First Selection Test: Exam 1

31-iii-2007

Problem 1 Let n be a natural number. We want to colour each natural number red or blue so that the following conditions are satisfied.

 (i) Infinitely many numbers are coloured red and infinitely many numbers are coloured blue.

 (ii) The sum of every n distinct red numbers is red and the sum of every n distinct blue numbers is blue.

Is such a colouring possible

 (a) if $n = 2007$?

 (b) if $n = 2008$?

Justify your answer.

Problem 2 Let triangle ABC have a right angle at A. Let D denote the foot of the perpendicular from A to BC. Let I and J be the respective incentres of triangles ABD and ADC. Draw the line IJ to meet AB at E and AC and F, Show that A is the circumcentre of triangle EDF.

Problem 3 Find all positive integers m, n, p, q such that

$$p^m q^n = (p + q)^2 + 1.$$

Time allowed: 4 hours 30 minutes