National Committee for Mathematical Contests

Second International Selection Test

Reading, 23rd April 1988

Time allowed : 3½ hours

Please write on one side of the paper only, use separate sheets for each question, and put your name on each sheet.

1. The triangle ABC is not equilateral and the usual notation is used for its sides and angles. Show that the equations

$$\frac{\sin (A - \theta)}{a^3} = \frac{\sin (B - \theta)}{b^3} = \frac{\sin (C - \theta)}{c^3}$$

are satisfied by a unique number θ in the interval $0 < \theta < \frac{\pi}{6}$. [It may help to work in terms of a, b, c and the area Δ of the triangle.]

2. Find the solutions in integers of the equation

$$x^4 + y^4 - 48xy = -79,$$

showing that you have them all.

3. Twelve delegates attend a conference at which they are seated around a circular table. Each delegate brings with him to the conference three copies of a document which he has prepared. He gives one copy to each of his two immediate neighbours at the table and keeps the third copy himself. While the delegates are away at coffee, having left the papers in question on their chairs, a thief rushes into the room and snatches one document at random from each chair. What is the probability that the thief procures a complete set of the twelve documents. ?

P.T.O.

4. The equilateral triangle ABC is inscribed in a circle K. T is a point of K lying on the smaller arc AB. Prove that $AT + BT = CT$.

A second circle k touches K at T and lies inside K. The lengths of the tangents from A, B, C to k are a, b, c respectively. Prove that $a + b = c$.

5. Find all pairs c, P consisting of a real number c and a non-zero polynomial P such that the identity

$$P(x^4 + x^2 + x) = (x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)\, P(cx)$$

holds.