Q2 (a) Show that every analytic function f(z)=u(x,y)+iv(x,y) defines two families of curves u(x,y)=C_1 and v(x,y)=C_2 which form an orthogonal system.

Answer

Since f(z)=u(x,y)+iv(x,y) is an analytic function

$$\therefore \frac{\partial y}{\partial x}=\frac{\partial v}{\partial y} \textit{and} \frac{\partial x}{\partial y}=-\frac{\partial v}{\partial x}.............(1)$$

$$\textit{staping} \text{ enreg } \mathrm{u(x, y)}x \text{ is given by } \mathrm{m}_1=\frac{dy}{dx}=-\frac{\partial 4 \big/ \partial x}{\partial 4 \big/ \partial y}...........(2)$$

$$"""""""""""""""" v(x,y)x \text{ is given by } \mathrm{m}_2=\frac{dy}{dv}=-\frac{\partial v/\partial x}{\partial v/\partial y}............(3)$$

$$\textit{product} \text{ of stapes of two curves} = \mathrm{m}_1 \times \mathrm{m}_2=\frac{\partial 4 \big/ \partial x}{\partial 4 \big/ \partial y}\times\frac{\partial v/\partial x}{\partial v/\partial y}=-1, \text{using (1)}$$

Aemer two farmily of curves $\mathrm{u}=\mathrm{c}_1$ and $\mathrm{v}=\mathrm{c}_2$ *formorthaonalsystem*

Q2 (b) Find the bilinear transformation which maps the points 1, i, –1 into the points 0, 1, ∞.

Answer

Let bilinear transformation be $w=\dfrac{az+b}{ez+d}$...........(1)

B_1 Bilinear transformation preserves the cross rotational four points

$$\frac{(z_1+z_2)(z_3-z_4)}{(z_1-z_4)(z_3-z_2)}=\frac{(w_1-w_2)(w_3-w_4)}{(w_1-w_4)(w_3-w_2)}.............(2)$$

$(1, i, -1, z)\textit{map}\sin \textit{to}$

$0,1,\infty, w$

$$\therefore \frac{(1-i)(-1-z)}{(1-z)(-1-i)}=\frac{(0-1)(\infty-w)}{(0-w)(\infty-1)}=to$$

$$\therefore w=\frac{(1-z)(1+i)}{(z+1)(1-i)}=\frac{1-z}{z+1}=\frac{1-1+2i}{1+1}=i\frac{1-z}{1+z}$$

n real transformation

Q3 (b) Find the Laurent's series expansion of $\dfrac{Z^2-6Z-1}{(Z-1)(Z-3)(Z+2)}$ **in the region**

3<$|Z+2|$<5

Answer

$$\frac{Z^2-6Z-1}{(Z-1)(Z-3)(Z+2)}=\frac{1}{z-1}-\frac{1}{z-3}+\frac{1}{z+2}$$

$$=\frac{1}{z+2-3}-\frac{1}{z+2-5}+\frac{1}{z+2}=\frac{1}{z+2}\left(1-\frac{3}{z+2}\right)^{-1}+\frac{1}{5}\left(1-\frac{z+2}{5}\right)^{-1}$$

$$=\frac{1}{z+2}\left[1+\frac{3}{z+2}+\frac{3^2}{(z+2)^2}+\frac{3^3}{(z+2)^3}+.........\right]+\frac{1}{5}\left[1+\frac{z+2}{5}+\frac{(z+2)^2}{5^2}+\left(\frac{z+2}{5}\right)^3+..........\right]$$

$$=\frac{2}{z+2}+\frac{3}{(z+2)^2}+\frac{3^2}{(z+2)^3}+..........+\frac{1}{5}\left[1+\frac{z+2}{5}+\left(\frac{z+2}{5}\right)^2+.................\right]$$

Q4 (a) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $Z=x^2+y^2-3$ at the point (2, –1, 2)

Answer

Vector normal to surface $\varphi_1 = x^2+y^2+z^2$ ....*isgivenby*

$\nabla\varphi_1 = 2xi+2yf+2zk$........................(1)

Vector normal to surface $\varphi_2 = -z+x^2+y^2+3$...................*isgivenby*

$\nabla\varphi_2 = +2xi+2yf-k$........................(2)

At the point (2,-1, 2), $\nabla\varphi_1 = 4i-2f+4k$

$\nabla\varphi_2 = +4i-2j-k$

$\therefore$ if φ line angle between two surfaces,

$$\cos\varphi=\frac{\nabla\varphi_1.\nabla\varphi_2}{|\nabla\varphi_1||\nabla\varphi_2|}=\frac{+16+4-4}{\sqrt{16+4+16}\sqrt{16+4+1}}=\frac{16}{6\sqrt{21}}$$

$$=\frac{3}{3\sqrt{21}}$$

hence

$$\varphi=\cos^{-1}\left(\frac{3}{3\sqrt{21}}\right)$$

Q5 (a) Apply Green's theorem to evaluate $\int[(y-\sin x)dx+\cos x dy]$ where C is the plane triangle enclosed by the lines y=0, $y=\frac{2}{\Pi}x$ and $x=\frac{\Pi}{2}$.

Answer

By Green`s theorem,

$$\int[(y-\sin x)dx+\cos x dy]=\int\int\left[\frac{\partial}{\partial x}(\cos x)-\frac{\partial}{\partial y}(y-\sin x)dx/dy\right]$$

$$=\int\int(-\sin x-1)dx/dy$$

Where E is create but creat by closed curve c

$$\int_{x=0}^{x=\frac{\pi}{2}}-\int_{y=0}^{y=2/\pi x}(1+\sin x)dy/dx$$

$$=-\int_0^{\pi/2}(1+\sin x)\frac{2x}{\pi}dx$$

$$=-\frac{2}{\pi}\left[\frac{x^2}{2}\bigg|_0^{\pi/2}+x(-\cos x)\,|_0^{\pi/2}+\int_0^{\pi/2}j.\cos x dx\right]$$

$$=-\frac{2}{\pi}\left[\frac{\pi^2}{8}+1\right]=-\left(\frac{\pi}{4}+\frac{2}{\pi}\right)$$

Q5 (b) For any closed surface S, use Divergence theorem to evaluate $\int_S[x(y-z)i+y(z-x)j+z(x-y)k].ds$

Answer

By Divergence theorem,

$$\int_s[x(y-z)i+y(z-x)j+z(x-y/k].ds$$

$$=\iiint_v Div[x(y-z)i+y(z-x)j+z(x-y)k].dk$$

$$=\iiint_v(y-z+z-x+x-y)dx=0$$

Q6 (a) Find an approximate value of $\log_e 5$ by calculating to 4 decimal places by Simpson's $\frac{1}{3^{rd}}$ rule. $\int_0^5 \frac{dx}{4x+5}$ dividing the range into ten equal parts.

Answer

Here f(x) = $\frac{1}{4x+5}$ and range n from x=0 to x=5. Dividing range into ten equal parts, we have

X	0	½	1	3/2	2	5/2	3	7/2	4	9/2	10
F(x)	1/5	1/7	1/9	1/11	1/13	1/15	1/17	1/19	1/21	1/23	1/25

is by Simpson's 1/3 rd rules

$$\int_0^5 \frac{dx}{4x+5} = \frac{\frac{1}{2}}{3}\left[\left(\frac{1}{5}+\frac{1}{25}\right)+4\left(\frac{1}{7}+\frac{1}{11}+\frac{1}{15}+\frac{1}{19}+\frac{1}{23}\right)+2\left(\frac{1}{9}+\frac{1}{13}+\frac{1}{17}+\frac{1}{21}\right)\right]$$

$$= 0.4025$$

By exact integration,

$$\int_0^5 \frac{1}{4x+5}dx = \frac{1}{4}\log(4x+5)\Big|_0^5 = \frac{1}{4}\left[\log 25 - \log 5\right]$$

$$= \frac{1}{4}\log 5$$

$$Hence \frac{1}{4}\log_e 5 = 0.4025$$

or

$$\log_e 5 = 4 \times 0.4025 = 1.622$$

Q6 (b) Given the values

X	5	7	11	13	17
f(x)	150	392	1452	2366	5202

Evaluate f(9), using

(i) Lagrange's formula
(ii)Newton's divided difference formula

Answer

(i) Using Lagrange's formula

$$f(a)=\frac{(a-1)(a-11)(a-17)}{(5-7)(5-11)(5-13)(5-17)}\times 15U+\frac{(a-5)(a-7)(a-11)(a-17)}{(13-5)(13-7)(13-11)(13-1)}\times 2366$$

$$+\frac{(a-5)(a-7)(a-B)(a-17)}{(11-5)(11-7)(11-13)(11-17)}\times 1452+\frac{(a-5)(a-7)(a-11)(a-17)}{(13-5)(13-7)(13-11)(13-1)}\times 2366$$

$$+\frac{(a-5)(a-7)(a-11)(a-13)}{(17-5)(17-7)(17-11)(17-13)}\times 5202=810$$

(ii) Using divided differences table

Divided Difference table h

X	F(x)	14.DD	2^{nd} DD	3^{rd} DD	4^{th} DD
5	150				
		121			
7	392				
			24		
11	1452	265		1	0
13	2366	454	32		
17	5202	709	42	1	

By Newton Divided formula

F (a) =150+ (9-5)121+ (9-5) (9-7)24+ (9-5) (9-7) (9-11)*1=810

Q7 (a) Use Charpit's method to solve pxy +pq +qy = yz

Answer

Here f(x,y,z,p,q)=pxy+pq+qy-yz=0………………(1)

Charpit's Subsiquary equation are

$$\frac{dx}{-(xy+q)}=\frac{dy}{-(p+y)}=\frac{dz}{-xyp-pq-pq-yq}=\frac{dp}{py-py}=\frac{dq}{py+q-z-qy}$$

$$\therefore ap=0$$

$$or$$

$$p=c..................(ii)$$

$$solving(i)and(ii),$$

$$v=\frac{yz-cxy}{c+y}........................(iii)$$

subsitituting p and q in dz=pdx+qdy ,we get

$$dz = cdx + \frac{yz - cxy}{c + y} dy = cdx + \frac{y(z - cx)}{c + y} dy$$

or

$$\frac{dz - cdx}{z - cx} = \frac{y}{c + y} dy = \left(1 - \frac{c}{c + y}\right) dy$$

Integratary

$$\log(z - cn) = y - c\log(c + y) + b$$

or

$$(z - cn)(c + y)^e = Be^y$$

Q7 (b) Use method of separation of variables to solve $3\frac{\partial U}{\partial x} + 2\frac{\partial U}{\partial y} = 0$, given that $U(x,0)=4e^{-x}$

Answer

Let U=X(x) Y(y)

$\therefore$ Given equation becomes

$$3X^1Y + 2XY^1 = 0$$

$$OR$$

$$3\frac{X^1}{X} = K$$

$$OR$$

$$X = e_1 e^{\frac{K}{3}X}$$

$$and \text{ - } 2\frac{Y^1}{Y} = K$$

$$OR$$

$$Y = e_2 e^{-k/2y}$$

$$Hence$$

$$U = e_1 e^{\frac{k}{3}y} . e_2 e^{-\frac{k}{2}y} = ee^{k/3y - k/2y}$$

$$u \sin g \text{ equation}$$

$$\text{U(x,0)} = 4\text{e}^{-x}, \text{we get c} = 4, \text{k} = -3$$

$$\text{hence reqd solution "U} = 4\text{e}^{-x+3/2y}$$

Q8 (a) Two persons ‘A’ and ‘B’ toss an unbiased coin alternately on the understanding that the first who gets the head wins. If ‘A’ starts the game, compare their chances of winning.

Answer

Probability of throwing head with an unbiased coin =1/2=p
“””””””””” of not “”””=1-1/2=1/2=q

If A starts the game, he wins of he throws head or if he does not throw head then B gets his turn and he also does not get start A get again his turn and he throws head or again A does not throw head, B does not those head A does not turn head, B does not throw head and then A throw head and so on

∴ prob of A’s winning =p+qqp+qqqqpe…………
=1/2+1/21/21/2+1/21/21/21/21/2+………………
=1/2/1-4=2/3
Since either y the two has to win, so chances of B’s winning
=1-1/3=1/3
∴ Their enhence of winning are 2/3**:**1/3
or
2:1

Q8 (b) An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of accident is 0.01, 0.03 and 0.15 respectively. One of the insured person meets an accident. What is the probability that he is a scooter driver?

Answer

Let P (B_1) =prob. of insured person be scooter driver =2000/12000=2/12
P (B_2) =----------------“”------------------car driver =4000/12000=4/12
P (B_3) = ----------------“””----------------Truck driver =6000/12000=6/12
P(A/ B_1)=polar of accident of scooter driver =.01=1/10v
P(A/ B_2 **)=**-------------“””-------------------- car driver =.03=3/100
P(A/ B_3 **)=**-------------“”””-----------------truck driver …15=15/100

∴. By Bays theorem,
P(B_1 /A)=Prob. of unshared person to the car driver

$$\frac{P(B_1)P(A/B_1)}{P(B_1)P(A/B_1)+P(B_2)P(A/B_2)+P(B_3)P(A/B_2)}$$

$$=\frac{\frac{2}{12}\times}{\frac{2}{12}.\frac{1}{100}+\frac{4}{12}.\frac{3}{100}+\frac{6}{12}.\frac{15}{100}}=\frac{2}{104}=\frac{1}{52}$$

Q9 (a) The diameter of an electric cable is assumed to be a continuous variate with probability density function given by f(x) = Kx(1–x), $0 \le x \le 1$. Find the number K. Also find the mean and the variance.

Answer

Since f(x)=kx(1-x), $0 \le x \le 1$ is a probability density function,

$$\therefore \int_0^1 f(x)dx = 1 = \int_0^1 kx(1-x)dx = k\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$\therefore K = 6$$

$$mean = \bar{x} = \int_0^1 xf(x)dx = \int_0^1 x.6x(1-x)dx = \frac{1}{2}$$

$$\operatorname{var} iance = \int_0^1 (x-\bar{x})^2 f(x)dx = 6\int_0^1 (x-\bar{x})^2 x(1-x)dx = \frac{1}{20}$$

<u>**Text Book**</u>

1. Higher Engineering Mathematics –Dr. B.S.Grewal, 40th Edition 2007, Khanna Publishers, Delhi.

2. A Text book of engineering Mathematics – N.P. Bali and Manish Goyal , 7th Edition 2007, Laxmi Publication(P) Ltd.