-1) = h(6) = 16$ |
|---|

* Accept candidate's value of $p$ from (i) above.

Blunders (– 3)

B1 Mathematical errors, e.g. $(-1)^2 = 2$

B2 Sign errors, e.g. $(-1)^2 = -1$

B3 Stops after obtaining $k(-1)$.

B4 $(k \circ h)(-1)$

Attempts (2 marks)

A1 Some effort at evaluating either $k(-1)$ or $h(-1)$ and stops.

**(iii)** **5 marks** **Att 2**

| | | |
|---|---|---|
| $h(x) + k(x) = 0$ | $\Rightarrow 3x-2+4x^2+2 = 0$ | $\Rightarrow 3x+4x^2 = 0$ |
| | $\Rightarrow x(3+4x) = 0$ | $\Rightarrow x = 0$ or $x = -0.75$ |

* Accept candidate's value of $p$ from (i) above.

Blunders (– 3)

B1 Mathematical / sign errors.

B2 One solution where there should be two.

B3 Correct factors and stops.

B4 Stops at $3x + 4x^2 = 0$.

B5 Transposition errors.

Attempts (2 marks)

A1 Incorrect factors and stops.

A2 Sets up equation and stops.

# QUESTION 6

| | | |
|---|---|---|
| **Part (a)** | **10 marks** | **Att 4** |
| **Part (b)** | **20 marks** | **Att 7** |
| **Part (c)** | **20 marks** | **Att 7** |

**Part (a)** **10 marks** **Att (2, 2)**

A = {1, 2, 3, 4, 5}, B = {4, 5, 6, 7}, C = {3, 4, 7, 8}.
List the elements of:

**(i)** A Δ B

**(ii)** (A \ B) Δ C.

**(i)** **5 marks** **Att 2**

A Δ B = {1, 2, 3, 6, 7}

* Accept correct answer and no work.
* Accept correct indication on Venn diagram.

Blunders (– 3)
B1 Each element incorrect or omitted with or without work,
e.g. A Δ B = {1, 2, 3, 4, 6} ➔ 2 marks

Attempts (2 marks)
A1 Symmetric difference defined or illustrated.
A2 Any Venn diagram showing intersecting sets.
A3 Any answer with at least 1 correct element.

Worthless (0)
W1 An answer with no correct elements, e.g. {4, 5, 8}.

**(ii)** **5 marks** **Att 2**

(A \ B) Δ C = {1, 2, 3} Δ {3, 4, 7, 8} = {1, 2, 4, 7, 8}

* Accept correct answer and no work.

Blunders (– 3)
B1 A \ B correct and stops.
B2 Each element incorrect or omitted with or without work.
B3 A \ B incorrect and continues.

Attempts (2 marks)
A1 Symmetric difference defined or illustrated.
A2 Any Venn diagram showing intersecting sets.
A3 A \ B with at least 1correct element and stops.

Do not award marks for these attempts if already awarded in (i) above.

**Part (b)** **20 marks** **Att 7**

Solve the simultaneous equations:

$$2x - y = 5$$

$$x + 3y = \frac{x-4}{2}.$$

$x + 3y = \frac{x-4}{2} \Rightarrow 2x + 6y = x - 4 \Rightarrow x + 6y = -4$

$2x - y = 5$

$\underline{2x + 12y = -8}$

$-13y = 13 \Rightarrow y = -1$

$2x - (-1) = 5 \Rightarrow 2x = 4 \Rightarrow x = 2$

* Accept $x = 2$ and $y = -1$, if verified in <u>both</u> equations.

Blunders (–3)

B1 Incorrect common denominator.

B2 Each different error in transposing or signs.

B3 Does not multiply every term of equation, each time, e.g. $2x + 12y = -4$, but $x + 3y = x - 4$ → 2 Blunders.

B4 Mathematical error, e.g. $2x + 2x = 0$.

B5 Calculates the value of $x$ **or** $y$ correctly and stops. → 17 marks.

Slips (–1)

S1 Finds $x = 2$ but subs in some other value of $x$ to find $y$, e.g. $x = -2$.

S2 Numerical to a max. of 3.

Attempts (7 marks)

A1 $x = 2$ **and** $y = -1$ without work.

A2 Some correct relevant work, e.g. $-2x + y = -5$ and stops.

A3 Writes $x$ in terms of $y$ or vice-versa, e.g. $y = 2x - 5$ .

A4 Graphical solution.

Worthless (0)

W1 $x = 2$ **or** $y = -1$ without work.

W2 Invents value for 1 variable and continues, e.g $x = 1$ to get $y = -3$.

**Part (c)** **20 marks** **Att (2, 2, 3)**

A prize fund of €1000 was shared equally between $x$ people.
If there had been one person less, each person would have received €50 more.
Write an equation in $x$ to represent this information.
Solve this equation for $x$ and verify your answer.

**One correct expression** **5 marks** **Att 2**

| |
|---|
| $x$ people, each prize $\frac{1000}{x}$; **or** $(x-1)$ people, each prize $\frac{1000}{x-1}$ |

Blunders (–3)

B1 Value of prize inverted, e.g. $\frac{x}{1000}$ or $\frac{x-1}{1000}$, but penalise once only.

B2 Uses $x + 1$.

**Second expression & set up equation** **5 marks** **Att 2**

| |
|---|
| $\frac{1000}{x} + 50 = \frac{1000}{x-1}$ or equivalent. |

Blunders (–3)

B1 Sign error in setting up equation, e.g. $\frac{1000}{x} - \frac{1000}{x-1} = 50$.

Attempts (2 marks)

A1 No equation, but expression for other prize correct.

**Solve equation** **10 marks** **Att 3**

$$\Rightarrow 1000(x-1) + 50(x)(x-1) = 1000x$$

$$\Rightarrow 1000x - 1000 + 50x^2 - 50x = 1000x$$

$$\Rightarrow 50x^2 - 50x - 1000 = 0 \Rightarrow x^2 - x - 20 = 0 \Rightarrow (x-5)(x+4) = 0$$

$$\Rightarrow x = 5 \text{ or } x = -4$$

5 people, each prize 200, 4 people each prize 250, 50 more.

Blunders (-3)

B1 Error in the distributive law.
B2 Errors in transposition.
B3 Mathematical / sign errors.
B4 One solution where there should be two, i.e. where there are 2 positive solutions.
B5 Correct factors and stops, but **note** will also incur B7 below.
B6 Incorrect factors each time and continues, but **note** $(x + 5)(x - 4)$ in only **one** Blunder.
B7 Failure to verify answer.

Attempts (3 marks)

A1 No quadratic due previous errors, merits attempt at most.
A2 Incorrect factors and stops.
A3 Stops at $x^2 - x - 20 = 0$.
A4 $x = 5$ without work and verifies.

Worthless (0)

W1 $x = 5$ without work and stops.

# MARKING SCHEME

# JUNIOR CERTIFICATE EXAMINATION 2002

# MATHEMATICS

# HIGHER LEVEL

# PAPER 2

## GENERAL GUIDELINES FOR EXAMINERS

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled as B1, B2, B3,......, S1, S2, S3,...., M1, M2, etc. Note that these lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), it is essential to note that
   - any correct relevant step in a part of a question merits *at least* the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,....etc.

4. The *same* error in the *same* section of a question is penalised *once* only.

5. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

6. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks only.

7. The phrase "and stops" means that no more work is shown by the candidate.

8. Early rounding off, which affects the final answer, is a blunder.

9 All sign errors (unless otherwise indicated) are blunders

# QUESTION 1

| Each Part | 10 marks | Att 3 |
|---|---|---|
| **Part (i)** | **10 marks** | **Att 3** |

Calculate $\frac{3}{7}$ of 98 and express your answer as a fraction of 56.

Give your answer in its simplest form.

$\frac{98 \text{ x } 3}{7} = 14 \text{ x } 3 \quad = 42$ (step 1)

$\frac{42}{56}$ (step 2) $= \frac{3}{4}$ (step 3)

* Accept correct answer without work

*Blunders (-3)*

B1 Each step omitted or incorrect, e.g. $\frac{98 \text{ x } 7}{3}$ for step1, and $\frac{56}{42}$ for step 2

B2 Interchanges 56 and 98 & continues (answer $\frac{12}{49}$)

B3 $\frac{98}{56} \text{ x } \frac{3}{7}$ & stops

*Slips (-1)*

S1 $\frac{6}{8}$ or $\frac{21}{28}$

S2 Each numerical slip

S3 $\frac{42}{56} \text{ x } 100 = .75$

*Attempts (3 marks)*

A1 98 x $\frac{7}{3}$ & stops

A2 98 x 3 or 98 / 7 & stops

| **Part (ii)** | **10 marks** | **Att 3** |
|---|---|---|

€225 is shared among three people in the ratio $1 : \frac{3}{2} : 2$. Calculate the largest share.

$1 : \frac{3}{2} : 2. \quad = \quad 2 : 3 : 4$ (step 1)

$\frac{225}{9} = 25$ (step 2) 25 x 4 = €100 (step 3)

* Accept correct answer without work

*Blunders (-3)*

B1 Each step omitted or incorrect

B2 $\frac{9 \times 225}{4}$ (€506.25)

*Slips (-1)*

S1 Each numerical slip

S2 Any other share (50, 75) or all three shares

*Attempts (3 marks)*

A1 $4\frac{1}{2}$ & stops

A2 $1+\frac{3}{2}$ & stops

**Part (iii)** **10 marks** **Att 3**

The height of a cone is twice the radius. The volume of the cone is $\frac{16}{3}\pi$ cm$^3$.

Calculate the radius.

| | | | | | |
|---|---|---|---|---|---|
| $\frac{\pi \times r^2 \times h}{3}$ | = | $\frac{16\pi}{3}$ | (step 1) | | |
| $\pi \times r^2 (2r)$ | = | $16\pi$ | (step 2) | $\Rightarrow r = \sqrt[3]{8}$ or 2 cm | (step 3) |

* No penalty for substitution for $\pi$ or cm$^3$ omitted.

*Blunders (-3)*

B1 Each step omitted or incorrect

B2 Incorrect formula e.g. $\pi r^2 h$ or $2\pi rh$

*Slips (-1)*

S1 $r = \frac{8}{3}$

S2 $r^3 = 8$ and stops

*Worthless (0)*

W1 Vol = $\frac{1}{3}\pi r^2 h$ & stops

W2 Correct answer, no work

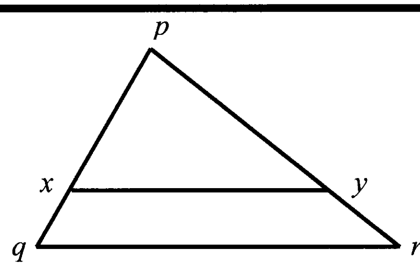
Note: $r^2 h = 16$ and stops merits 4 marks (double blunder)

**Part (iv)** **10 marks** **Att 3**

In the triangle $pqr$, $xy$ is parallel to $qr$.

$|pq| = 14$ cm, $|qr| = 21$ cm and $|xq| = 4$ cm.

Find $|xy|$.




$|px| = 10$ and / or $\dfrac{|px|}{|pq|} = \dfrac{|xy|}{|qr|}$ (step 1)

$\dfrac{10}{14} = \dfrac{|xy|}{21}$ (step 2) $\Rightarrow$ $|xy| = \dfrac{10}{14} \times 21 = 15$ cm. (step 3)

*Blunders (-3)*

B1 Each step omitted or incorrect

B2 Incorrect ratio e.g. $\dfrac{|pq|}{|px|} = \dfrac{|xy|}{|qr|}$ (gives $|xy| = 29.4$)

*Attempts (3 marks)*

A1 States or indicates similar triangles

A2 $\dfrac{|px|}{|xq|} = \dfrac{|py|}{|yr|}$

A3 Some correct indicated equal angles

A4 $\frac{21}{14}$ & stops

*Worthless (0)*

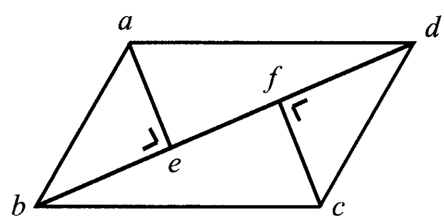
W1 Diagram with the given measurements

**Part (v)** **10 marks** **Att 3**

$abcd$ is a parallelogram.

$ae$ and $cf$ are perpendicular to $bd$ as shown.

Prove the triangles $abe$ and $dcf$ are congruent.



In triangles *abe* and *dcf*:
$|\angle bea| = |\angle dfc|$ (given) (step 1)
$|\angle abe| = |\angle cdf|$ (alt) (step 2)
$|ab| = |dc|$ (parm.) (step 3)

Hence,ASA, triangles *abe* and *dcf* are congruent.

* No penalty for omitting reasons
* Diagram only: note B3

*Blunders (-3)*
B1 Each step omitted or incorrect
B2 States $|\angle bae| = |\angle dcf|$ without justification
B3 Correct 3 steps shown on a diagram with no reason for congruency stated (ASA,AAS)

*Attempts (3 marks)*
A1 States alternate angles are equal
A2 States the opposite sides of a parallelogram are equal & stops

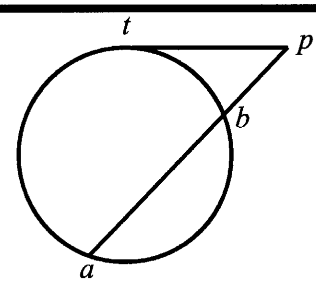
**Part (vi)** **10 marks** **Att 3**

*pt* is a tangent to the circle at *t*.

$|pt| = 8$ cm and $|ab| = 12$ cm.

Find $|pb|$.

[ Hint: Let $|pb| = x$.]




$x(x+12) = 8^2$ (step 1) $\Rightarrow x^2 + 12x = 64$ and/or $x^2 + 12x - 64 = 0$ (step 2)
$\Rightarrow (x-4)(x+16) = 0 \Rightarrow x - 4 = 0$ or $x + 16 = 0$
$\Rightarrow x = 4$ or $x = -16$, which is impossible (step 3)

* Accept $x(x+12) = 8^2 \Rightarrow x = 4$ for full marks

*Blunders (-3)*
B1 Each step omitted or incorrect

*Misreadings (-1)*
MR1 $x(x+8) = 12^2$ mark by slips and blunders

*Attempts (3 marks)*
A1 $|ab|.|bp| = |pt|^2$ & continues $(x = 5\frac{1}{3})$
A2 x + 12 & stops
A3 x (x + 12) & stops
A4 $|pb|.|pa| = |pt|^2$ & stops
A5 64 or $8^2$
A6 4 with no work or trial and error

**Part (vii)** **10 marks** **Att 3**

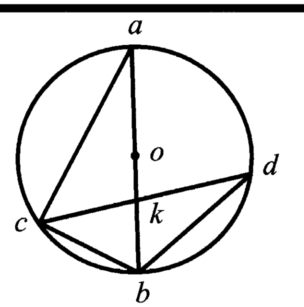
[$ab$] is a diameter of the circle of centre $o$.

$c$ and $d$ are points on the circle.

[$ab$] and [$cd$] intersect at $k$.

$|\angle cdb| = 38°$ and $|\angle ckb| = 80°$.

Write down $|\angle cab|$ and then find $|\angle dcb|$.




| | | |
|---|---|---|
| $\lvert\angle cab\rvert = \lvert\angle cdb\rvert = 38°$ | (step 1) | |
| $\lvert\angle cba\rvert = 52°$ | (step 2) | or $\lvert\angle ack\rvert = 80° - 38° = 42°$ |
| $\lvert\angle dcb\rvert = 180° - (80° + 52°) = 48°$ | (step 3) | $\lvert\angle dcb\rvert = 90° - 42° = 48°$ |

* $|\angle dcb|$ can be found without reference to step 1

*Blunders (-3)*

B1 Each step omitted or incorrect

B2 $|\angle bac| = 40°$ (i.e. half of 80°) & continues

*Attempts (3 marks)*

A1 Statement or use of any relevant theorem

A2 Joins ad & stops

**Part (viii)** **10 marks** **Att 3**

The line $2x - 3y + 12 = 0$ cuts the $x$-axis at $p$ and the $y$-axis at $q$.
Find the coordinates of the midpoint of [$pq$].

$2x - 3y + 12 = 0$

$y = 0 \Rightarrow 2x + 12 = 0 \Rightarrow x = -6;$ $p(-6,0)$ (step 1)

$x = 0 \Rightarrow -3y + 12 = 0 \Rightarrow y = 4;$ $q(0,4)$ (step 2)

midpoint $= \left(\dfrac{-6+0}{2}, \dfrac{0+4}{2}\right) = (-3,2)$ (step 3)

*Blunders (-3)*

B1 Each step omitted or incorrect

B2 Uses point (-6,4) from steps 1 & 2

B3 Incorrect relevant formula e.g. minus for plus in both (but one is a slip)

B4 p (0,-6) q (4,0) & continues

*Slips (-1)*

S1 One sign incorrect in mid-point formula (c.f. B3)

*Attempts (3 marks)*

A1 Correct mid-point formula without substitution & stops

A2 x = 0 and/or y = 0 & stops

*Worthless (0)*

W1 Any use of irrelevant formula

Note: No penalty for correct p & q without work, but marks may be lost in arriving at the correct co-ordinates for p & q if work is shown.

**Part (ix)** **10 marks** **Att 3**

Verify that the point (1, –1) is on the line $3x+2y-1=0$.
Find the equation of the image of this line under the translation $(1,-1) \to (-2,3)$.

$3x+2y-1=0;\ \ 3(1)+2(-1)-1=3-3=0 \Rightarrow (1,-1)$ is on $3x+2y-1=0$. (step 1)

$3x+2y-1=0 \Rightarrow 2y=-3x+1 \Rightarrow$ slope $m=-\frac{3}{2}$ (step 2)

$y-3=-\frac{3}{2}(x+2)$ or $2y-6=-3x-6$ or $3x+2y=0$ (step 3)

May use 3x +2y + k = 0 etc. (step 2) and gets k = 0 (step 3)

*Blunders (-3)*

B1 Each step omitted or incorrect

B2 Uses (1,-1) for (-2,3) in step 3

B3 Incorrect substitution for both variables (for equation) (Note S1 below)

B4 Incorrect relevant formula e.g. $y+y_1=m(x+x_1)$

*Slips (-1)*

S1 Error in only one sign

*Attempts (3 marks)*

A1 Some effort at substituting in step 1 & stops

A2 Diag. showing two parallel lines

A3 Correct formula for equation with / without substitution & stops

A4 Down 3, up 4 recognised & stops

A5 Any other point found on 3x + 2y –1 = 0

*Worthless (0)*

W1 (1,-1) and (-2,3) plotted correctly & stops

**Part (x)** **10 marks** **Att 3**

$\sqrt{3}\tan 2A = 1$ where $0° \le A \le 90°$. Find $A$.

$\tan 2A = \frac{1}{\sqrt{3}} (0.5774)$ (step 1) $\Rightarrow$ $2A = 30° (\frac{\pi}{6})$ (step 2) $\Rightarrow$ $A = 15°(\frac{\pi}{12})$ (step 3)

*Blunders (-3)*

B1 Each step omitted or incorrect (but note A1 below)

B2 Taking $1° = 100'$

B3 Reading wrong page of tables

B4 Decimal error reading tables e.g. $Tan^{-1} 0.05774$

B5 Ignores square root sign & continues correctly ( $\tan 2A = \frac{1}{3}, 2A = 18°26', A = 9°13'$)

*Slips (-1)*

S1 Incorrect column read in tables

S2 Each numeric slip e.g. $\frac{30}{2}$

S3 Error in halving or not done (but only where 2A is found)

*Attempts (3 marks)*

A1 Ignores $\sqrt{3}$ (oversimplifying) & continues (Tan 2A = 1 $\Rightarrow 2A = 45°$ )

A2 $\sqrt{3} = 1.7321$, or $\frac{1}{\sqrt{3}}$ or 0.5774 only & stops

A3 $30°, 60°, 90°$ triangle

A4 Right angled triangle with I and square root of 3 shown

A5 Definition of tan

*Worthless (0)*

W1 1.7321 only

W2 Reads Tan 1° or Tan 2°

# QUESTION 2

| | | |
|---|---|---|
| **Part (a)** | **30 marks** | **Att 10** |
| **Part (b)** | **20 marks** | **Att 6** |

| | | |
|---|---|---|
| **Part (a)** | **30 (15,15) marks** | **Att (5,5)** |

€750 was invested for three years at compound interest.
The rate of interest for each of the first two years was 4% per annum.
**(i)** Calculate the amount of the investment at the end of the second year.
**(ii)** At the end of the third year the amount of the investment was €851.76.
Calculate the rate of interest for the third year.

| | | |
|---|---|---|
| **(i)** | **15 mark** | **Att 5** |

| | |
|---|---|
| Interest for year 1 = $750 \times 0.04 = 30$ | (step1) |
| Principal for year 2 = $750 + 30 = 780$ | (step 2) |
| Interest for year 2 = $780 \times 0.04 = 31.20$ | (step 3) |
| Amount at end of year 2 = $780 + 31.2 = 811.20$ | (steps 4) |

*Blunders (-3)*

B1 Each step omitted or incorrect
B2 Subtracts interest (once only)
B3 Decimal blunder
B4 Uses T = 3 in $\frac{P.T.R}{100}$
B5 Takes 4% as a quarter & continues

*Attempts (5 marks)*

A1 $4\% = \frac{1}{25}$ or 0.04 & stops
A2 $\frac{P.T.R}{100}$ & stops
A3 Calculates S.I. for two years

Note: If the principal for year 2 is not different from the principal for year 1, candidate loses the marks for steps 3 & 4

| | | |
|---|---|---|
| **(ii)** | **15 marks** | **Att 5** |

Interest for year 3 = $851.76 - 811.20 = 40.56$ (step 1) $R = \frac{100.I}{P.T}$ (step 2)

$\frac{40.56 \times 100}{811.20 \times 1}$ (step 3) $= 5\%$ (step 4)

***OR***

€ 811.2 = 100% (step1) $€\ 1 = \frac{100}{811.2}\%$ (step2)

$€\ 851.76 = \frac{100 \text{ x } 851.76}{811.2}\%$ (step3) R = 5% (step4)

*Blunders (-3)*

B1 Works with $\dfrac{811.2}{40.56 \times 100}$ or $\dfrac{811.2}{851.76 \times 100}$

B2 $\dfrac{40.56 \times 100}{750 \times 1}$ & continues

*Slips (-1)*
S1 105 % as the answer

*Attempts (5 marks)*
A1 Arrives at some % by trial and error (including 5%)

*Worthless (0)*
W1 Correct answer without work

**Part (b)** **20 (10,10) marks** **Att (3,3)**

Given that $4xp - 3t = 5p$,
(i) express $x$ in terms of $p$ and $t$.
(ii) find the value of $x$ when $t = \dfrac{2p}{3}$ .

**(i)** **10 mark** **Att 3**

$4xp - 3t = 5p \Rightarrow 4xp = 5p + 3t$ (step 1) $\Rightarrow x = \dfrac{5p+3t}{4p}$ (step 2) or $\dfrac{5}{4} + \dfrac{3t}{4p}$

*Blunders (-3)*
B1 Each step omitted or incorrect
B2 Error in transposition

*Attempts (3 marks)*
A1 4xp = 5p – 3t

*Misreadings (-1)*
MR1 Expresses p or t in terms of the other variables

**(ii)** **10 marks** **Att 3**

$t = \dfrac{2p}{3} \Rightarrow x = \dfrac{5p + 3(\frac{2p}{3})}{4p}$ (step 1) $\Rightarrow \dfrac{5p+2p}{4p} \Rightarrow \dfrac{7p}{4p}$ (step 2) $\Rightarrow \dfrac{7}{4}$ (step 3)

* Candidates may do this part correctly without doing part (i)

*Blunders (-3)*
B1 Each step omitted or incorrect
B2 Error in substitution
B3 Error in multiplying with fractions

*Slips (-1)*
S1 Numerical slips

# QUESTION 3

| | | |
|---|---|---|
| **Part (a)** | **20 marks** | **Att 6** |
| **Part (b)** | **30 marks** | **Att 9** |

| | | |
|---|---|---|
| **Part (a)** | **20 marks** | **Att 6** |

Prove that any point on the perpendicular bisector of a given line segment is equidistant from the end points of the line segment.

Given: Line segment [ab] & perpendicular bisector xm

RTP: $|ax| = |bx|$

Con: Join $[xa]$ and $[xb]$ (step 1)

Proof:

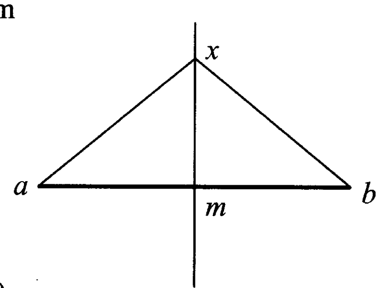
$|am| = |mb|$ (given) (step 2)

$|xm| = |xm|$ (common) (step 3)

$|\angle amx| = |\angle xmb|$ (90°) (step 4)

Triangles *xam* and *xmb* are congruent or SAS (step 5)

Thus, $|ax| = |bx|$ (step 6)




* No penalty if no reasons given

* Steps (i) to (iv) could be indicated on a diagram

* Step 6 implies RTP but not vice versa

* No diagram – full marks if all 6 steps are fully stated (including Given and RTP)

*Blunders (-3)*

B1 Each step omitted or incorrect

B2 Steps in an illogical order, but steps 2,3 and 4 could be interchanged

*Attempts (6 marks)*

A1 Diagram showing construction of a perpendicular bisector

A2 Diagram showing xm through mid point of ab & stops

*Worthless (0)*

W1 Line segment [ab] with nothing else

Alternative proofs, (by symmetry in *xm*):

| (Mapping triangles) | | (Mapping line segments) |
|---|---|---|
| Step 1 as before | | Step 1 as before |
| $x \to x$ | step 2 | $x \to x$ |
| $a \to b$ | step 3 | $\|am\| = \|mb\|$ & $\|\angle amx\| = 90°$ |
| $m \to m$ | step 4 | $\Rightarrow$ $a \to b$ |
| $\Rightarrow$ $\Delta xam \to \Delta xbm$ | step 5 | $\Rightarrow$ $[xa] \to [xb]$ |
| $\Rightarrow$ $[xa] \to [xb]$ $\Rightarrow \|xa\| = \|xb\|$ | step 6 | $\Rightarrow$ $\|xa\| = \|xb\|$ |

* "Given" not written, allow if diagram is done

* If no diagram, then candidate must have "Given"

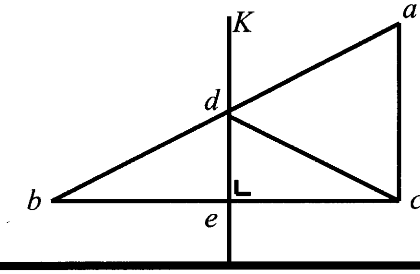
| Part (b) | 30 (10,10,10) marks | Att (3,3,3) |
|---|---|---|

In the triangle $abc$, $ac \perp bc$ and $|\angle abc| = 30°$.
$K$ is the perpendicular bisector of $[bc]$
and $K$ intersects $[ab]$ at $d$.

**(i)** Find $|\angle dcb|$.

**(ii)** Prove $|dc| = |da| = |ac|$.

**(iii)** Find the ratio $\dfrac{\text{area } \Delta\, dbe}{\text{area } \Delta\, abc}$.




| (i) | 10 marks | Att 3 |
|---|---|---|

$|\angle dcb| = |\angle dbc| = 30°$

* Accept the correct answer without work

*Blunders (-3)*

B1 States or indicates on diagram that $|\angle dbc| = |\angle dcb|$ & stops

*Attempts (3 marks)*

A1 $|db| = |dc|$ and stops

| (ii) | 10 marks | Att 3 |
|---|---|---|

$|\angle dca| = 60° \quad (90° - 30°)$ (step 1)

$|\angle bac| = 60° \quad (90° - 30°)$

$\Rightarrow |\angle adc| = 60°$ (step 2)

Thus, triangle $adc$ is equilateral

Or, $|dc| = |da| = |ac|$. (step 3)

* Note: One or two $60°$ angles found in triangle $acd$, or shown on diagram merits 4 marks.

*Attempts (3 marks)*

A1 $|\angle bdc| = 120°$ & stops

| (iii) | 10 marks | Att 3 |
|---|---|---|

| | | |
|---|---|---|
| $\dfrac{\text{area } \Delta\, dbe}{\text{area } \Delta\, abc} = \dfrac{0.5\,|be|.|de|}{0.5\,|bc|.|ac|}$ | (step 1) | Draws a line through d // bc |
| $= \dfrac{|be|.|de|}{2\,|be|.2\,|de|}$ | (step 2) | Indicates four congruent triangles |
| $= \dfrac{1}{4}$. | (step 3) | Ratio = 1:4 |

*Blunders (-3)*

B1 Any step omitted or incorrect

*Attempts (3 marks)*

A1 Area $\Delta dbe = \frac{1}{2}|be||ed|(or\ h)$ & stops A2 Area $\Delta abc = \frac{1}{2}|bc||ac|(or\ h)$ & stops

A3 Correct Answer without work

A4 $\dfrac{1}{2}$ab sinC with some relevant substitution & stops

# QUESTION 4

| | | |
|---|---|---|
| **Part (a)** | **30 marks** | **Att 10** |
| **Part (b)** | **20 marks** | **Att 6** |

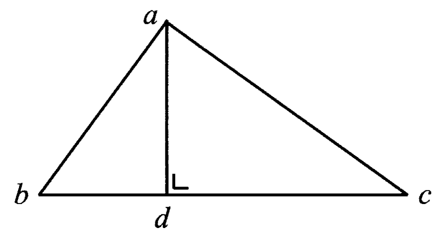
**Part (a)** **30 marks** **Att 10**

Prove that in a right-angled triangle the area of the square on the hypotenuse is the sum of the areas of the squares on the other two sides.

*Given:* $\Delta abc$ where $|\angle bac| = 90°$

*RTP:* $|bc|^2 = |ab|^2 + |ac|^2$

*Const:* Draw $ad \perp bc$ (step 1)




*Proof:*

$|\angle cab| = |\angle bda|$ and $|\angle abc| = |\angle abd|$

$\therefore$ triangles *abc* and *abd* are similar (step 2)

$\frac{|bc|}{|ab|} = \frac{|ab|}{|bd|} \Rightarrow |ab|^2 = |bc|.|bd|$ (step 3)

Likewise, the triangles *abc* and *adc* are similar, so that:

$\frac{|bc|}{|ac|} = \frac{|ac|}{|dc|} \Rightarrow |ac|^2 = |bc|.|dc|$ (step 4)

(step 3) + (step 4) $\Rightarrow |ab|^2 + |ac|^2 = |bc|.|bd| + |bc|.|dc|$ (step 5)

$= |bc|(|bd| + |dc|) = |bc|.|bc|$ or $|bc|^2$ (step 6)

* Step 6 implies RTP but not vice versa

* Accept "Similarly $|ac|^2 = |bc|.|dc|$ " for step 4

*Blunders (-3)*

B1 Each step omitted or incorrect

B2 Steps in an illogical order but steps 3 & 4 maybe interchanged

*Attempts (3 marks)*

A1 Right angled triangle with *ad* drawn

A2 An illustration of Pythagoras' Theorem

A3 States or illustrates a special case e.g. 3,4,5

*Worthless (0)*

W1 Right angled triangle drawn & stops

***Method 2***

*Given:* Right angled triangle *abc* with side lengths *x,y,z*, where $z$ is the hypotenuse.

*RTP:* $z^2 = x^2 + y^2$

*Const:* Draw a square of side $(x + y)$ as shown and join the inner quadrilateral (step 1)

*Proof:* The 4 triangles are congruent, by S.A.S.

$\Rightarrow$ each side of quad *bcgh* $= z$ (step 2)

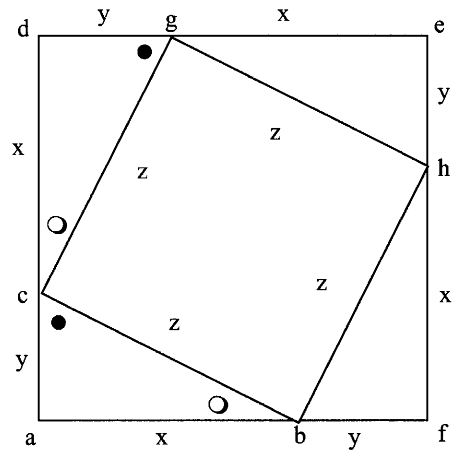
At vertex *c*, ● + ❍ = 90° $\Rightarrow |\angle gcb| = 90°$

Thus quad bcgh is a square (step3)

$(x + y)^2 = 4(\frac{1}{2}x.y) + z^2$ (step 4)

$x^2 + 2xy + y^2 = 2xy + z^2$ (step 5)

$x^2 + y^2 = z^2$ (step 6)




*Blunders (-3)*

B1 Each step omitted or incorrect

B2 Steps in an illogical order

*Attempts (3 marks)*

A1 Diagram with triangle *abc* and quad *afed* drawn (With or without quad *cbhg*)

***Method 3***

*Given:* $\Delta$ abc with $|\angle \text{bac}| = 90°$

*RTP:* $|bc|^2 = |ac|^2 + |ab|^2$

*Const:* Circle with centre *b* and radius $|\text{ba}| = r$ is drawn on a diagram (step 1)

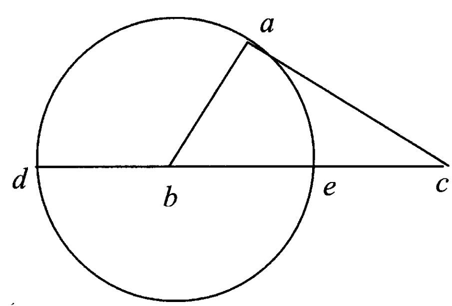
*Proof:* $|\angle bac| = 90° \Rightarrow$ *ca* is a tangent (step 2)

Thus, $|ce|.|cd| = |ca|^2$ (step 3)

$(|cb| - r)(|cb| + r) = |ca|^2$ (step 4)

$\Rightarrow |cb|^2 - r^2 = |ca|^2$ (step 5)

$\Rightarrow |bc|^2 = |ac|^2 + r^2 \quad \Rightarrow |bc|^2 = |ac|^2 + |ab|^2$ (step 6)




*Blunders (-3)*

B1 Each step omitted or incorrect

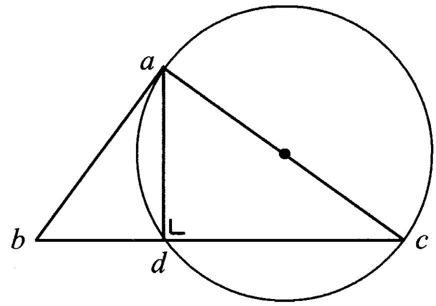
B2 Steps in an illogical order

***Method 4***

*Given:* $\Delta abc$ where $|\angle bac| = 90°$

*RTP:* $|bc|^2 = |ab|^2 + |ac|^2$

*Const:* Circle on [*ac*] as diameter, (step 1)




*Proof:*

Angle in semicircle = 90° ⇒ circle intersects [*bc*] at *d*, where $ad \perp bc$.

Also, $|\angle bac| = 90° \Rightarrow$ *ba* is a tangent (step 2)

$\therefore \quad |ab|^2 = |bc|.|bd|$ (step 3)

Likewise, by constructing a circle on [*ab*], it can be shown that:

$|ac|^2 = |bc|.|dc|$ (step 4)

steps 5 & 6 as in method 1.

*Blunders (-3)*

B1 Each step omitted or incorrect

B2 Steps in an illogical order

**Part (b)** **20 marks** **Att 6**

In the triangle $xyz$, $|\angle xyz| = 90°$.

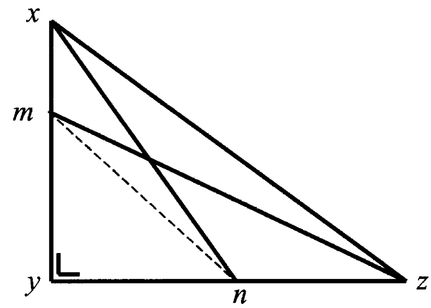
$m$ is a point on $[xy]$ and $n$ is a point on $[yz]$.

**(i)** Prove that

$$|xz|^2 - |mz|^2 = |xy|^2 - |my|^2 .$$

**(ii)** Deduce that

$$|xz|^2 - |mz|^2 = |xn|^2 - |mn|^2 .$$




**(i)** **10 marks** **Att 3**

| | |
|---|---|
| $\|xz\|^2 = \|xy\|^2 + \|yz\|^2$ | (step 1) |
| $\|mz\|^2 = \|my\|^2 + \|yz\|^2$ | (step 2) |
| $\|xz\|^2 - \|mz\|^2$ (step 1 - step 2) $= \|xy\|^2 - \|my\|^2$ | (step 3) |

*Blunders (-3)*

B1 Each step omitted or incorrect

B2 Incorrect use of Pythagoras, once only

*Attempts (3 marks)*

A1 Any correct use of Pythagoras or attempt at use of theorem

*Worthless (0)*

W1 Use of Pythagoras in a non-right angled triangle

**(ii)** **10 marks** **Att 3**

| | |
|---|---|
| $\|xn\|^2 = \|xy\|^2 + \|yn\|^2$ | |
| $\|mn\|^2 = \|my\|^2 + \|yn\|^2$ | (step 1) |
| $\|xn\|^2 - \|mn\|^2 = \|xy\|^2 - \|my\|^2$ | (step 2) |
| Hence, $\|xz\|^2 - \|mz\|^2 = \|xn\|^2 - \|mn\|^2$ . | (step 3) |

Slips and blunders as above

## QUESTION 5

| | | |
|---|---|---|
| **Part (i)** | **10 marks** | **Att 3** |
| **Part (ii)** | **10 marks** | **Att 3** |
| **Part (iii)** | **10 marks** | **Att 3** |
| **Part (iv)** | **10 marks** | **Att 3** |
| **Part (v)** | **10 marks** | **Att 3** |

| | | |
|---|---|---|
| **Part (i)** | **10 marks** | **Att 3** |

$a(-1, 4)$, $b(3, 1)$ and $c(2, 0)$ are three points.
Find $|ab|$.

$|ab| = \sqrt{(3+1)^2 + (1-4)^2}$ (step 1)

$= \sqrt{16+9}$ (step 2)

$= \sqrt{25}$ or 5 (step 3)

* Accept correct answer without work

*Blunders (-3)*

B1 Any step omitted or incorrect
B2 Incorrect relevant formula (see S2 )
B3 Any mathematical error

*Misreadings (-1)*

MR $|ac|$ or $|bc|$ found

*Slips (-1)*

S1 Each numeric slip
S2 One sign incorrect in formula
S3 Distance formula with a minus between the two brackets

*Attempts (3 marks)*

A1 Distance formula without substitution
A2 Reference to "over 4 and/or down 3"
A3 Correct square of a number
A4 a and b plotted & stops
A5 Correct square root of a number

*Worthless (0)*

W1 Incorrect answer without work

**Part (ii)** **10 marks** **Att 3**

| Find the slope of *ab*. |
|---|

| |
|---|
| Slope $= \dfrac{1-4}{3+1} = -\dfrac{3}{4}$ |

* Accept correct answer without work

*Blunders (-3)*
B1 Fails to simplify after substitution
B2 Incorrect relevant formula (see S1)

*Misreadings (-1)*
MR Slope of *ac* or *bc* found

*Slips (-1)*
S1 One sign only incorrect in formula

*Attempts (3 marks)*
A1 Correct formula & stops with or without partial substitution
A2 One or both differences & stops e.g. reference to "over 4 and/or down 3"
A3 Plots *a* and *b* (if A4 in part (i) was not applied)

**Part (iii)** **10 marks** **Att 3**

| The line *L* passes through the point *c* and is perpendicular to *ab*. Find the equation of *L*. |
|---|

| | | |
|---|---|---|
| Slope of $L = \dfrac{4}{3}$. | (step 1) | $m = \dfrac{4}{3}$ |
| $y - y_1 = \dfrac{4}{3}(x - x_1)$ | (step 2) | $y = \dfrac{4}{3}x + c$ |
| $y - 0 = \dfrac{4}{3}(x-2)$ | (step 3) | $0 = \dfrac{4}{3}(2) + c \Rightarrow c = -\dfrac{8}{3}$ |

* Accept candidate's slope from (ii)

*Blunders (-3)*
B1 Incorrect or arbitrary perpendicular slope used and continues
B2 Partial or incorrect substitution in equation formula (but note S1 below)
B3 Incorrect relevant formula e.g. both signs wrong or $3x \pm 4y + k = 0$ & continues

*Misreadings (-1)*
MR Point *a* or *b* used instead of *c*

*Slips (-1)*
S1 One sign incorrect in formula i.e. for $x_1$ or $y_1$

*Attempts (3 marks)*
A1 Line formula (either) no substitution & stops
A2 $m_1.m_2 = -1$ only and stops
A3 Plots *c*

**Part (iv)** **10 marks** **Att 3**

Calculate the area of the triangle *abc*.

| | | | | |
|---|---|---|---|---|
| (-1,4) → (-3,4) | (3,1) → (1,1) | (2,0) → (0,0) | (step 1) | (Or other translation) |
| Area $=\frac{1}{2}\lvert(-3)(1)-(4)(1)\rvert$ | | | (step 2) | |
| $= 3.5$ | | | (step 3) | |

* Long area formula used – mark by slips and blunders
* No penalty for negative area

*Blunders (-3)*
B1 Each step omitted or incorrect

*Slips (-1)*
S1 Numerical slips in calculation (maximum 3)

*Attempts (3 marks)*
A1 Coordinate Geometry Area formula, no substitution, & stops
A2 Accurate diagram with measurements used on it

*Worthless (0)*
W1 Scale diagram, no work, e.g. no measurements shown
W2 Correct answer with no work at all

Note: Long Area Formula $\frac{1}{2}\lvert x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\rvert$

This can also appear in a tabulated form:

e.g:. $$\frac{1}{2}\begin{vmatrix}-1 & 4\\ 3 & 1\\ 2 & 0\\ -1 & 4\end{vmatrix}=\frac{1}{2}\lvert(-1+0+8)-(12+2+0)\rvert=\frac{1}{2}\lvert 7-14\rvert=3\tfrac{1}{2}$$

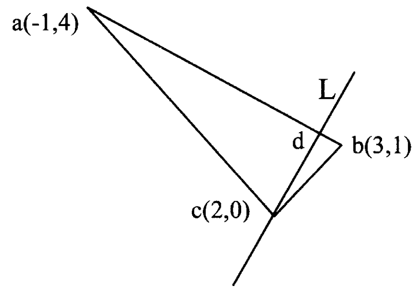
**Part (v)** **10 marks** **Att 3**

The line $L$ intersects $ab$ in $d$. Use the area of the triangle $abc$ to find $|cd|$.

Diagram with c and d identified / labelled (step 1)

Area $= 0.5|ab|.|cd| = \frac{1}{2} \times 5 \times |cd| = 3\frac{1}{2}$ (step 2)

$\Rightarrow \quad |cd| = \dfrac{7 \times 2}{2 \times 5} = 1.4.$ (step 3)




* Accept candidate's answer from parts (i) and (iv)

* Accept calculation to $\frac{7}{5}$ or $\frac{14}{10}$ for full marks

*Blunders (-3)*

B1 Each step omitted or incorrect

B2 Incorrect relevant formula and continues

B3 Error in transposition

B4 Correct answer, with work, by alternative method

*Slips (-1)*

S1 Numerical slips

S2 Stops at $\dfrac{7 \times 2}{2 \times 5}$ or $\dfrac{3\frac{1}{2}}{2\frac{1}{2}}$ or $\dfrac{3\frac{1}{2}}{\frac{1}{2} \times 5}$

*Attempts (3 marks)*

A1 Sketch drawn & stops

A2 Any reference to "base" or "height"

A3 $3\frac{1}{2}$ written, no other worthwhile work

A4 Indication of '5' or '7' found or appearing

A5 Correct distance formula with or without substitution

A6 Finds or attempts to find the coordinates of d

# QUESTION 6

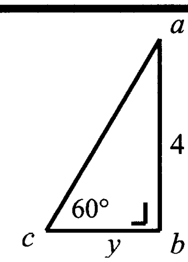
| | | |
|---|---|---|
| **Part (a)** | **10 marks** | **Att 3** |
| **Part (b)** | **20 marks** | **Att 6** |
| **Part (c)** | **20 marks** | **Att 6** |

| | | |
|---|---|---|
| **Part (a)** | **10 marks** | **Att 3** |

The triangle *abc* is right-angled as shown.

Calculate $y$ and the area of the triangle *abc*.

Give your answers in surd form.




$\text{Tan } 60° = \frac{4}{y} = \sqrt{3}$ (step 1) $\Rightarrow 4 = \sqrt{3}y \Rightarrow y = \frac{4}{\sqrt{3}}$ (step 2)

$\text{Area } abc = \frac{1}{2} \times \frac{4}{\sqrt{3}} \times 4 = \frac{8}{\sqrt{3}}$ (step 3)

## To be applied to all parts, (a), (b) and (c)

*Blunders (-3)*

- B1 Each step omitted (max of 3 marks can be lost in any step)
- B2 Incorrect ratio (sin, cos or tan)
- B3 Incorrect ratio in Sine Rule
- B4 Error in cross multiplication
- B5 Takes $1° = 100'$ (or not $= 60°$)
- B6 Incorrect transposition
- B7 Decimal error
- B8 Reading wrong page of tables

*Slips (-1)*

- S1 Numerical slips
- S2 Slips in reading tables e.g. reading wrong column.

To be applied to part (a):

*Slips (-1)*

- S3 Answer not in surds
- S4 Incorrect column read from tables

*Attempts (3 marks)*

- A1 Ignores square root and continues
- A2 Writes down sin, cos or tan ratio & stops
- A3 Some substitution in area formula
- A4 30° appearing & stops
- A5 Gets $|ac|$ & stops ($4.6188$ or $\frac{8}{\sqrt{3}}$)

**Note:**

For step 1: $\frac{4}{y} = 1.732$ is a slip

For step 2: $y = \frac{4}{1.732}$ & stops is att. 3

For step 2: y = 2.3 & stops merits 6 marks

For step 3: 2 x 2.3 = 4.6 merits 9 marks

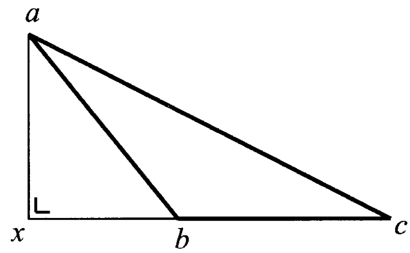
For step 3: $\frac{8}{1.732}$ & stops merits 6 marks

**Part (b)** **20 (10,10)marks** **Att (3,3)**

In the triangle *abc*,
$|\angle acb| = 28°41'$, $|\angle bac| = 23°35'$
and $|bc| = 15$ cm.

**(i)** Calculate $|ab|$.

**(ii)** $x$ is on $cb$ such that $ax \perp xb$ as shown.
Calculate $|ax|$, correct to the nearest cm.




**(i)** **10 marks** **Att 3**

$$\frac{15}{\sin 23°35'} = \frac{|ab|}{\sin 28°41'} \qquad \text{(step1)}$$

$$\Rightarrow \frac{15}{0.4} = \frac{|ab|}{0.48} \qquad \text{(step 2)}$$

$$\Rightarrow |ab| = \frac{15 \times 0.48}{0.4} = 18 \qquad \text{(step 3)}$$

Slips and blunders as listed in (a) and the box on page 52. Additionally:

*Slips (-1)*
S5 Fails to calculate

*Attempts (3 marks)*
A5 Partly filled in Sine Rule & stops

*Worthless (0)*
W1 Treats triangle abc as right angled triangle

**(ii)** **10 marks** **Att 3**

| | | |
|---|---|---|
| $\lvert\angle abx\rvert = 23°35' + 28°41' = 52°16'$ | (step 1) | Same |
| $\sin 52°16' = \frac{\lvert ax\rvert}{18} = 0.7909$ | (step 2) | $\frac{\lvert ax\rvert}{0.7909} = \frac{18}{1}$ |
| $\lvert ax\rvert = 0.7909 \times 18 = 14.2362 = 14$ | (step 3) | Same |

* Accept candidate's $|ab|$ from (b) (i)

Slips and blunders as listed in (a), (b)(i) and the box on page 52. Additionally:

*Slips (-1)*
S6 Failure to round off

*Misreadings (-1)*
MR1 Gets $|xb|$ instead of $|ax|$

*Attempts (3 marks)*
A6 Gets 51°76' & stops

**Part (c)** **20 marks** **Att (3, 3)**

$x, y, z$ are points on the circle of centre $o$.

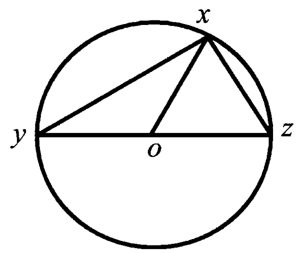
The radius of the circle is 10 cm.

The triangle $xoz$ is an equilateral triangle.

Find

**(i)** area of triangle $xoz$

**(ii)** area of triangle $xyz$.




**(i)** **10 marks** **Att 3**

| | | | |
|---|---|---|---|
| Area of triangle $xoz$ | = | $\frac{1}{2}\times 10\times 10\times \sin 60°$ | (step 1) |
| | = | $\frac{1}{2}\times 10\times 10\times \frac{\sqrt{3}}{2}$ | (step 2) |
| | = | $50\frac{\sqrt{3}}{2}$ or $25\sqrt{3}$ | (step 3) |

* If candidate uses $\sin 60° = 0.8660$ the answer is 43.3

Slips and blunders as before, and:

*Blunders (-3)*
B9 Takes $r = 20$ and continues
B10 $50 \times 0.8660$ or $25 \times 1.732$ & stops
B11 Treats *ab* as one line $\frac{1}{2}$x10 x sin60°

*Misreadings (-1)*
MR2 Takes $r = 5$ and continues

*Attempts (3 marks)*
A7 $|xz| = 10$ & stops
A8 Recognition of any of the 3 angles = 60° and stops

**(ii)** **10 marks** **Att 3**

Area of triangle xyz = $\frac{1}{2}$ x10 x 20 sin60° (step 1)

$= \frac{1}{2}$ x 10 x 20 x $\frac{\sqrt{3}}{2}$ (step 2) $= 100\frac{\sqrt{3}}{2}$ or $50\sqrt{3}$ or 86.6 (step 3)

* Accept without work $50\sqrt{3}$ or 86.6

Slips and blunders as before

*Attempts (3 marks)*
A9 $|zy| = 20$ or $|oy| = 10$