Q.2 a. State and explain Gauss Law for electrostatic and derive its differential form.

Answer: 3.2 of Text Book I

b. Define potential and electric flux density. Also derive expression for a point charge in electric field. Point charges 1 mC and -2 mC are located at (3, 2, -1) and (-1, -1, 4), respectively. Calculate the electric force on a 10-nC charge located at (0, 3, 1) and the electric field intensity at that point.

Answer: 3.1 of Text Book I

$$\mathbf{F} = \sum_{k=1,2} \frac{QQ_k}{4\pi\varepsilon_o R^2}\mathbf{a}_R = \sum_{k=1,2} \frac{QQ_k(\mathbf{r} - \mathbf{r}_k)}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}_k|^3}$$

$$= \frac{Q}{4\pi\varepsilon_o}\left\{\frac{10^{-3}[(0, 3, 1) - (3, 2, -1)]}{|(0, 3, 1) - (3, 2, -1)|^3} - \frac{2.10^{-3}[(0, 3, 1) - (-1, -1, 4)]}{|(0, 3, 1) - (-1, -1, 4)|^3}\right\}$$

$$= \frac{10^{-3}\cdot 10\cdot 10^{-9}}{4\pi\cdot\frac{10^{-9}}{36\pi}}\left[\frac{(-3, 1, 2)}{(9 + 1 + 4)^{3/2}} - \frac{2(1, 4, -3)}{(1 + 16 + 9)^{3/2}}\right]$$

$$= 9\cdot 10^{-2}\left[\frac{(-3, 1, 2)}{14\sqrt{14}} + \frac{(-2, -8, 6)}{26\sqrt{26}}\right]$$

$$\mathbf{F} = -6.507\mathbf{a}_x - 3.817\mathbf{a}_y + 7.506\mathbf{a}_z \text{ mN}$$

At that point,

$$\mathbf{E} = \frac{\mathbf{F}}{Q}$$

$$= (-6.507, -3.817, 7.506)\cdot\frac{10^{-3}}{10\cdot 10^{-9}}$$

$$\mathbf{E} = -650.7\mathbf{a}_x - 381.7\mathbf{a}_y + 750.6\mathbf{a}_z \text{ kV/m}$$

Q.3 a. Derive the relationship between normal and tangential components at the boundary region in case of perfect dielectrics.

Answer: 6.2 of Text Book I

b. A wire of diameter 1 mm and conductivity $5\times10^{7}\,\text{S/m}$ has 10^{29} free electron s/m^3 when an electric field of 10 mV/m is applied. Determine

(i) The charge density of free electrons
(ii) The current density
(iii) The current in the wire
(iv) The drift velocity of the electrons. Take the electronic charge as $e = -1.6\times10^{-19}\,\text{C}$.

Answer:

(In this particular problem, convection and conduction currents are the same.)

(a) $\rho_v = ne = (10^{29})(-1.6 \times 10^{-19}) = -1.6 \times 10^{10}\ \text{C/m}^3$

(b) $J = \sigma E = (5 \times 10^7)(10 \times 10^{-3}) = 500\ \text{kA/m}^2$

(c) $I = JS = (5 \times 10^5)\left(\frac{\pi d^2}{4}\right) = \frac{5\pi}{4}\cdot 10^{-6}\cdot 10^5 = 0.393\ \text{A}$

(d) Since $J = \rho_v u$, $u = \frac{J}{\rho_v} = \frac{5 \times 10^5}{1.6 \times 10^{10}} = 3.125 \times 10^{-5}$ m/s.

Q.4 a. Derive the Poisson's and Laplace Equation. Represent Laplace Equation in all three co-ordinate systems.

Answer: 7.1 of Text Book I

Q.5 a. State and explain Ampere's Circuit law.

Answer: 8.2 of Text Book I

b. Explain the concept of Scalar and Vector Magnetic potential.

Answer: 8.6 of Text Book I

Q.6 b. Let us $\mu_1 = 4\times10^{-6}$ H/m in Region 1 :- z > 0, $\mu_2 = 7\times10^{-6}$ H/m in Region 2 :- z < 0. Let $K = .80\hat{o}_x$ A/m on the surface z = 0 and Field $B_1 = 2\hat{a}_x - 3\hat{a}_y + \hat{a}_z$ mT in Region 1. Find B_2 in Region 2.

Answer: Page Number 283 of Text Book I

Q.7 a. Express Maxwell's equation in both differential and integral form for a time varying field and explain it.

Answer: 10.3-10.4 of Text Book I

b. The electric field in free space is given by

$$\mathbf{E = 50 \cos (10^8 t + \beta\, x)\ \hat{a}_y\ V/m}$$

(i) Find the direction of wave propagation.
(ii) Calculate β and the time it takes to travel a distance of (λ/2).

Answer:

(a) From the positive sign in $(\omega t + \beta x)$, we infer that the wave is propagating along $-\mathbf{a}_x$. This will be confirmed in part (c) of this example.

(b) In free space, $u = c$.

$$\beta = \frac{\omega}{c} = \frac{10^8}{3 \times 10^8} = \frac{1}{3}$$

or

$$\beta = 0.3333 \text{ rad/m}$$

If T is the period of the wave, it takes T seconds to travel a distance λ at speed c. I travel a distance of $\lambda/2$ will take

$$t_1 = \frac{T}{2} = \frac{1}{2}\frac{2\pi}{\omega} = \frac{\pi}{10^8} = 31.42 \text{ ns}$$

Alternatively, because the wave is traveling at the speed of light c,

$$\frac{\lambda}{2} = ct_1 \qquad \text{or} \qquad t_1 = \frac{\lambda}{2c}$$

But

$$\lambda = \frac{2\pi}{\beta} = 6\pi$$

Hence,

$$t_1 = \frac{6\pi}{2(3 \times 10^8)} = 31.42 \text{ ns}$$

Q.8 With reference to ionosphere and skywave propagation, explain the following terms:

(i) The virtual height
(ii) The critical frequency
(iii) The maximum usable frequency
(iv) The skip distance

Answer: Page Number 241-242 of Text Book II

TEXTBOOK

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2. Electronic Communication Systems, George Kennedy and Bernard Davis, Fourth Edition (1999), Tata McGraw Hill Publishing Company Ltd.