

## TEAM SELECTION TEST 2

TUESDAY 29 MAY 2001

08.30-13.00

1. Let  $a, b, c, x, y, z$  be positive reals, with  $a \geq b \geq c$  and  $x \geq y \geq z$ . Prove that

$$\frac{a^2x^2}{(by + cz)(bz + cy)} + \frac{b^2y^2}{(cz + ax)(cx + az)} + \frac{c^2z^2}{(ax + by)(ay + bx)} \geq \frac{3}{4}.$$

2. We have 10 points in the plane, with all distances between them distinct. For each point, we mark red the point nearest to it. What is the smallest number of points that can be marked red?

3. Let  $n$  be a fixed positive integer, and let  $S$  be an infinite collection of  $n$ -tuples of non-negative integers with the following property: whenever  $x = (x_1, \dots, x_n)$  and  $y = (y_1, \dots, y_n)$  are  $n$ -tuples of non-negative integers, with  $x_i \leq y_i$  for all  $i$ , and  $y$  is a member of  $S$ , then also  $x$  is a member of  $S$ . For each non-negative integer  $k$ , let  $f(k)$  be the number of  $n$ -tuples  $(x_1, \dots, x_n)$  in  $S$  with  $x_1 + \dots + x_n = k$ . Prove that the function  $f$  is eventually a polynomial (in other words, there exists a polynomial  $g$ , with real coefficients, and a positive integer  $K$  such that  $f(k) = g(k)$  for all  $k \geq K$ ).