

## DiplETE – ET/CS

Time: 3 Hours

**DECEMBER 2013**

Max. Marks: 100

**PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.**

**NOTE: There are 9 Questions in all.**

- Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

**Q.1 Choose the correct or the best alternative in the following: (2×10)**

a.  $\lim_{x \rightarrow 0} \frac{\sin x^0}{x}$

(A)  $\frac{180}{\pi}$

(B)  $\frac{\pi}{360}$

(C)  $\frac{360}{\pi}$

(D)  $\frac{\pi}{180}$

b. If  $y = \tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right)$  then  $\frac{dy}{dx}$  is

(A)  $-\frac{1}{2}$

(B)  $\frac{1}{2}$

(C) 1

(D) -1

c.  $\int \cos^3 x dx$  is

(A)  $\frac{\cos 3x}{12} + \frac{3}{4} \cos x + C$

(B)  $\frac{\sin 3x}{12} - \frac{3}{4} \sin x + C$

(C)  $\frac{\sin 3x}{12} + \frac{3}{4} \sin x + C$

(D)  $\frac{\cos 3x}{12} - \frac{3}{4} \cos x + C$

d. If  $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ , then  $A^2 - 4A + 7I$  is

(A)  $\begin{bmatrix} 1 & 6 \\ 2 & 4 \end{bmatrix}$

(B)  $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(C)  $\begin{bmatrix} -1 & 8 \\ 0 & 0 \end{bmatrix}$

(D)  $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

e. If  $\Delta = \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$  and  $\omega$  is a cube root of unity, then the value of  $\Delta$  is:

- (A) -1 (B) 3  
(C) 0 (D) 2

f. The value of  $\cos 52^\circ + \cos 68^\circ + \cos 172^\circ$  is

- (A) -1 (B) 0  
(C) 2 (D) 1

g. The term independent of  $x$  in the expansion of  $\left(2x^2 - \frac{1}{x}\right)^{12}$  is

- (A) 7820 (B) 4595  
(C) 8410 (D) 7920

h. If  $\cos(A-B) = 3\cos(A+B)$ , then  $\cot A \cot B$  is:

- (A) -2 (B) 2  
(C) 1 (D) 3

i. The area of a triangle whose vertices are (6, 3), (-3, 5) and (4, -2) is:

- (A) 24.5 sq units (B) 23 sq units  
(C) 24 sq units (D) 26.4 sq units

j. The equation of the line which makes intercepts -4 and 5 on the axis is:

- (A)  $4x + 5y + 20 = 0$  (B)  $4x - 5y + 20 = 0$   
(C)  $5x - 4y + 20 = 0$  (D)  $5x + 4y + 20 = 0$

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**Answer any FIVE Questions out of EIGHT Questions.**  
**Each question carries 16 marks.**

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**Q.2** a. Prove that  $\cos \alpha + \cos \left(\alpha + \frac{2\pi}{3}\right) + \cos \left(\alpha + \frac{4\pi}{3}\right) = 0$  (8)

b. Prove that  $\frac{\sec 8A - 1}{\sec 4A - 1} = \frac{\tan 8A}{\tan 2A}$  (8)

**Q.3** a. Find the coefficient of  $x^{18}$  in the expansion of  $\left(x^2 + \frac{3a}{x}\right)^{15}$  (8)

- b. If the first term of an A.P is 2 and the sum of first five terms is equal to one-fourth of the sum of the next five terms, then (i) show that  $T_{20} = -112$  (ii) find the sum of first 30 terms. (8)

**Q.4** a. Show that

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c) \quad (8)$$

- b. Using determinants solve the following system of equations: (8)

$$\begin{aligned} 2y - 3z &= 0 \\ x + 3y &= -4 \\ 3x + 4y &= 3 \end{aligned}$$

**Q.5** a. Find the equation of the right bisector of the segment joining A(1, 1) and B(2, 3) (8)

- b. Find the equation of the lines through the origin and making an angle of  $60^\circ$  with the line  $x + y\sqrt{3} + 3\sqrt{3} = 0$  (8)

**Q.6** a. Find the equation of the circle which passes through the points (5, -8), (2, -9) and (2, 1). Find also the co-ordinates of its centre and radius. (8)

- b. Find the length of major and minor axis, eccentricity, the co-ordinates of vertices and foci, directrices and the length latus rectum of the ellipse  $3x^2 + 2y^2 = 6$ . (8)

**Q.7** a. If  $y = \log(x + \sqrt{1+x^2})$ , Prove that

$$(1+x^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = 0 \quad (8)$$

- b. Find the equation of the tangent to the curve  $y = \frac{x-7}{(x-2)(x-3)}$  at the point where it cuts x-axis. (8)

**Q.8** a. Evaluate  $\int \frac{4x+1}{x^2+3x+2} dx$  (8)

- b. Evaluate  $\int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx$  (8)

**Q.9** a. Solve the initial value problem

$$x \frac{dy}{dx} + \cot y = 0, \text{ when } y(\sqrt{2}) = \pi/4 \quad (8)$$

b. Solve  $(x^2 + xy)dy = (x^2 + y^2)dx$  (8)