INTERNATIONAL INDIAN SCHOOL-DAMMAM

FIRST TERMINAL EXAMINATION-JUNE 2012

MATHEMATICS

SET A

GRADE: XII MAX MARKS: 100

TIME: 3 HOURS

GENERAL INSTRUCTIONS:

1. All questions are compulsory.

2. This question paper consists of 29 questions divided into three sections A, B and C. Section A contains 10 questions of 1 mark each. Section B contains 12 questions of 4 marks each and section C contains 7 questions of 6 marks each.

3. There is no overall choice. However internal choice has been provided for 4 questions of 4 marks each and 2 questions of 6 mark each.

SECTION -A

1. Find the approximate change in the volume of a cube of side x meters caused by increasing the side by 3%.

2. Check the injectivity and surjectitivity of f: R→ R ,given by f(x) = $[x]$

3. Find g of and fog, if f(x) = $8x^3$ and g(x) = $x^{\frac{1}{3}}$

4. Find $\frac{dy}{dx}$, if y = $\sin(x\sqrt{x})$

5. If f(x) = x^2-4 x + I, find f (A) when A = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

6. Find the principal value of
$\text{cosec}^{-1}(\text{cosec}\frac{3\pi}{4})$

7. Find $\frac{dy}{dx}$, if $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2$

8. A=$\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$, is such that $A^2 = I$, find the relation between α , β & γ.

9. Find the approximate value of $\sqrt{80}$

10. Differentiate $\cos x$, with respect to e^x

SECTION-B

11. If f: R→ R be a function defined by f(x) = $4x^3 - 7$ .Show that f is one-one and onto. Find inverse of f.

OR

Show that the relation R in the set A=$\{x \in Z : 0 \le x \le 12\}$,given by R=$\{(a,b): |a-b| \text{ is multiple of } 4\}$ is an equivalence relation.

12. The volume of the cube is increasing at the rate of 8 cubic cm/sec .How fast is the surface area increasing when the length of the cube is 16 cm?

13. Using elementary transformations, find the inverse of A, where A = $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$

14. If the function f(x) = $\begin{cases} \frac{\sin x + x\cos x}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous. Find k.

15. Express $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$ in the simplest form.

16. By using properties of determinants, show that:

$$\begin{vmatrix} a^2 & bc & ac + c^2 \\ a^2 + ab & b^2 & ac \\ ab & b^2 + bc & c^2 \end{vmatrix} = 4\, a^2\, b^2\, c^2$$

OR

$$\begin{vmatrix} a + bx & c + dx & p + qx \\ ax + b & cx + d & px + q \\ u & v & w \end{vmatrix} = (1- x^2) \begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$$

17. If $\cos y = x\cos(a+y)$, prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$.

18. Find the intervals in which the function f(x) = $\frac{x}{2} + \frac{2}{x}$ is increasing or decreasing

OR

Find the intervals in which the function f(x) = $x^3 - 12x^2 + 36x + 17$ is increasing or decreasing

19. If y = a sin($\log x$) + bcos($\log x$), then show that $x^2\, y_2 + x\, y_1 + y = 0$

20. If A = $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then prove that $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$, where n ∈ N

21. If A= $\begin{bmatrix} 1 & 6 & 9 \\ 2 & 1 & 5 \end{bmatrix}^T$ and B= $\begin{bmatrix} 9 & 3 & 8 \\ 0 & 5 & -11 \end{bmatrix}$, verify $(AB)^T = (B)^T(A)^T$

22. Solve for x: $\tan^{-1} x + 2\cot^{-1} x = \frac{2\pi}{3}$

OR

Show that $\sin^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}=\pi$

SECTION C

23. Find the equations of the tangent and normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at the point (x_0, y_0).

24. Find $\frac{dy}{dx}$ of the function: $x^y+y^x+x^x = a^b$.

25. If A = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify $A^3-6A^2+9A-4I = 0$; hence find A^{-1}

26. If y = log tan $\left(\frac{\pi}{4}+\frac{x}{2}\right)$, show that $\frac{dy}{dx}$ - sec x = 0

OR

If y = $\left[x+\sqrt{x^2+a^2}\right]^n$, prove that $\frac{dy}{dx} = \frac{ny}{\sqrt{x^2+a^2}}$

27. Let A= Q X Q and let * be a binary operation on A defined by (a , b) *(c , d) = (ac , b + ad) ; ∀ (a , b), (c , d) ∈ A, then with respect to * on A (i)Is * commutative (ii)Is * associative (iii)Find the identity element of A (iv)Find invertible element of A.

28. If length of the three sides of a trapezium other than the base is equal to 10cm then find the area of the trapezium when it is maximum.

OR

Prove that the height of the right circular cone of maximum volume which can be inscribed in a sphere of radius R is $\frac{4R}{3}$.Also prove that the volume of the cone is $\frac{8}{27}$ of the volume of the sphere.

29. Solve the system of equations by using matrix method:

3x –y+ z = 5; 2x - 2y + 3z = 7; x + y – z = -1

END

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4. Find $\frac{dy}{dx}$, if $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2$

5. Find g o f and f o g, if f(x) = $8x^3$ and g(x) = $x^{\frac{1}{3}}$

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