ROLL NO. ___________________

Code: AE56/AC56/AT56 Subject: ENGINEERING MATHEMA[TICS]

AMIETE – ET/CS/IT (NEW SCHEME)

Time: 3 Hours **JUNE 2012** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. Cauchy-Riemann evaluations in polar form are given by

(A) $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$, $\frac{\partial u}{\partial \theta} = -r\frac{\partial v}{\partial r}$

(B) $\frac{\partial u}{\partial r} = r\frac{\partial v}{\partial \theta}$, $\frac{\partial u}{\partial \theta} = -\frac{1}{r}\frac{\partial v}{\partial r}$

(C) $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$, $\frac{\partial u}{\partial \theta} = -\frac{1}{r}\frac{\partial v}{\partial r}$

(D) None of these

b. The value of the integral $\int_C (x+y)dx + x^2 y dy$ along $y = x^2$, having $(0,0)$, $(3,9)$ as end points, is

(A) 513 **(B)** 243

(C) $\frac{513}{4}$ **(D)** $\frac{513}{2}$

c. The value of the integral $\int_C \frac{1}{z}\cos z dz$, where C is the ellipse $9x^2 + 4y^2 = 1$, is

(A) 2π **(B)** πi

(C) $2\pi i$ **(D)** $\frac{1}{2}\pi i$

d. A unit vector normal to the surface $x^2 + 3y^2 + 2z^2 = 6$ at $P(2,0,1)$ is

(A) $(\hat{i} + \hat{j} + \hat{k})$ **(B)** $\frac{1}{\sqrt{2}}(\hat{i} + \hat{k})$

(C) $\frac{1}{\sqrt{2}}(\hat{i} - \hat{k})$ **(D)** $(\hat{i} - \hat{j} + \hat{k})$

e. If a force $\vec{F} = 2x^2y\hat{i} + 3xy\hat{j}$ displaces a particle in the xy-plane from (0,0) to (1, 4) along a curve $y = 4x^2$, then the work done is

(A) $\frac{5}{104}$ **(B)** $\frac{104}{5}$

(C) 104 **(D)** 52

f. The value of $\Delta \log f(x)$ is

(A) $\log\left[1 - \frac{\Delta f(x)}{f(x)}\right]$ **(B)** $\log[1 + \Delta f(x)]$

(C) $\log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$ **(D)** $\log[1 - \Delta f(x)]$

g. The value of $\left(\frac{\Delta^2}{E}\right)x^3$ is

(A) 1-6x **(B)** -6x

(C) 1+6x **(D)** 6x

h. The partial differential evaluation obtained from $y = f(x - at) + F(x + at)$ is

(A) $\frac{\partial^2 y}{\partial t^2} = \frac{1}{a^2}\frac{\partial^2 y}{\partial x^2}$ **(B)** $\frac{\partial^2 y}{\partial t^2} = a^2\frac{\partial^2 y}{\partial x^2}$

(C) $\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}$ **(D)** $\frac{\partial^2 y}{\partial t^2} = -\frac{1}{a^2}\frac{\partial^2 y}{\partial x^2}$

i. A husband and wife appear in an interview for two vacancies in the same post. The probability of husband's selection is $\frac{1}{7}$ and that of wife's selection is $\frac{1}{5}$. Then the probability that only one of them will be selected is

(A) $\frac{1}{7}$ **(B)** $\frac{1}{35}$

(C) $\frac{2}{35}$ **(D)** $\frac{2}{7}$

j. Ten percent of screws produced in a certain factory turn out to be defectiv Then the probability that in a sample of 10 screws chosen at random, exactly two will be defective is

(A) 1.937 **(B)** 0.1937
(C) 0.397 **(D)** 0.731

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Using the Cauchy-Riemann evaluations, show that $f(z)=z^3$ is analytic in the entire z-plane. **(8)**

b. Show that the transformation $\omega=\frac{3-z}{z-2}$ transforms the circle with centre $\left(\frac{5}{2},0\right)$ and radius $\frac{1}{2}$ in the z-plane into the imaginary axis in the ω-plane and the interior of the circle into the right half of the plane. **(8)**

Q.3 a. Evaluate the following integral using Cauchy integral formula $\int_C \frac{4-3z}{z(z-1)(z-2)}dz$ where C is the circle $|z|=\frac{3}{2}$. **(8)**

b. Evaluate the following integral using residue theorem $\int_C \frac{1+z}{z(2-z)}dz$ where C is the circle $|z|=1$. **(8)**

Q.4 a. Find the directional derivative of the scalar function $f(x,y,z)=x^2+xy+z^2$ at the point $A(1,-1,-1)$ in the direction of the line AB, where B has co-ordinates (3, 2, 1). **(8)**

b. Prove that $\nabla\times(\vec{F}\times\vec{G})=\vec{F}(\nabla\cdot\vec{G})-\vec{G}(\nabla\cdot\vec{F})+(\vec{G}\cdot\nabla)\vec{F}-(\vec{F}\cdot\nabla)\vec{G}$ **(8)**

Q.5 a. Use Green's theorem to evaluate $\int_C (x^2+xy)dx+(x^2+y^2)dy$ where C is the square formed by the lines $y=\pm1, x=\pm1$ **(8)**

b. Use Divergence theorem to evaluate $\iint_S \vec{F}\cdot\overrightarrow{dS}$, where $\vec{F}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$ and S is the surface bounding the region $x^2+y^2=4$, z=0 and z=3. **(8)**

ROLL NO. ___________________

Q.6 a. Use Newton-Gregory formula for interpolation to find the net premium at the age 25 from the table given below: **(8)**

Age	20	24	28	32
Annual Net Premium	0.01427	0.01581	0.01772	0.01996

b. Find $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}$ and $\frac{3}{8}$ rule, dividing the range (0, 1) into 6 equal parts. Hence obtain the approximate value of π in each case. **(8)**

Q.7 a. Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$ **(8)**

b. Find a complete integral of $q = (z + px)^2$ using Charpit's method. **(8)**

Q.8 a. An urn A contains 2 white and 4 black balls. Another urn B contains 5 white and 7 black balls. A ball is transferred from the urn A to the urn B, then a ball is drawn from urn B. Find the probability that it is white. **(8)**

b. A six-faced dice is so biased that, when thrown, it is twice as likely to show an even number than an odd number. If it is thrown twice, what is the probability that the sum of two numbers thrown is odd. **(8)**

Q.9 a. If the probability that an individual suffer a bad reaction from a certain injection is 0.001, determine the probability that out of 2000 individuals
(i) exactly 3
(ii) more than 2 individuals
(iii) None
(iv) more than one individual will suffer a bad reaction. **(8)**

b. A manufacturer of envelopes knows that the weight of the envelopes is normally distributed with mean 1.9 gm and variance 0.01 gm. Find how many envelopes weighing
(i) 2 gm or more
(ii) 2.1 gm or more, can be expected in a given pocket of 1000 envelopes.
[Given: if t is the normal variable, then $\phi(0 \le t \le 1) = 0.3413$ and $\phi(0 \le t \le 2) = 0.4772$] **(8)**