Faculty of Actuaries **Institute of Actuaries**

Subject CT5 — Contingencies
Core Technical

EXAMINERS' REPORT

April 2009

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

R D Muckart
Chairman of the Board of Examiners

June 2009

Comments

Where relevant, comments for individual questions are given after each of the solutions that follow.

1 $_{5|10}q_{[40]+1}$ is the probability that a life now aged 41 exact and at the beginning of the second year of selection will die between the ages of 46 and 56 both exact.

Value is:

$$
\begin{aligned}
&(l_{46} - l_{56}) / l_{[40]+1} \\
&= (9786.9534 - 9515.104) / 9846.5384 \\
&= 0.02761
\end{aligned}
$$

2

$$
\begin{aligned}
&l_x = 110 - x \\
&\Rightarrow d_x = l_x - l_{x+1} \\
&\qquad = (110 - x) - (110 - x - 1) \\
&\qquad = 1 \quad \text{for all } x
\end{aligned}
$$

(i)

$$
\begin{aligned}
A^{1}_{40:\overline{20|}} &= \left(\sum_{0}^{19} v^{t+1} * d_{40+t} \right) / l_{40} \\
&= \left(\sum_{0}^{19} v^{t+1} \right) / l_{40} \\
&= a_{\overline{20|}} / l_{40} \\
&= 13.5903 / (110 - 40) \\
&= 0.19415
\end{aligned}
$$

(ii)

$$
\begin{aligned}
A_{40:\overline{20|}} &= A^{1}_{40:\overline{20|}} + v^{20} * l_{60} / l_{40} \\
&= 0.19415 + 0.45639 * (110 - 60) / (110 - 40) \\
&= 0.52014
\end{aligned}
$$

3 Assuming contributions are payable continuously we make the approximation that they are payable on average half-way through the year. The present value of contributions in the year t to $t + 1$ is:

$$
(0.04).\left(S \frac{s_{x+t}}{s_{x-1}} - 5000 \right).\frac{v^{x+t+0.5}}{v^{x}}.\frac{l_{x+t+0.5}}{l_x}
$$

Define the following parameters and commutation functions:

$\frac{s_{x+t}}{s_x}$ represents the ratio of a member's earnings in the year of age $x+t$ to $x+t+1$ to their earnings in the year x to $x+1$.

$D_{x+t} = v^{x+t} l_{x+t}$

$\overline{D}_{x+t} = v^{x+t+0.5} l_{x+t+0.5}$

${}^{s}D_x = s_{x-1} v^x l_x$

${}^{s}\overline{D}_{x+t} = s_{x+t} v^{x+t+0.5} l_{x+t+0.5}$

$\overline{N}_x = \sum_{t=0}^{t=NRA-x-1} \overline{D}_{x+t}$

${}^{s}\overline{N}_x = \sum_{t=0}^{t=NRA-x-1} {}^{s}\overline{D}_{x+t}$

Then the present value of all future contributions is:

$$(0.04).\left(S \frac{{}^{s}\overline{N}_x}{{}^{s}D_x} - 5000 \frac{\overline{N}_x}{D_x} \right)$$

4 (a) Different groups or classes of policyholders may have higher or lower lapse rates for all major risk factors (age, duration, gender etc.) than other classes. An example would be where a class of policyholders is defined as those who purchased their policies through a particular sales outlet (e.g. broker versus newspaper advertising).

(b) Lapse rates may vary by policy duration as well as age for shorter durations. At shorter durations lapse rates may be the result of "misguided" purchase by policyholder whereas at longer durations the policy has become more stable.

(c) Lapse rates vary with calendar time for all major risk factors, e.g. economic prosperity varies over time and this results in a similar variation in lapse rates.

Other valid comments were credited. Many students ignored lapses altogether attempting to answer the question from a mortality standpoint only. No credit was given for this.

5 Assumptions

- Equal forces in the multiple and single decrement tables
- Uniform distribution of all decrements across year of age

Then

$$(aq)_x^{\beta} = \int_0^1 {}_t(ap)_x . \mu_{x+t}^{\beta} . \frac{{}_t p_x^{\beta}}{{}_t p_x^{\beta}} . dt = \int_0^1 ({}_t p_x^{\beta} \mu_{x+t}^{\beta}) . \frac{{}_t(ap)_x}{{}_t p_x^{\beta}} . dt$$

Our assumptions give us:

$${}_t p_x^{\beta} \mu_{x+t}^{\beta} = q_x^{\beta}$$

$$\frac{{}_t(ap)_x}{{}_t p_x^{\beta}} = {}_t p_x^{\alpha} = 1 - t.q_x^{\alpha}$$

Therefore:

$$(aq)_x^{\beta} = \int_0^1 q_x^{\beta} . (1 - t.q_x^{\alpha}) . dt$$

$$= q_x^{\beta} \left[t - \frac{t^2}{2} q_x^{\alpha} \right]_0^1 = q_x^{\beta} \left(1 - \frac{1}{2} q_x^{\alpha} \right)$$

$$= \left(\frac{1}{3} + \frac{1}{4} q_x^{\alpha} \right) \left(1 - \frac{1}{2} q_x^{\alpha} \right) = \frac{1}{3} + \frac{1}{12} q_x^{\alpha} - \frac{1}{8} \left(q_x^{\alpha} \right)^2$$

Alternatively the solution can be expressed in terms of q_x^{β}:

$$= q_x^{\beta} \left(1 - \frac{1}{2} * 4 * \left(q_x^{\beta} - \frac{1}{3} \right) \right)$$

$$= q_x^{\beta} \left(\frac{5}{3} - 2 q_x^{\beta} \right)$$

This question was essentially course bookwork plus a substitution. To gain good credit it was necessary to work though the solution as above.

6 $$E[T_{xy}] = \int_0^\infty t.{}_tp_x.{}_tp_y(\mu_{x+t} + \mu_{y+t})dt$$

$$= \int_0^\infty t.e^{-.02t}e^{-.03t}(0.02 + 0.03)dt$$

$$= 0.05\int_0^\infty t.e^{-.05t}dt$$

Integrating by parts:

$$= 0.05([-t.e^{-.05t} / .05]_0^\infty + 1/.05 * \int_0^\infty e^{-.05t}dt)$$

$$= 0.05(0 - 20/.05[e^{-.05t}]_0^\infty)$$

$$= 20$$

Alternatively:

$$E[T_{xy}] = \int_0^\infty {}_tp_x.{}_tp_y dt$$

$$= \int_0^\infty e^{-.02t}e^{-.03t}dt$$

$$= \int_0^\infty e^{-.05t}dt$$

$$= [-1/.05 * e^{-.05t}]_0^\infty$$

$$= 20$$

The alternative solution above in essence belongs to the Course CT4 but students who used this were given full credit. The first solution is that which applies to the CT5 Course.

7 Consider the continuous version

This would be

$$50000\int_0^\infty v^{t+5}(1 - {}_tp_{70}^m)\,{}_{t+5}p_{60}^f dt$$

$$= 50000v^5\,{}_5p_{60}^f\int_0^\infty v^t(1 - {}_tp_{70}^m)\,{}_tp_{65}^f dt$$

$$= 50000v^5\,{}_5p_{60}^f(\bar{a}_{65}^f - \bar{a}_{70:\,65}^{m\ \ f})$$

The monthly annuity equivalent SP is:

$$= 50000 v^5 {}_5p_{60}^{f} (\ddot{a}_{65}^{(12)f} - \ddot{a}_{70:65}^{(12)m\ \ f})$$
$$= 50000 v^5 {}_5p_{60}^{f} (\ddot{a}_{65}^{f} - \ddot{a}_{70:65}^{m\ \ f}) \text{ (note the monthly adjustment cancels out)}$$
$$= 50000 * 0.82193 * 9703.708 / 9848.431 * (14.871 - 10.494)$$
$$= 177236$$

Other methods were credited. Students who developed the formulae without recourse to continuous functions were given full credit.

8 (i) Final salary – rate of salary at retirement

Final average salary – salary averaged over a fixed period (usually 3 to 5 years) before retirement

Career average salary – salary averaged over total service

.

(ii) $\dfrac{s_{x+t}}{s_x}$ represents the ratio of a member's earnings in the year of age $x+t$ to $x+t+1$ to their earnings in the year x to $x+1$.

$z_x = \dfrac{s_{x-1} + s_{x-2} + \ldots.. + s_{x-y}}{y}$ is defined as a y-year final average salary scale.

Other versions credited. Strictly speaking Final Salary is not an average but this caused no confusion and was fully credited

9 $$25000 \int_0^{10} e^{-\delta t} ({}_tp_{55}^{aa} \mu_{55+t} + {}_tp_{55}^{ai} \nu_{55+t}) dt$$
$$+ \int_0^{10} e^{-\delta t} (0.{}_tp_{55}^{aa} + 1000\, {}_tp_{55}^{ai}) dt$$

where:

δ= the force of interest

${}_tp_{55}^{aa}$ = the probability that an able life age 55 is able at age $55 + t$

${}_tp_{55}^{ai}$ = the probability that an able life age 55 is ill at age 55 + t

10 This is the same as:

If y dies in 10 years, then 50000 is paid if x is alive, 200000 if x is dead,
If x dies in 10 years, then 100000 is paid

So the expected present value =

$$\int_0^{10} {}_t p_y \mu_{y+t} (50000\, {}_t p_x + 200000\, {}_t q_x).e^{-\delta t} dt$$

$$+100000 \int_0^{10} {}_t p_x . \mu_{x+t} . e^{-\delta t} dt$$

$${}_t p_x = e^{-\int_0^t 0.02 dr} = e^{-.02t}$$

$${}_t p_y = e^{-\int_0^t 0.03 dr} = e^{-.03t}$$

Therefore value =

$$\int_0^{10} e^{-.03t} 0.03(50000 e^{-.02t} + 200000(1 - e^{-.02t})).e^{-.04t} dt$$

$$+100000 \int_0^{10} e^{-.02t} 0.02.e^{-0.4t} dt$$

$$= 6000 \int_0^{10} e^{-.07t} dt + 2000 \int_0^{10} e^{-.06t} dt - 4500 \int_0^{10} e^{-.09t} dt$$

$$= \frac{6000}{-0.07}\left[e^{-0.7} - 1\right] + \frac{2000}{-0.06}\left[e^{-0.6} - 1\right] - \frac{4500}{-0.09}\left[e^{-0.9} - 1\right]$$

$$= 28{,}518$$

11 (i) Annual premium for endowment with £75,000 sum assured given by:

$$P = \frac{75,000 A_{[45]:\overline{20|}}}{\ddot{a}_{[45]:\overline{20|}}} = \frac{75,000 \times 0.46982}{13.785} = 2556.15$$

Reserves at the end of the eighth year:

for endowment with £75,000 sum assured is given by:

$$_8V = 75,000 \times A_{53:\overline{12|}} - 2556.15\ddot{a}_{53:\overline{12|}}$$

$$= 75,000 \times 0.63460 - 2556.15 \times 9.5 = 23,311.58$$

for temporary annuity paying an annual benefit of £18,000 is given by:

$$_8V = 18,000\ddot{a}_{53:\overline{12|}} = 18,000 \times 9.5 = 171,000.00$$

Death strain at risk:

Endowment: DSAR = 75,000 – 23,311.58 = 51,688.42
Immediate annuity DSAR = – 171,000.00

(ii) Mortality profit = EDS – ADS

For endowment assurance

$$EDS = (5000 - 65) \times q_{52} \times 51,688.42$$
$$= 4935 \times 0.003152 \times 51,688.42 = 804,019.58$$

$$ADS = 10 \times 51,688.42 = 516,884.20$$

mortality profit = 287,135.38

For immediate annuity

$$EDS = (2500 - 30) \times q_{52} \times -171,000.00$$
$$= 2470 \times 0.003152 \times -171,000.00 = -1,331,310.24$$

$$ADS = 5 \times -171,000.00 = -855,000.00$$

mortality profit = – 476,310.24

Hence, total mortality profit = 287,135.38 – 476,310.24 = –189,174.86
(i.e. a mortality loss)

12 (i) For a unit-linked life assurance contract, we have:

the ***unit fund*** that belongs to the policyholder. This fund keeps track of the premiums allocated to units and benefits payable from this fund to policyholders are denominated in these units. This fund is normally subject to unit fund charges.

the ***non-unit fund*** that belongs to the company. This fund keeps track of the premiums paid by the policyholder which are not allocated to units together with unit fund charges from the unit-fund. Company expenses will be charged to this fund together with any non-unit benefits payable to policyholders.

(ii) It is a principle of prudent financial management that once sold and funded at outset, a product should be self-supporting. However, some products can give profit signatures which have more than one financing phase. In such cases, reserves are required at earlier durations to eliminate future negative cash flows, so that the office does not expect to have to input further money in the future.

(iii)

Year t	$q_{[50]+t-1}$	$p_{[50]+t-1}$
1	0.001971	0.998029
2	0.002732	0.997268
3	0.003152	0.996848
4	0.003539	0.996461

$$ {}_3V = \frac{118.0}{1.055} = 111.85 $$

$$ {}_2V \times 1.055 - p_{52} \times {}_3V = 136.2 \Rightarrow {}_2V = 234.78 $$

$$ {}_1V \times 1.055 - p_{[50]+1} \times {}_2V = 152.0 \Rightarrow {}_1V = 366.01 $$

13 Multiple decrement table constructed using $(aq)_x^d = q_x^d \left[1 - \tfrac{1}{2}(q_x^s + q_x^m) + \tfrac{1}{3} q_x^s \times q_x^m\right]$ etc. which assumes that the decrements in each single decrement table are uniformly distributed over each year of age

x	q_x^d	q_x^s	q_x^m	$(aq)_x^d$	$(aq)_x^s$	$(aq)_x^m$
40	0.0009370	0.10	0.05	0.0008683	0.0974547	0.0474781
41	0.0010140	0.10	0.05	0.0009396	0.0974510	0.0474763
42	0.0011040	0.10	0.05	0.0010230	0.0974466	0.0474742

Using an arbitrary radix of 1,000,000, we can construct the following multiple decrement table

X	$(al)_x$	$(ad)_x^d$	$(ad)_x^s$	$(ad)_x^m$
40	1,000,000	868.3	97,454.7	47,478.1
41	854,198.9	802.6	83,242.5	40,554.2
42	729,599.6	746.4	71,097.0	34,637.2
43	623,119.0			

Let P be the annual premium for the contract.

Then equation of value gives:

PV of premiums = PV of death benefits + PV of surrender benefits + PV of survival benefits + PV of expenses

PV of premiums

$$= P\left(1 + \frac{854{,}198.9}{1{,}000{,}000} \times v_{0.05} + \frac{729{,}599.6}{1{,}000{,}000} \times v_{0.05}^2\right)$$
$$= P(1 + 0.813523 + 0.661768) = 2.475291P$$

PV of expenses = 0.005 × 2.475291P = 0.0123765P

PV of death benefits

$$= 15{,}000 \times (1.05)^{1/2} \times \left(\frac{868.3}{1{,}000{,}000} \times v_{0.05} + \frac{802.6}{1{,}000{,}000} \times v_{0.05}^2 + \frac{746.4}{1{,}000{,}000} \times v_{0.05}^3\right)$$
$$= 15{,}370.4262(0.00082695 + 0.00072798 + 0.00064477) = 33.8103$$

PV of withdrawal benefits =

$$= P\left(1 \times \frac{97{,}454.7}{1{,}000{,}000} v_{0.05} + 2 \times \frac{83{,}242.5}{1{,}000{,}000} v_{0.05}^2 + 3 \times \frac{71{,}097.0}{1{,}000{,}000} v_{0.05}^3\right) \times 1.05^{1/2}$$
$$= P(0.092814 + 0.1510068 + 0.1842488) \times 1.024695 = 0.4386408P$$

PV of marriage benefits =

$$= P\left(\frac{47,478.1}{1,000,000}\times \ddot{s}_{\overline{1}|}^{0.04}\times v_{0.05}+\frac{40,554.2}{1,000,000}\times \ddot{s}_{\overline{2}|}^{0.04}\times v_{0.05}^{2}+\frac{34,637.2}{1,000,000}\times \ddot{s}_{\overline{3}|}^{0.04}\times v_{0.05}^{3}\right)\times\left(\frac{1.05}{1.04}\right)^{1/2}$$

$$= P\left(0.0470259+0.0780406+0.0971372\right)\times 1.0047962 = 0.2232694P$$

PV of survival benefits =

$$5000\times\frac{623,119.0}{1,000,000}v_{0.05}^{3} = 2691.3681$$

Equation of value becomes

$2.475291P = 33.8103 + 0.4386408P + 0.2232694P + 2,691.3681 + 0.0123765P$

$\Rightarrow P = 2725.1784/1.801004 = 1513.14$

14 (i) Let P be the annual premium payable. Then equation of value gives:

PV of premiums = PV of benefits + PV of expenses

i.e.

$$P\ddot{a}_{[60]:\overline{5}|} = 10,000A_{[60]:\overline{5}|} + 400(IA)_{60:\overline{5}|} + 0.05P\ddot{a}_{[60]:\overline{5}|} + 0.55P \quad at \quad 6\%$$

where $(IA)_{[60]:\overline{5}|} = (IA)_{[60]} - \frac{l_{65}}{l_{[60]}}\times v_{0.06}^{5}\left(5A_{65}+(IA)_{65}\right) + 5\times\frac{l_{65}}{l_{[60]}}\times v_{0.06}^{5}$

$$= 5.4772 - 0.7116116(5\times 0.40177 + 5.50985) + 5\times 0.7116116 = 3.684864$$

and $\frac{l_{65}}{l_{[60]}} = \frac{8821.2612}{9263.1422}$

$$\Rightarrow P(0.95\ddot{a}_{[60]:\overline{5}|} - 0.55) = 10,000A_{[60]:\overline{5}|} + 400(IA)_{60:\overline{5}|}$$

$$\Rightarrow P(0.95\times 4.398 - 0.55) = 10,000\times 0.75104 + 400\times 3.684864$$

$$P = \frac{8984.3456}{3.6281} = 2476.32$$

(ii) Reserves required on the policy at 4% interest are:

$$_1V_{60:\overline{5}|} = 10,400A_{61:\overline{4}|} - NP\ddot{a}_{61:\overline{4}|}$$

$$= 10,000\left(1 - \frac{\ddot{a}_{61:\overline{4}|}}{\ddot{a}_{60:\overline{5}|}}\right) + 400A_{61:\overline{4}|} = 10,000\left(1 - \frac{3.722}{4.550}\right) + 400 \times 0.85685 = 2162.52$$

$$_2V_{60:\overline{5}|} = 10,000\left(1 - \frac{\ddot{a}_{62:\overline{3}|}}{\ddot{a}_{60:\overline{5}|}}\right) + 800A_{62:\overline{3}|} = 10,000\left(1 - \frac{2.857}{4.550}\right) + 800 \times 0.89013 = 4432.98$$

$$_3V_{60:\overline{5}|} = 10,000\left(1 - \frac{\ddot{a}_{63:\overline{2}|}}{\ddot{a}_{60:\overline{5}|}}\right) + 1200A_{63:\overline{2}|} = 10,000\left(1 - \frac{1.951}{4.550}\right) + 1200 \times 0.92498 = 6822.06$$

$$_4V_{60:\overline{5}|} = 10,000\left(1 - \frac{\ddot{a}_{64:\overline{1}|}}{\ddot{a}_{60:\overline{5}|}}\right) + 1600A_{64:\overline{1}|} = 10,000\left(1 - \frac{1.000}{4.550}\right) + 1600 \times 0.96154 = 9340.66$$

Year t	*Prem*	*Expense*	*Opening reserve*	*Interest*	*Death Claim*	*Mat Claim*	*Closing reserve*	*Profit vector*
1	2476.32	1485.79	0	69.34	83.43	0	2145.17	−1168.73
2	2476.32	123.82	2162.52	316.05	97.30	0	4393.04	340.73
3	2476.32	123.82	4432.98	474.98	113.25	0	6753.08	394.13
4	2476.32	123.82	6822.06	642.22	131.59	0	9234.20	450.99
5	2476.32	123.82	9340.66	818.52	152.59	11847.41	0	511.68

Year t	$_{t-1}p$	*Profit signature*	*Discount factor*	*NPV of profit signature*
1	1.0	−1168.73	.91743	−1072.23
2	0.991978	338.00	.84168	284.49
3	0.983041	387.45	.77218	299.18
4	0.973101	438.85	.70843	310.89
5	0.962062	492.27	.64993	319.94

NPV of profit signature = £142.28

Year t	*Premium*	$_{t-1}p$	*Discount factor*	*NPV of premium*
1	2476.32	1.0	1	2476.32
2	2476.32	0.991978	.91743	2253.63
3	2476.32	0.983041	.84168	2048.92
4	2476.32	0.973101	.77218	1860.73
5	2476.32	0.962062	.70843	1687.74

NPV of premiums = £10,327.34

Profit margin = $\dfrac{142.28}{10,327.34} = 0.0138 \quad i.e. \;\; 1.38\%$

END OF EXAMINERS' REPORT