ROLL NO. ________________

Code: AC65 **Subject: DISCRETE STRUCTURES**

AMIETE – CS

Time: 3 Hours **DECEMBER 2013** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. Every cyclic group is

(A) Abelian **(B)** Non Abelian
(C) Klein – 4 **(D)** None of these

b. The compound proposition $\neg(p \wedge q) \leftrightarrow [(\neg P) \vee (\neg q)]$ is

(A) Contradiction **(B)** Contingency
(C) Tautology **(D)** All of these

c. Addition theorem of probability for Mutually exclusive event A and B is

(A) $P(A \cup B) = P(A) + P(B)$ **(B)** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
(C) $P(A \cap B) = P(A) + P(B)$ **(D)** $P(A \cap B) = P(A) + P(B) - P(A \cap B)$

d. An Abelian group must satisfy

(A) Associative law **(B)** Commutative law
(C) Inverse law **(D)** Identity law

e. A box contains 3 red, 4 white and 5 green balls. 3 balls are selected at random. The probability of selecting one of each colour is

(A) $\frac{6}{220}$ **(B)** $\frac{1}{2}$
(C) $\frac{2}{3}$ **(D)** 0

f. A function $f : A \rightarrow B$ is invertible if and only if f is

(A) one - one **(B)** onto
(C) Both one – one and onto **(D)** None of these

g. If G is a finite group and H is a sub group of G, then the Lagranse's theorem is

(A) $O(H)/O(G)$ **(B)** $O(G)/O(H)$
(C) $O(H) = O(G)$ **(D)** All are incorrect

h. The Biconditional $p \leftrightarrow q$ will have a truth value of T whenever

(A) Both p and q have Identical truth values
(B) p is truth and q is false
(C) p is false and q is True
(D) p is false and q is False.

i. A Bijective function is

(A) one - one **(B)** onto
(C) Neither one – one or onto **(D)** Both one –one and onto

j. If A and B are Mutually exclusive events, then

(A) $A \cap B = \phi$ **(B)** $A \cap B \neq \phi$
(C) $A \cup B = \phi$ **(D)** $A \cup B \neq \phi$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. A Computer company requires 30 programmers to handle system programming Jobs and 40 programmers for application programming. If the company appoints 55 programmers to carry out these Jobs, how many of these perform Job of both types ? How many can handle only system programming jobs ? How many can handle application programming. **(8)**

b. Three students x, y, z write an examination. Their chances of passing are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. Find the probability that.
(i) all of them pass (ii) atleast one of them passes. **(8)**

Q.3 a. Check whether the compound proposition
$\neg(P \vee q) \vee [(\neg P) \wedge q] \vee P$ is a tautology or not. **(8)**

b. Show that $\neg \forall x \; [P(x) \rightarrow Q(x)]$ and $\exists x [P(x) \wedge \neg Q(x)]$ are logically equivalent. **(8)**

Q.4 a. State any Four Rules of Inference and explain. **(8)**

b. Show that the hypothesis" If you send me an e-mail message, then I will finish writing the program" "If you don't send me an e-mail message, then I will go to sleep early". And If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion " If I do not finish writing the program, then I will wake up feeling refreshed. **(8)**

Q.5 a. Prove the following statement by mathematical induction. If a set has n elements then its powerset has 2^n elements. **(8)**

b. Suppose U is a universal set and $A, B_1, B_2 ......B_n \subseteq U$ prove that
$A \cap (B_1 \cup B_2 \cup .... \cup B_n) = (A \cap B_1) \cup (A \cap B_2) \cup ....(A \cap B_n)$ **(8)**

Q.6 a. Define the following (i) Reflexive (ii) Symmetric (iii) Transitive properties of Relation with an example.
What is an equivalence relation? **(8)**

b. Define the partial order and POSET. If R is a Relation on the set A = {1,2,3,4,} defined by $R = \{(x, y) | x, y \in A \text{ and } x \text{ divides } y\}$ prove that (A, R) is a POSET and draw its Hasse diagram. **(8)**

Q.7 a. If ‘o’ is an operation on z defined by x o y = x + y +1 prove that (z, o) is an abelian group. **(8)**

b. Prove that any two left (or right) cosets of a sub group H of a group G are either disjoint or identical. **(8)**

Q.8 a. If f: $A \rightarrow B$ and g : $B \rightarrow$ C are Bijective functions then prove that $(gof)^{-1} = f^{-1} o g^{-1}$. **(8)**

b. If A = B = C = R, the set of all real numbers. Let $f : A \rightarrow B$;
$g : B \rightarrow C$ & f(a) = 2a+1 ; g(b) = b/3 find (i) f o g(-2) (ii) g o f(-1) (iii) verify that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$ **(8)**

Q.9 a. The parity – check Matrix for an encoding function $E : Z_2^3 \rightarrow Z_2^6$ is given by

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

(i) Determine the associated generator Matrix
(ii) Does this code correct all single errors is transmission. **(8)**

b. Define a Ring.
Find all integers k and m for which $(z, \oplus, \Theta)$ is a Ring under the binary operations
$x \oplus y = x + y - k, \; x \; \Theta \; y = x + y - m\,x\,y$ **(8)**