Q.2 a. Define Quantization. Derive the signal-to-quantization noise ratio for sinusoidal signals.

Answer: Topic 4.8.3, Page Number 219 of Text Book 1

b. In the system shown in Fig.1, $X_c(j\Omega)$ and $H(e^{j\omega})$ are as shown

and $1/T_1 = 30000$, $1/T_2 = 10000$ respectively. Sketch and label the Fourier transforms of $y_d[n]$ and $y_c(t)$.

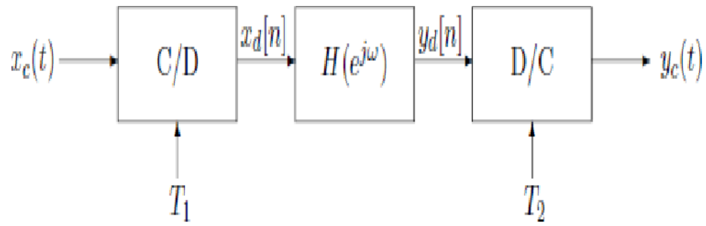


Fig.1

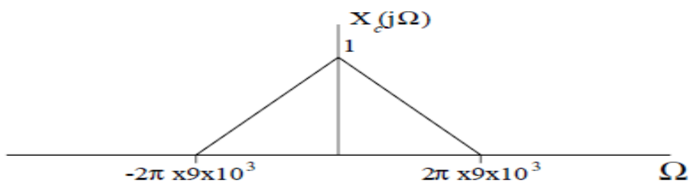


Fig.2

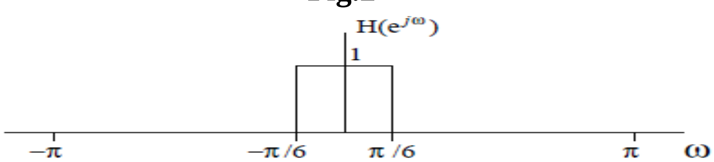
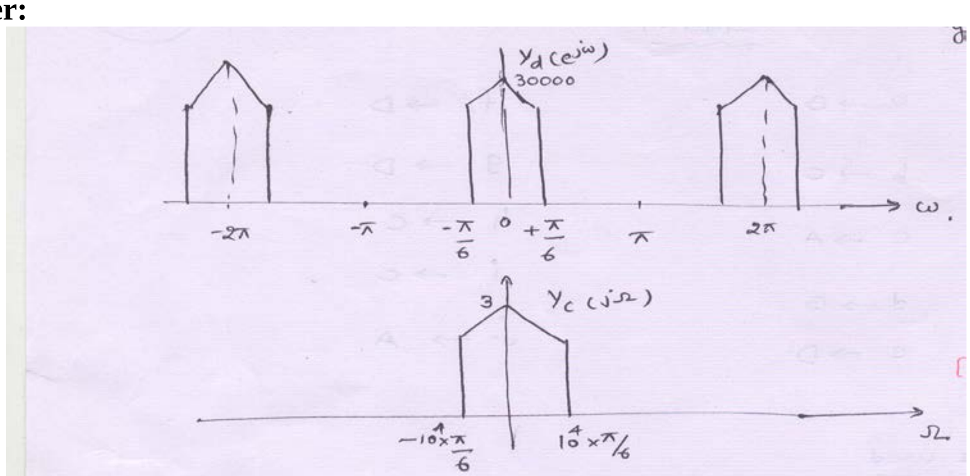


Fig.3

Answer:



Q.3 a. Consider the LTI system with input x[n] and output y[n], which are related through the difference equation: y[n] -5/2 y[n-1] + y[n-2] = x[n]

(i) Obtain the system function and its ROC
(ii) Draw its pole-zero plot
(iii) Comment on the causality and stability of this system

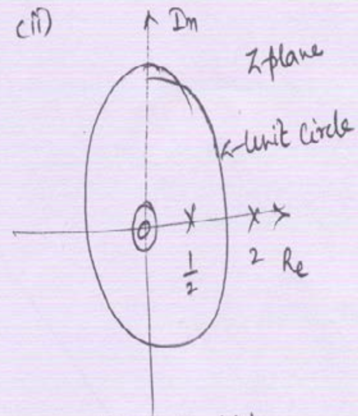
Answer:

Q.3 (a) $y(n) - \frac{5}{2} y(n-1) + y(n-2) = x(n)$

(i) obtain system fn & its ROC (ii)

$$\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{5}{2} z^{-1} + z^{-2}}$$

$$= \frac{1}{(1 - \frac{1}{2} z^{-1})(1 - 2z^{-1})}$$



Pole-zero plot.

(iii) ROC: three choices

If system, assumed, causal → ROC is outside the outmost pole i.e. $|z| > 2$
and unstable.

If system is, stable (Assume) → ROC will be $\frac{1}{2} < |z| < 2$.
and noncausal

For ROC $|z| < \frac{1}{2}$, System will be ~~neither Stable or causal~~, anticausal and unstable.

b. A discrete-time causal LTI system has the system function

$$H(z) = \frac{(1 + 0.2z^{-1})(1 - 9z^{-2})}{(1 + 0.81z^{-2})}$$

Find expression for a minimum-phase system $H_1(z)$ and an all- pass system $H_{ap}(z)$ such that $H(z) = H_1(z)\, H_{ap}(z)$.

Answer:

Q3(b)

$$H(z) = \frac{(1+0.2z^{-1})(1-9z^{-2})}{(1+0.81z^{-2})}$$

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$$= \left[\frac{(1+0.2z^{-1})}{(1+0.81z^{-2})}(1-3z^{-1})(1+3z^{-1})\right]\left[\frac{(1-\frac{1}{3}z^{-1})(1+\frac{1}{3}z^{-1})}{(1-\frac{1}{3}z^{-1})(1+\frac{1}{3}z^{-1})}\right]$$

$$= \left[\frac{(1+0.2z^{-1})(1-\frac{1}{3}z^{-1})(1+\frac{1}{3}z^{-1})}{(1+0.81z^{-2})}\right]\left[\frac{(1-3z^{-1})(1+3z^{-1})}{(1-\frac{1}{3}z^{-1})(1+\frac{1}{3}z^{-1})}\right]$$

$$H(z) = \left[\frac{(1+0.2z^{-1})(1-\frac{1}{9}z^{-2})}{(1+0.81z^{-2})}\right]\left[\frac{1-9z^{-2}}{1-\frac{1}{9}z^{-2}}\right]$$

3b

$$H(z) = H_1(z)\,H_{ap}(z)$$

where $H_1(z) = \dfrac{(1+0.2z^{-1})(1-\frac{1}{9}z^{-2})}{(1+0.81z^{-2})}$ ... minimum-phase system

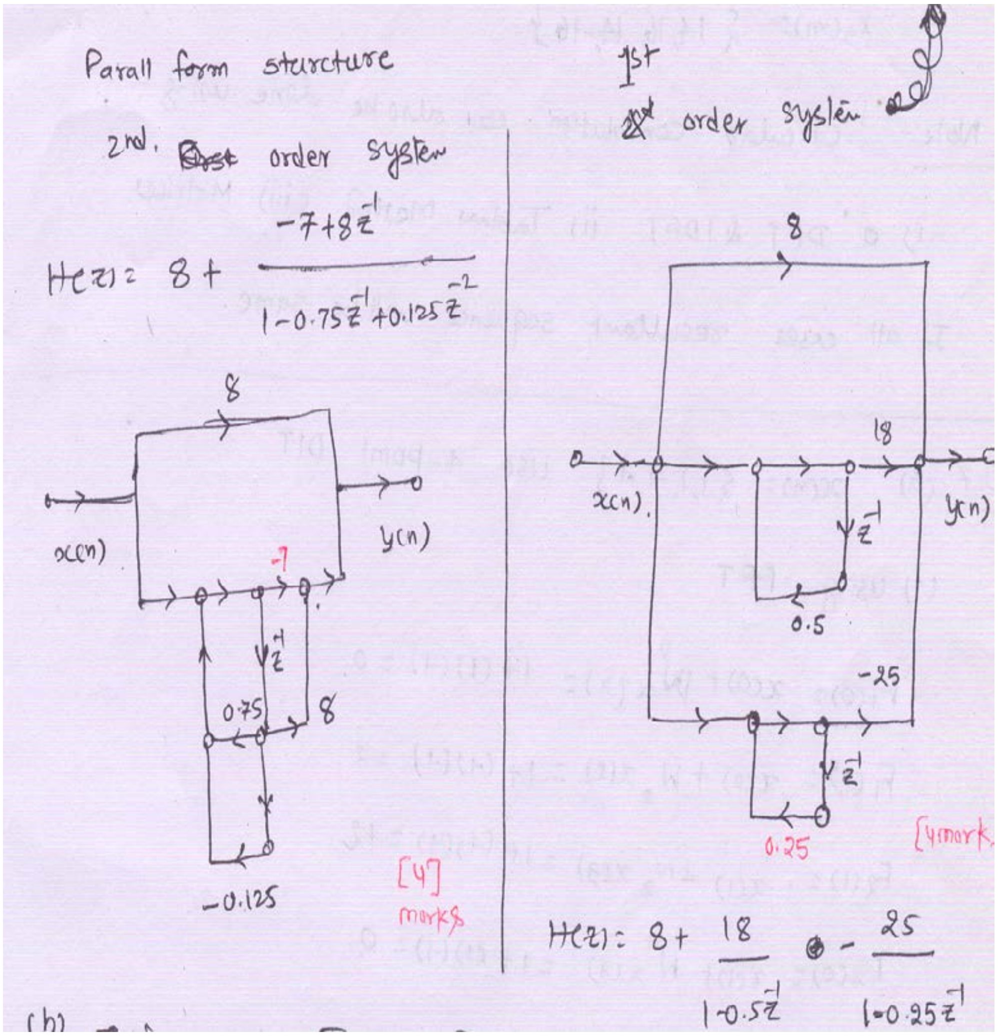
[08 mar

$H_{ap}(z) = \dfrac{1-9z^{-2}}{1-\frac{1}{9}z^{-2}}$ ... all-pass system.

Q.4 a. Obtain the parallel-form structure of the given H(z) for first-order and second order systems.

$$H(z) = \frac{(1+2z^{-1}+z^{-2})}{(1-0.75z^{-1}+0.125z^{-2})}$$

Answer:



b. Describe the signal flow graph representation of linear constant coefficient difference equations.

Answer: Topic 6.2 of Text Book 1

Q.5 a. With an example, design a differentiator using Kaiser Window concept.

Answer: Topic 7.3.2 of Text Book 1

b. Discuss the Parks- McClellan algorithm for type I low pass filter.

Answer: Topic 7.4.3 of Text Book 1

Q.6 a. Discuss and prove the following properties of Discrete Fourier Transform. (i) Duality (ii) Symmetry

Answer: Topic 8.6.3 and 8.6.4 of Text Book 1

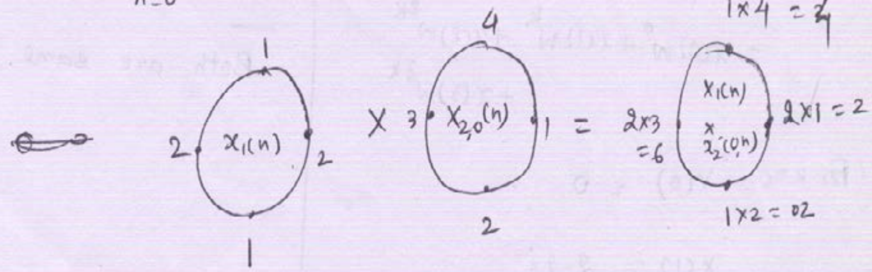
b. Perform the Circular Convolution of the two sequences $x_1(n) = \{\underline{\mathbf{2}}, 1, 2, 1\}$ and $x_2(n) = \{\underline{\mathbf{1}}, 2, 3, 4\}$.

Answer:

(b) circular convolution $x_1(n) = \{\underset{\uparrow}{2},1,2,1\}$ & $x_2(n) = \{\underset{\uparrow}{1},2,3,4\}$

$$x_3(m) = \sum_{n=0}^{N-1} x_1(n)\cdot x_2(m-n) = \sum_{n=0}^{N-1} x_1(n)\cdot x_{2,m}(n)$$

$$m=0, \quad x_3(0) = \sum_{n=0}^{3} x_1(n)\cdot x_{2,0}(n)$$



$\therefore x_3(0) = 4+2+2+6 = 14$

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Similary $m=1 \quad x_3(1) = 16$

$m=2 \quad x_3(2) = 14$

$m=3 \quad x_3(3) = 16$.

$\therefore x_3(m) = \{14, 16, 14, 16\}$

Note:- Circular convolution can also be done using

i) a DFT & IDFT ii) Tabluar Method (iii) Matrices.

In all cases resultant sequence will be same.

Q.7 a. For x(n) = (1,1,-1,-1) use 4-point DIT, algorithm for FFT and cross check the result using DFT.

Answer:

Q:7 (a) $x(n) = \{1, 1, -1, -1\}$ use 4-point DIT

(i) using FFT

$F_1(0) = x(0) + W^0 x(2) = 1 + (1)(-1) = 0.$

$F_1(1) = x(0) + W_2 x(2) = 1 + (-1)(-1) = 2$

$F_2(1) = x(1) + W_2 x(3) = 1 + (-1)(-1) = +2$

$F_2(0) = x(1) + W^0 x(3) = 1 + (1)(-1) = 0$

$\therefore X(0) = F_1(0) + W_4^0 F_2(0) = 0 + (1)(0) = 2.$

$X(1) = F_1(1) + W_4^1 F_2(1) = 2 + (-j)(2) = 2 - 2j$

$X(2) = F_1(0) + W_4^2 F_2(0) = 0 + (-1)(0) = 0$

$X(3) = F_1(1) + W_4^3 F_2(1) = 2 + j(2) = 2 + 2j$

[4 marks]

$\therefore X(k) = \{0, 2-2j, 0, 2+2j\}.$

Cross check using DFT

$$X(k) = \sum_{n=0}^{3} x(n) W^{nk}$$

$$= x(0)W^0 + x(1)W^k + x(2)W^{2k} + x(3)W^{3k}$$

For $k=0$. $X(0) = 0$

$X(1) = 2 - 2j$

$X(2) = 0$

$X(3) = 2 + 2j$

hence $X(k) = \{0, 2-2j, 0, 2+2j\}$

Both are same.

[4 marks]

Q.8 a. Discuss the effect of windowing on Fourier analysis of sinusoidal signals.
Answer: Section 10.2, Page Number 723 of Text Book 1

b. Discuss the time-dependent Fourier transform with a suitable example.

Answer: Topic 10.3 of Text Book 1

Q.9 b. For a real, causal sequence x(n) for which $X_R(e^{jw}) = \frac{5}{4} - \cos\omega$. Obtain

(i) The original sequence x(n) and

(ii) Imaginary part of the Fourier transform $X_I(e^{jw})$.

Answer:

Q.9(b) (i) $X_R(e^{j\omega}) = \frac{5}{4} - \cos\omega = \frac{5}{4} - \frac{1}{2}e^{j\omega} - \frac{1}{2}e^{-j\omega}$

we know that $x_e(n) \longleftrightarrow X_R(e^{j\omega})$

$$x_e(n) = \text{IDTFT}\left[X_R(e^{j\omega})\right]$$

$$x_e(n) = \text{IDTFT}\left[\frac{5}{4} - \frac{1}{2}e^{j\omega} - \frac{1}{2}e^{-j\omega}\right]$$

$$x_e(n) = \frac{5}{4}\delta(n) - \frac{1}{2}\delta(n+1) - \frac{1}{2}\delta(n-1)$$

$$\therefore \quad x(n) = 2x_e(n)u(n) - x_e(0)\delta(n)$$

$$x(n) = \left[\frac{5}{2}\delta(n) - \delta(n-1)\right] - \frac{5}{4}\delta(n)$$

$$x(n) = \frac{5}{4}\delta(n) - \delta(n-1)$$

$$X(e^{j\omega}) = \frac{5}{4} - e^{-j\omega}$$

$$= \left[\frac{5}{4} - \cos\omega\right] + j\sin\omega$$

$$X(e^{j\omega}) = X_R(e^{j\omega}) + jX_I(e^{j\omega})$$

$$\therefore \quad X_I(e^{j\omega}) = \sin\omega$$

TEXT BOOKS

Discrete-Time Signal Processing (1999), Oppenheim, A. V., and Schafer, R. W., with J. II R.Buck, Second Edition, Pearson Education, Low Price Edition.