ROLL NO. ___________

Code: AE51/AC51/AT51 Subject: ENGINEERING MATHEMATICS – I

AMIETE – ET/CS/IT

Time: 3 Hours

DECEMBER 2013

Max. Marks: 100

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions, answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. If $u = \frac{x}{y+z} + \frac{y}{z+x} + \frac{z}{x+y}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$ is equal to

(A) 0 **(B)** 1
(C) 2 **(D)** x+y+z

b. The rank of the matrix $\begin{bmatrix} 2 & -1 & 3 & 1 \\ 1 & 4 & -2 & 1 \\ 5 & 2 & 4 & 3 \end{bmatrix}$ is

(A) 1 **(B)** 2
(C) 3 **(D)** 4

c. The value of the double integral $\int_0^{2a} \int_0^{\frac{x^2}{4a}} xy\, dy\, dx$ is

(A) a^4 **(B)** $\frac{a^4}{2}$
(C) $\frac{a^4}{3}$ **(D)** $\frac{a^4}{4}$

d. Iterative formula for finding approximate root of f(n) = 0, using Newton-Raphson method, is

(A) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ **(B)** $x_{n+1} = x_n - \frac{f'(x_n)}{f(x_n)}$

(C) $x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}$ **(D)** $x_{n+1} = x_n + \frac{f'(x_n)}{f(x_n)}$

e. The differential equation $\left(y^2 e^{xy^2} + 4x^3\right)dx + \left(Kxye^{xy^2} - 3y^2\right)dy = 0$ is exact if K is equal to

(A) 0 **(B)** 1
(C) 2 **(D)** 3

f. The solution of differential equation $4\frac{d^3y}{dx^3} + 4\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

(A) $y = \left(c_1 + c_2x + c_3x^2\right)e^{\frac{x}{2}}$ **(B)** $y = (c_1 + c_2x)e^{\frac{x}{2}} + c_3$

(C) $y = \left(c_1 + c_2x + c_3x^2\right)e^{\frac{-x}{2}}$ **(D)** $y = (c_1 + c_2x)e^{\frac{-x}{2}} + c_3$

g. Particular Integral (P.I.) of the differential equation $\left(D^2 + 3D + 2\right)y = 5$ is equal to

(A) $\frac{2}{5}$ **(B)** $\frac{1}{5}$

(C) 0 **(D)** $\frac{5}{2}$

h. $\beta(m+1, n) + \beta(m, n+1)$ is equal to

(A) $\beta(m+1, n+1)$ **(B)** $\beta(m, n)$
(C) $\beta(m-1, n-1)$ **(D)** None of these

i. $J_2(x)$ interms of $J_1(x)$ and $J_0(x)$ is

(A) $J_1(x) + \frac{2}{x}J_0(x)$ **(B)** $J_1(x) - \frac{2}{x}J_0(x)$

(C) $\frac{2}{x}J_1(x) + J_0(x)$ **(D)** $\frac{2}{x}J_1(x) - J_0(x)$

j. Legendre's polynomial $P_2(x) = \frac{1}{2}\left(Kx^2 - 1\right)$ where K is equal to

(A) 1 **(B)** 2
(C) 3 **(D)** 4

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If Z is a homogeneous function of degree n in x and y, show that

$$x^2\frac{\partial^2 z}{\partial x^2}+2xy\frac{\partial^2 z}{\partial x\partial y}+y^2\frac{\partial^2 z}{\partial y^2}=n(n-1)z$$ **(8)**

b. Use method of differentiation under integral sign to show that

$$\int_0^\infty \frac{\tan^{-1}ax}{x(1+x^2)}dx=\frac{\pi}{2}\log(1+a), a\ge 0$$ **(8)**

Q.3 a. Change the order of integration and then evaluate $\int_0^{4a}\int_{y^2/4a}^{2\sqrt{ay}} dx\ dy$ **(8)**

b. Find the volume bounded by the xy-plane, the cylinder $x^2+y^2=1$ and the plane x+y+z = 3. **(8)**

Q.4 a. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}$ **(8)**

b. Determine the rank of the following matrices:

(i) $\begin{bmatrix}1 & 2 & 3\\ 1 & 4 & 2\\ 2 & 6 & 5\end{bmatrix}$ (ii) $\begin{bmatrix}0 & 1 & -3 & -1\\ 1 & 0 & 1 & 1\\ 3 & 1 & 0 & 2\\ 1 & 1 & -2 & 0\end{bmatrix}$ **(8)**

Q.5 a. Use Regula-Falsi method to compute real root of $xe^x=2$ correct to three decimal places. **(8)**

b. Find by Runge-Kutta method of order four, an approximate value of y at x = 0.2 for the equation $\frac{dy}{dx}=\frac{y-x}{y+x}, y(0)=1$. Take h = 0.2. **(8)**

Q.6 a. Solve the differential equation $(x+1)\frac{dy}{dx}+1=2e^{-y}$ **(8)**

b. Find the orthogonal trajectories of family of curves $ay^2 = x^3$ **(8)**

Q.7 a. Solve the differential equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = e^{2x} + x + \text{Sin}x$ **(8)**

b. Solve the equation $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = 12\frac{\log x}{x^2}$ **(8)**

Q.8 a. Obtain the series solution of $(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$ **(8)**

b. Show that $\beta(m, n) = \frac{\overline{|m}\,\overline{|n}}{\overline{|m+n}}$ **(8)**

Q.9 a. State and prove Rodrigue's formula. **(8)**

b. Prove that $J_{n+1}(n) + J_{n-1}(n) = \frac{2n}{x}J_n(n)$ **(8)**