ROLL NO. ___________________

Code: DE55/DC55 Subject: ENGINEERING MATHEMATICS

DipIETE – ET/CS

Time: 3 Hours **DECEMBER 2013** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. The value of the limit $\underset{x\to 0}{\mathrm{Lt}} \dfrac{\sin x^\circ}{x}$ is equal to

(A) π **(B)** 1

(C) $\dfrac{\pi}{180}$ **(D)** 0

b. The value of definite integral $\int_0^{\pi/2} \sin^7 x dx$ is equal to

(A) $\dfrac{14}{35}$ **(B)** $\dfrac{16}{35}$

(C) $\dfrac{17}{35}$ **(D)** $\dfrac{11}{35}$

c. The complementary function for the differential equation $\dfrac{d^2x}{dt^2} + \dfrac{g}{\ell}x = \dfrac{g}{\ell}L$ where g, ℓ,L are constants, is given by

(A) $c_1 \cos\sqrt{\dfrac{\ell}{g}}\, t + c_2 \sin\sqrt{\dfrac{\ell}{g}}\, t$ **(B)** $c_1 \cos\sqrt{\dfrac{g}{\ell}}\, it + c_2 \sin\sqrt{\dfrac{g}{\ell}}\, it$

(C) $c_1 \cos\sqrt{\dfrac{g}{\ell}}\, t + c_2 \sin\sqrt{\dfrac{g}{\ell}}\, t$ **(D)** $c_1 \cos\sqrt{\dfrac{g}{\ell}}\, t + ic_2 \sin\sqrt{\dfrac{g}{\ell}}\, t$

d. The principal argument of -2 i is equal to

(A) $-\pi/3$ **(B)** $-\pi/2$

(C) $\pi/2$ **(D)** $\pi/3$

e. If $Z = 1 + i\sqrt{3}$, then $Z^2 + 4$ is equal to

(A) $Z\sqrt{3}$ **(B)** 3Z
(C) 2Z **(D)** 4Z

f. $L\{4\cos 5t\}$ is equal to

(A) $\frac{5S}{S^2+16}$ **(B)** $\frac{2S}{S^2+16}$
(C) $\frac{4S}{S^2+16}$ **(D)** $\frac{4S}{S^2+25}$

g. $L^{-1}\left\{\frac{5}{S+3}\right\}$ is equal to

(A) $3e^{-5t}$ **(B)** $5e^{3t}$
(C) $5e^{-3t}$ **(D)** $3e^{5t}$

h. The period of the function of $|\cos x|$ is equal to

(A) π **(B)** 2π
(C) 3π **(D)** 4π

i. If $\vec{a} = 3i + 2j + 9k$ and $\vec{b} = i + \lambda j + 3k$ are perpendicular to each other then λ is equal to

(A) -15 **(B)** 27
(C) -27 **(D)** 15

j. The voltage and current of a circuit are given by the complex number 3 + 4j and 2 – 5j respectively then complex number of the impedance of the circuit is equal to

(A) $\frac{14}{29} - \frac{23}{29}j$ **(B)** $\frac{7}{29} - \frac{15}{29}j$
(C) $\frac{7}{29} + \frac{15}{29}j$ **(D)** $\frac{-14}{29} + \frac{23}{29}j$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Evaluate $\underset{x \to 0}{LT} \frac{\sin 2x + \sin 6x}{\sin 5x - \sin 3x}$ **(8)**

b. If f is a real function defined by $f(x)=\frac{x-1}{x+1}$ then prove tha $f(2x)=\frac{3f(x)+1}{f(x)+3}$ **(8)**

Q.3 a. Find the volume of the right circular cone formed by the revolution of a right angled triangle about a side which contains the right angle. **(8)**

b. Find the length of the curve $y^2=x^3$ from origin to the point (1, 1). **(8)**

Q.4 a. If n is a positive integer then show that $\left(\sqrt{3}+i\right)^n+\left(\sqrt{3}-i\right)^n=2^{n+1}\cos\frac{n\pi}{6}$ where i = $\sqrt{-1}$ **(8)**

b. A resistance of 20 ohms and inductance of 0.2 H and a capacitance of $100\,\mu F$ are connected in series a cross 220 Volt, 50cycle/sec main. Determine:
(i) impedance (ii) current
(iii) voltage across L,R and C (iv) power in watt
(v) power factor **(8)**

Q.5 a. A rigid body is spinning with an angular velocity of 27 radian/second about an axis parallel to 2i +j–2k passing through the point i+3j-k. Find the velocity of the point whose position vector is 4i+8j+k. **(8)**

b. Find the area of the triangle formed by the point whose position vectors are 3i+j, 5i+2j+k, i-2j+3k. **(8)**

Q.6 a. Solve $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=6e^{3x}+7e^{-2x}-\log 2$ **(8)**

b. Solve $\frac{d^2y}{dx^2}+9y=\sec 3x$ **(8)**

Q.7 a. Expand f(x) = e^x in a cosine series over (0, 1) **(8)**

b. Find the Fourier Series of the function **(8)**

$$f(t)=\begin{cases}0 & \text{when } -2<t<-1\\ K & \text{"} \quad -1<t<1\\ 0 & \text{"} \quad 1<t<2\end{cases}$$

Q.8 a. Evaluate $L\left\{te^{-t}\cosh t\right\}$ **(8)**

b. Evaluate $L\left\{\int_0^t \frac{e^t \sin t}{t}\,dt\right\}$ **(8)**

Q.9 a. Show that $L^{-1}\left\{\frac{S^2}{S^4+4a^4}\right\} = \frac{1}{2a}(\cosh at.\sin at + \sinh at.\cos at)$ **(8)**

b. Evaluate $L^{-1}\left\{\log\frac{s+1}{s-1}\right\}$ **(8)**