

Q2 (a) Evaluate $\lim_{x \rightarrow 0} \frac{\log \sin 2x}{\log \sin x}$

Answer

$$\begin{aligned}
 & \lim_{x \rightarrow 0} \frac{\log \sin 2x}{\log \sin x} \\
 & \Rightarrow \lim_{x \rightarrow 0} \left(\frac{\frac{1}{\sin 2x} \cdot 2 \cos 2x}{\frac{1}{\sin x} \cdot \cos x} \right) \\
 & \Rightarrow \lim_{x \rightarrow 0} \left(\frac{\sin x}{\sin 2x} \cdot \frac{2 \cos 2x}{\cos x} \right) \\
 & \Rightarrow \lim_{x \rightarrow 0} \left(\frac{\sin x}{2 \sin x \cos x} \cdot \frac{2 \cos 2x}{\cos x} \right) \\
 & \Rightarrow \lim_{x \rightarrow 0} \left(\frac{\cos 2x}{\cos^2 x} \right) = \frac{\cos 0}{\cos^2 0} = \frac{1}{1} = 1
 \end{aligned}$$

Q2 (b) Expand $\cos x$ in powers of $(x - \pi/4)$ upto 4 terms. (Using Taylor's Expansion).

Answer

We have

$$\begin{aligned}
 f(x) &= f\left\{(x - \pi/4) + \pi/4\right\} = f(\pi/4) + (x - \pi/4)f'(\pi/4) \\
 &+ \frac{1}{2!}(x - \pi/4)^2 f''(\pi/4) + \dots \\
 &\Rightarrow \text{now } f(x) = \cos x, f'(x) = -\sin x. \\
 &\Rightarrow f''(x) = -\cos x, f'''(x) = \sin x, \dots \\
 &\Rightarrow \therefore f(\pi/4) = \cos \pi/4 = 1/\sqrt{2}, f'(\pi/4) = -\sin(\pi/4) = -1/\sqrt{2} \\
 &\Rightarrow f''(\pi/4) = -\cos \pi/4 = -1/\sqrt{2}, f'''(\pi/4) = \sin \pi/4 = 1/\sqrt{2}, \dots \\
 &\text{Substituting in (1), we get} \\
 \cos x &= \frac{1}{\sqrt{2}} + (x - \pi/4) \cdot \left(-\frac{1}{\sqrt{2}}\right) + \frac{1}{2!}(x - \pi/4)^2 \left(\frac{1}{\sqrt{2}}\right) + \dots \\
 &= \frac{1}{\sqrt{2}} \left\{1 - (x - \pi/4) + \frac{(x - \pi/4)^2}{2!} - \frac{(x - \pi/4)^3}{3!} + \dots\right\}
 \end{aligned}$$

Q3 (a) Evaluate $\int_0^{2a} x^2 \sqrt{2ax - x^2} dx$.

Answer

$$\text{let } I = \int_0^{2a} x^2 \sqrt{2ax - x^2} dx$$

$$\Rightarrow \int_0^{2a} x^2 \cdot x^{1/2} \sqrt{2a - x} dx$$

$$\Rightarrow \int_0^{2a} x^{5/2} \sqrt{2a - x} dx$$

$$\text{let } x = 2a \sin^2 Q, \therefore dx = 4a \sin Q \cos Q dQ$$

$$\text{when } x = 0, Q = 0; \& \text{ when } x = 2a, Q = \pi/2$$

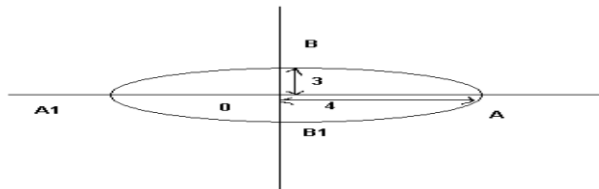
$$\therefore I = \int_0^{\pi/2} (2a)^{5/2} \sin^5 Q \sqrt{2a - 2a \sin^2 Q} \cdot 4a \sin Q \cos Q dQ$$

$$= 32a^4 \int_0^{\pi/2} \sin^6 Q \cos^2 Q dQ$$

$$= 32a^4 \left(\frac{5 \cdot 3 \cdot 1 \cdot 1}{8 \cdot 6 \cdot 4 \cdot 2} \right) \cdot \pi/2 = \frac{5\pi a^4}{8}$$

Q3 (b) Find the volume generated by revolving the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ about the x-axis.

Answer



$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

$$\Rightarrow \therefore \frac{y^2}{9} = 1 - \frac{x^2}{16}$$

$$\Rightarrow \frac{16 - x^2}{16}$$

$$\Rightarrow y^2 = \frac{9}{16}(16 - x^2)$$

$$\therefore \text{Requird volume} = 2\pi \int_0^4 y^2 dx$$

$$= 2\pi \int_0^4 \frac{9}{16}(16 - x^2) dx$$

$$= \frac{9\pi}{8} \left[16x - \frac{x^3}{3} \right]_0^4$$

$$= \frac{9\pi}{8} \left(64 - \frac{64}{3} \right)$$

$$= \frac{9\pi}{8} \left(\frac{192 - 64}{3} \right) = \frac{9\pi}{8} = \frac{128}{3} = 48\pi$$

Q4 (a) If $x + iy = \sqrt{\frac{a+ib}{c+id}}$, **prove that** $(x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$.

Answer

$$\text{we have } x + iy = \sqrt{\frac{a+ib}{c+id}} \dots\dots\dots(1)$$

taking conjugate

$$\Rightarrow x + iy = \sqrt{\frac{a-ib}{c-id}} \dots\dots\dots(2)$$

multiplying (i) & (ii) we get

$$(x + iy)(x - iy) = \sqrt{\frac{a-ib}{c-id}} * \sqrt{\frac{a-ib}{c-id}} = \sqrt{\frac{(a+ib)(a-ib)}{(c-id)(c+id)}}$$

$$\Rightarrow \sqrt{\frac{(a^2 - i^2 b^2)}{(c^2 - i^2 d^2)}} = \sqrt{\frac{(a^2 + b^2)}{(c^2 + d^2)}}$$

Q4 (b) Prove that $(1+i)^n + (1-i)^n = 2^{(n/2)+1} \cos\left(\frac{n\pi}{4}\right)$

Answer

$$(1+i)^n + (1-i)^n = 2^{(n/2)+1} \cos\left(\frac{n\pi}{4}\right)$$

$$\Rightarrow 1+i = r(\cos\phi + i\sin\phi). \text{ then}$$

$$\Rightarrow r = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\& \tan Q = 1/1 = 1$$

$$\Rightarrow Q = \frac{\pi}{4}$$

$$\therefore 1+i = \sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$$

$$\Rightarrow (1+i)^n = (\sqrt{2})^n \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)^n$$

$$\Rightarrow (2)^{n/2} \left(\cos \frac{n\pi}{4} + i \sin \frac{n\pi}{4} \right) \dots \dots \dots (1)$$

let the polar form of $1-i$ be $r(\cos\phi + i\sin\phi)$.

$$\text{then, } r = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

$$\& \tan \phi = \left(\frac{-1}{1} \right) = -1 = \tan\left(-\frac{\pi}{4}\right)$$

$$\therefore 1-i = \sqrt{2} \left(\cos \frac{\pi}{4} - i \sin \frac{\pi}{4} \right)$$

$$(1-i)^n = (\sqrt{2})^n \left(\cos\left(-\frac{\pi}{4}\right) + i \sin\left(-\frac{\pi}{4}\right) \right)^n$$

$$= (\sqrt{2})^n \left(\cos\left(-\frac{x\pi}{4}\right) + i \sin\left(-\frac{x\pi}{4}\right) \right)$$

$$= (2)^{n/2} \left(\cos\left(\frac{x\pi}{4}\right) - i \sin\left(\frac{x\pi}{4}\right) \right)$$

Adding 1 & 2

$$= (1+i)^n + (1-i)^n = 2^{n/2} \left(\cos \frac{n\pi}{4} + i \sin \frac{n\pi}{4} + \cos \frac{n\pi}{4} - i \sin \frac{n\pi}{4} \right)$$

$$= 2^{n/2} \left(2 \cos \frac{n\pi}{4} \right)$$

$$= 2^{n/2+1} \left(\cos \frac{n\pi}{4} \right)$$

Q5 (a) What is the unit vector perpendicular to each of the vectors $2\hat{i} - \hat{j} + \hat{k}$ & $3\hat{i} + 4\hat{j} - \hat{k}$? Calculate the sine of the angle between these two vectors

Answer

The vector obtained by cross multiplying the given vectors is perpendicular to each of the given vectors. When $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ & $\vec{b} = 3\hat{i} + 4\hat{j} - \hat{k}$

$$= \vec{a} \times \vec{b}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & 4 & -1 \end{vmatrix}$$

$$= (1 - 4)\hat{i} - (-2 - 3)\hat{j} + (8 + 3)\hat{k}$$

$$= -3\hat{i} + 5\hat{j} + 11\hat{k}$$

$$\text{the magnitude of this vector} = \sqrt{9 + 25 + 121} = \sqrt{155}$$

$\therefore$ Unit vectors $\perp$ to the given vectors

$$= +(-) \frac{1}{\sqrt{155}} (-3\hat{i} + 5\hat{j} + 11\hat{k})$$

let ϕ be the angle between vectors since the magnitude of $\vec{a} \times \vec{b}$ is $|\vec{a}| |\vec{b}| \sin \phi$, we have, magnitude of $(2\hat{i} - \hat{j} + \hat{k}) \times (3\hat{i} + 4\hat{j} - \hat{k})$

$$= |2\hat{i} - \hat{j} + \hat{k}| |3\hat{i} + 4\hat{j} - \hat{k}| \sin \phi$$

$$= \sqrt{4 + 1 + 1} \sqrt{9 + 16 + 1} \sin \phi$$

$$= \sqrt{156} \sin \phi$$

but the magnitude of $\vec{a} \times \vec{b}$ must be same as the magnitude of $-3\hat{i} + 5\hat{j} + 11\hat{k}$

$$= \sqrt{156} \sin \phi = \sqrt{155}$$

$$\therefore \sin \phi = \sqrt{155/156}$$

Q5 (b) A force is represented in magnitude and direction by the line joining the point A(1,-2,4) to the point B(5,2,3). Find its moment about the point (-2, 3, 5).

Answer

We have $\vec{F} = \vec{AB}$
 $= \text{P.V of B} - \text{P.V of A}$
 $= (5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) - (1\mathbf{i} + 2\mathbf{j} + 4\mathbf{k})$
 $= 4\mathbf{i} + 0\mathbf{j} - \mathbf{k}$
 let O be the point (2,3,5). then
 $\vec{r} = \vec{OA} = \text{P.V of A} - \text{P.V of O}$
 $= 1\mathbf{i} - 1\mathbf{j} - 2\mathbf{k}$
 then

$$\vec{m} = \vec{r} \times \vec{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & -2 \\ 4 & 0 & -1 \end{vmatrix}$$

$$= (5+4)\mathbf{i} - (-3+4\mathbf{j}) + (12+20)\mathbf{k}$$

$$= 9\mathbf{i} - \mathbf{j} + 32\mathbf{k}$$

Q6 (a) Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{3x}$

Answer

The given e.g $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{3x}$

Can be written as:

$$(D^2 - 5D + 6)y = e^{3x}$$

The A.E of the above equation is $(m^2 - 5m + 6) = 0$

Which gives $m = 2, 3$

Hence the complementary function is:

C.F. $= C_1 e^{2x} + C_2 e^{3x}$ where, C_1, C_2 are Arbitrary constants

Now P.I

$$\Rightarrow \frac{1}{(D^2 - 5D + 6)} e^{3x}$$

$$\Rightarrow \left(\frac{1}{D-2} - \frac{1}{D-3} \right) e^{3x}$$

$$\Rightarrow \left(\frac{1}{D-3} - \frac{1}{D-2} \right) e^{3x}$$

$$\begin{aligned}
 &\Rightarrow e^{3x} \int e^{-3x} e^{3x} dx - e^{2x} \int e^{-2x} e^{3x} dx \\
 \Rightarrow &e^{3x} \int dx - e^{2x} \int e^x dx \\
 &\Rightarrow xe^{3x} - e^{2x} \cdot e^x \\
 &\Rightarrow xe^{3x} - e^{3x} = (x-1)e^{3x} \\
 \Rightarrow &\text{Hence the genral solution is} \\
 &\Rightarrow y = C.F + P.I = Ge^{2x} + C_2 e^{3x} + e^{3x}(X-1) \\
 &\Rightarrow Ge^{2x} + (C_2-1)e^{3x} + xe^{3x} \\
 &\Rightarrow Ge^{2x} + C_3 e^{3x} + xe^{3x}, \text{Where } C_3 = C_2 - 1 \\
 &\text{new constant}
 \end{aligned}$$

Q6 (b) Solve $\frac{d^2 y}{dx^2} - \frac{dy}{dx} - 2y = 2x^2$, **given that** $y(0) = 0$ **and** $y'(0) = 0$.

Answer

The given differential equation is symbolic form will be

$$(D^2 - D - 2) = 2x^2$$

The auxiliary equation is $m^2 - m - 2 = 0$ or $(m-2)(m+1) = 0$

Which gives $m=2, -1$. $\therefore$ C.F = $C_1 e^{2x} + C_2 e^{-x}$

Now P.I

$$\begin{aligned}
&\Rightarrow \frac{1}{D^2 - D - 2} 2x^2 \\
&\Rightarrow 2 \frac{1}{-2(1 + \frac{D}{2} - \frac{D^2}{2})} x^2 \\
&\Rightarrow -1(1 + \frac{D}{2} - \frac{D^2}{2})^{-1} x^2 \\
&\Rightarrow -1[1 - \frac{D}{2} - \frac{D^2}{2}) + (\frac{D}{2} - \frac{D^2}{2})^2 - \dots] x^2 \\
&\Rightarrow [1 - \frac{D}{2} - \frac{D^2}{2} + \frac{D^2}{4} + \dots] x^2 \\
&\Rightarrow [1 - \frac{D}{2} - \frac{3}{4} D^2 + \dots] x^2 \\
&\Rightarrow -[x^2 - \frac{1}{2} Dx^2 + \frac{3}{4} D^2 x^2 + \dots] \\
&\Rightarrow -[x^2 - \frac{1}{2} 2x + \frac{3}{4}] \\
&\Rightarrow -[x^2 - x + \frac{3}{4}] \\
&\Rightarrow \text{So the genral solution is given by} \\
&\Rightarrow y = c_1 e^{2x} + c_2 e^{-x} - x^2 + x - 3/2 \dots \dots \dots (1) \\
&\text{Differentiating this w.r.t } x, \text{ we get} \\
&\frac{dy}{dx} = 2C_1 e^{2x} - C_2 e^{-x} - 2x + 1 \dots \dots \dots (2) \\
&\text{it is given that } y = 0 \text{ and } \frac{dy}{dx} = 0 \text{ at } x = 0. \\
&\Rightarrow \text{so, on putting } x = 0 \text{ in (i) and (ii), we get.} \\
&0 = C_1 - C_2 - \frac{3}{2} \\
&\& 0 = 2C_1 - C_2 + 1 \\
&\text{Solving these two equations : we get} \\
&C_1 = \frac{1}{6}, C_2 = \frac{4}{3} \\
&\text{putting values of } C_1 \& C_2 \text{ in (i), we get} \\
&y = 1/6 e^{2x} + 4/3 e^{-x} - x^2 + x - 3/2.
\end{aligned}$$

Q7 (a) Obtain a Fourier series representation for f(x) where

$$f(x) = \left(\frac{\pi - x}{2} \right)^2, 0 < x < 2\pi.$$

Answer

Let the fourier series of f(x) be

$$a_0/2 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$\text{where } a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x) dx, \quad a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos nx dx$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin nx dx.$$

$$\text{Now, } a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x) dx = \frac{1}{\pi} \int_0^{2\pi} \left(\frac{\pi - x}{2} \right)^2 dx$$

$$= \frac{1}{4\pi} \int_0^{2\pi} (\pi - x)^2 dx.$$

$$= \frac{1}{4\pi} \int_0^{2\pi} \left(\frac{(\pi - x)^3}{-3} \right)_0^{2\pi} dx.$$

$$\Rightarrow \frac{-1}{2\pi} ((\pi - 2\pi)^3 - (\pi - 0)^3)$$

$$\Rightarrow \frac{-1}{2\pi} ((-\pi)^3 - (\pi)^3)$$

$$\Rightarrow \frac{2\pi^3}{2\pi} = \frac{\pi^2}{6}$$

$$\Rightarrow a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos nx dx.$$

$$\Rightarrow \frac{1}{\pi} \int_0^{2\pi} \left(\frac{\pi - x}{2} \right)^2 \cos nx dx.$$

$$\Rightarrow \frac{1}{\pi} \left[\left(\left(\frac{\pi - x}{2} \right)^2 \frac{\sin 2x\pi}{x} \right)_0^{2\pi} - \int_0^{2\pi} \frac{1}{4} 2(\pi - x) \frac{\sin x}{x} dx \right]$$

$$\Rightarrow \frac{1}{\pi} \left[\left(\left(\frac{\pi - 2\pi}{2} \right)^2 \frac{\sin 2\pi}{x} - \left(\frac{\pi - 0}{2} \right)^2 \left(\frac{\sin 0}{x} \right)_0 + \frac{1}{2\pi} \int_0^{2\pi} (\pi - x) \sin nx dx \right) \right]$$

$$\Rightarrow \frac{1}{2n\pi} \int_0^{2\pi} (\pi - x) \sin nx dx$$

$$\Rightarrow \frac{1}{2n\pi} \left[(\pi - x) \left(\frac{-\cos nx}{x} \right)^{2\pi} - \int_0^{2\pi} (1) \frac{\cos nx}{x} dx \right]$$

$$\Rightarrow \frac{1}{2n\pi} \left[-((\pi - 2\pi) \frac{\cos 2n\pi}{x} - (\pi - 0) \frac{\cos 0}{x}) \frac{1}{x} \left(\frac{\sin nx}{x} \right)^{2\pi}_0 \right]$$

$$\Rightarrow \frac{1}{2n\pi} \left[\frac{\pi}{x} \cos 2n\pi + \frac{\pi}{x} \right] - \frac{1}{2n\pi(x)^2} (\sin 2x\pi - \sin 0)$$

$$\Rightarrow \frac{1}{2n\pi} \left[\frac{\pi}{x} (-1)^{2x} + \frac{\pi}{x} \right]$$

$$\Rightarrow \frac{1}{2n\pi} \left(\frac{2\pi}{x} \right) = \frac{1}{(x)^2}$$

$$\Rightarrow \frac{1}{\pi} \left[\left(\left(\frac{-2\pi}{x} \right)^2 \frac{\cos 2n\pi}{x} - \left(\frac{\pi - 0}{2} \right)^2 \frac{\cos 0}{x} \right) - 2/4x \int_0^{2\pi} (\pi - x) \cos nx dx \right]$$

$$\Rightarrow -\frac{1}{2n\pi} \left[-\left(\frac{\pi^2}{4x} - \frac{\pi^2}{4x} \right) - \frac{1}{2x} \int_0^{2\pi} (1) \frac{\sin nx}{x} dx \right]$$

$$\Rightarrow -\frac{1}{\pi} \left[-\left(\left(\frac{\pi - 2\pi}{4x} \right) \sin 2x\pi - \left(\frac{\pi - 0}{x} \right) \sin 0 \right) + 1/x \left(\frac{-\cos nx}{x} \right)^{2\pi}_0 \right]$$

$$\Rightarrow \frac{1}{2n\pi} \left[\frac{1}{x^2} (\cos 2n\pi - \cos 0) \right] = 0$$

=> Substituting the values of a_0, a_n and b_n in (i), we obtain the series

$$= \frac{\pi^2}{12} + \left[\frac{1}{1^2} \cos x + \frac{1}{2^2} \cos 2x + \frac{1}{3^2} \cos 3x + \dots \right]$$

$$=$$

Q7 (b) Find the Fourier sine series which represents

$$f(x) = \pi - x \text{ in the interval } (0, \pi)$$

Answer

A Fourier series consisting of sine terms alone is obtained only for an odd function. Hence we extended the function $f(x)$ on the interval $[-\pi, \pi]$ so that it becomes an odd function for this we define:

$$F(x) = \begin{cases} f(x), & \text{for } 0 < x < \pi \\ -f(-x), & \text{for } -\pi < x < 0 \end{cases}$$

$$= \begin{cases} \pi - x, & \text{for } 0 < x < \pi \\ -(\pi + x), & \text{for } -\pi < x < 0 \end{cases}$$

Now $f(x)$ is an odd function on $(-\pi, \pi)$, therefore its Fourier series is purely a sine series given by

$$= \sum_{n=1}^{\infty} b_n \sin nx \dots \dots \dots (i)$$

$$\text{where } b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx \, dx = \frac{2}{\pi} \int_0^{\pi} (\pi - x) \sin nx \, dx$$

$$= \frac{2}{\pi} \left[-(\pi - x) \cos nx / x - \sin nx / x \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left(\frac{\pi}{x} \right) = \frac{2}{x}$$

substituting the value of b_n in (i) we get

$$= \sum_{n=1}^{\infty} \frac{2}{x} \sin nx.$$

As the Fourier series for $f(x)$, since $f(x)$ is continuous in $(0, \pi)$. Consequently

$$\Rightarrow \pi - x = \sum_{n=1}^{\infty} \frac{2}{n} \sin nx$$

$$\Rightarrow 2 \left[\frac{\sin x}{1} + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \dots \right]$$

Q8 (a) Find the Laplace transform of $t^2 \cos at$

Answer

$$\begin{aligned}
&\Rightarrow \text{since } L\{\cos at\} = \frac{s}{s^2 + a^2} \\
&\Rightarrow \therefore L\{t^2 \cos at\} = (-1)^2 \frac{d^2}{ds^2} \left(\frac{s}{s^2 + a^2} \right) \\
&\Rightarrow \frac{d}{ds} \left(\frac{d}{ds} \left(\frac{s}{s^2 + a^2} \right) \right) \\
&= \frac{d}{ds} \left(\frac{s^2 + a^2 - s(2s)}{(s^2 + a^2)^2} \right) \\
&\Rightarrow \frac{d}{ds} \left(\frac{a^2 - s^2}{(s^2 + a^2)^2} \right) \\
&\Rightarrow -25(s^2 + a^2)^{-2} + (s^2 + a^2)(-2)(s^2 + a^2)^{-3}(2s) \\
&\Rightarrow \frac{-25}{(s^2 + a^2)^2} - \frac{45(a^2 - s^2)}{(s^2 + a^2)^3} \\
&\Rightarrow \frac{-25(s^2 + a^2) - 45(a^2 - s^2)}{(s^2 + a^2)^3} \\
&\Rightarrow \frac{-25s^3 - 25a^2 - 45a^2 + 45s^3}{(s^2 + a^2)^3} \\
&\Rightarrow \frac{-25s^3 + a^2}{(s^2 + a^2)^3} \Rightarrow \frac{25(s^2 + 3a^2)}{(s^2 + a^2)^3}
\end{aligned}$$

Q8 (b) Find Laplace transform of $\frac{1-e^{2t}}{t}$ **Answer**

$$\begin{aligned}
&\Rightarrow \text{we have } L\{1 - e^{2t}\} = \frac{1}{s} - \frac{1}{s-2} \\
&\Rightarrow \lim_{t \rightarrow 0} \frac{1 - e^{2t}}{t} = -2 \lim_{t \rightarrow 0} \frac{e^{2t} - 1}{2t} = -2 \times 1 = -2 \text{ exist} \\
&\Rightarrow \therefore L\left\{\frac{1 - e^{2t}}{t}\right\} = \int_s^\infty \left(\frac{1}{s} - \frac{1}{s-2} \right) ds \\
&\Rightarrow [\log s - \log(s-2)]_s^\infty \\
&\Rightarrow \left[\log\left(\frac{1}{s-2}\right) \right]_s^\infty \\
&\Rightarrow \lim_{s \rightarrow \infty} \log\left(\frac{1}{s-2}\right) - \log\left(\frac{1}{s-2}\right) \\
&\Rightarrow \log\left(\frac{s-2}{s}\right) - \lim_{s \rightarrow \infty} \log\left(\frac{s-2}{s}\right) \\
&\Rightarrow \log\left(\frac{s-2}{s}\right) - \lim_{s \rightarrow \infty} \log\left(1 - \frac{2}{s}\right) \\
&\Rightarrow \log\left(\frac{s-2}{s}\right) - \log 1 \\
&\Rightarrow \log\left(\frac{s-2}{s}\right)
\end{aligned}$$

Q9 (a) Find $L^{-1} \left\{ \frac{3s+9}{(s^2+2s+10)} \right\}$

Answer

$$\begin{aligned}
 &\Rightarrow L^{-1} \left\{ \frac{3s+9}{(s^2+2s+10)} \right\} = L^{-1} \left\{ \frac{3s+9}{(s+1)^2+(3)^2} \right\} \\
 &\Rightarrow L^{-1} \left\{ \frac{3(s+1)}{(s+1)^2+(3)^2} \right\} + L^{-1} \left\{ \frac{6}{(s+1)^2+(3)^2} \right\} \\
 &\Rightarrow 3L^{-1} \left\{ \frac{s+1}{(s+1)^2+(3)^2} \right\} + 6L^{-1} \left\{ \frac{1}{(s+1)^2+(3)^2} \right\} \\
 &\Rightarrow 3e^{-t} L^{-1} \left\{ \frac{s}{(s)^2+(3)^2} \right\} + 6e^{-t} L^{-1} \left\{ \frac{1}{(s)^2+(3)^2} \right\} \\
 &\Rightarrow 3e^{-t} \cos 3t + 6e^{-t} \left(\frac{\sin 3t}{3} \right) \\
 &\Rightarrow 3e^{-t} \cos 3t + 2e^{-t} \sin 3t \\
 &\Rightarrow e^{-t} (3 \cos 3t + 2 \sin 3t)
 \end{aligned}$$

Q9 (b) Use convolution theorem to find $L^{-1} \left\{ \frac{1}{(s^2-s-2)} \right\}$

Answer we have,

$$\begin{aligned}
 &L^{-1} \left\{ \frac{1}{(s^2-s-2)} \right\} = L^{-1} \left\{ \frac{1}{(s-2)(s+2)} \right\} \\
 &\& L^{-1} \left\{ \frac{1}{s-2} \right\} = e^{2t}, L^{-1} \left\{ \frac{1}{(s+2)} \right\} = e^{-t} \\
 &\therefore L^{-1} \left\{ \frac{1}{(s-2)(s+2)} \right\} = \int_0^t e^{-4} e^{2(t-u)} du \\
 &= e^{2t} \int_0^t e^{-3u} du \\
 &= e^{2t} \left[\frac{e^{-3u}}{-3} \right]_0^t \\
 &= \frac{-1}{3} e^{2t} (e^{-3t} - e^0) \\
 &= \frac{-1}{3} (e^{2t-3t} - e^{2t}) \\
 &= \frac{-1}{3} (e^{-t} - e^{2t}) = \frac{1}{3} (e^{2t} - e^{-t})
 \end{aligned}$$

Text Book

1. Engineering mathematics –Dr. B.S.Grewal, 12th edition 2007, Khanna publishers, Delhi.
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