Next Selection Test: Paper 4

Oundle School

1st June 2011

1. Let A be the set of all integers of the form $a^2 + 13b^2$, where a and b are integers and b is nonzero. Prove that there are infinitely many pairs of integers x, y such that $x^{13} + y^{13} \in A$ but $x + y \notin A$.

2. 2500 chess kings are to be placed on a 100×100 chess board so that:

   (a) no king can capture any other one (ie. no two kings are placed in two squares which share a common vertex);

   (b) each row and each column contains exactly 25 kings.

   In how many ways is this possible?

3. Let ABC be a scalene triangle. Let l_A be the tangent to the nine-point circle at the foot of the perpendicular from A to BC, and let l'_A be the tangent to the nine-point circle from the midpoint of BC. The lines l_A and l'_A intersect at A'; we define B' and C' similarly.

   Show that the lines AA', BB' and CC' are concurrent.

Each question is worth seven marks.
Time: 4 hours, 30 minutes.