ROLL NO. ___________________

Code: AE35/AC35/AT35 Subject: MATHEMATICS-II

AMIETE – ET/CS/IT (OLD SCHEME)

Time: 3 Hours **JUNE 2012** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. Eliminating a and b from the $(x-a)^2+(y-b)^2+z^2=c^2$ the partial differential equation is

(A) $z^2(p-q+1)=c^2$ **(B)** $z^2(p^2+q^2+1)=c^2$

(C) $z^2(p^2+q^2)=c^2$ **(D)** $z^2(p-q)=c^2$

b. Solution of $xp+yq=3z$ is

(A) $z=x^3f\left(\frac{x}{y}\right)$ **(B)** $x=y^3f\left(\frac{x}{y}\right)$

(C) $y=x^3f\left(\frac{x}{y}\right)$ **(D)** $x=z^3f\left(\frac{z}{y}\right)$

c. Residue of $\frac{\cos z}{z}$ at z = 0 is

(A) 1 **(B)** –1

(C) 2 **(D)** 0

d. The function $w=\bar{z}$ is not differentiable if the value of z is equal

(A) –1 **(B)** 1

(C) 2 **(D)** 0

e. If f(z) is analytic in a simply connected domain D and c is any simple closed path then the value of $\int_c f(z)dz$ is given by

(A) 1 **(B)** 2

(C) 0 **(D)** 3

f. The angle between the tangent to the curve $\overline{R} = t^2\bar{i} + 2t\bar{j} - t^3\overline{k}$ at the point t=1and t = –1

(A) $\cos^{-1}(9/17)$ **(B)** $\cos(9/17)$
(C) $\sin^{-1}(9/17)$ **(D)** $\sin(9/17)$

g. A unit normal to $x^2 + y^2 + z^2 = 5$

(A) $\frac{1}{\sqrt{5}}(i + j + k)$ **(B)** $\frac{1}{\sqrt{5}}(i + j - k)$
(C) $\frac{1}{\sqrt{5}}(j + 2k)$ **(D)** $\frac{1}{\sqrt{5}}(i - j + k)$

h. If $u = x^2 + y_2 + z_2, \vec{V} = xi + yj + zk,$ then value of $\nabla.\left(u\vec{V}\right)$ is equal to

(A) 2u **(B)** –u
(C) 3u **(D)** 5u

i. $\int_c \overline{F}.d\overline{R}$ is independent of the path joining any two points if it is

(A) irrotational field **(B)** solenoidal field
(C) rotational field **(D)** vector field

j. In a Poisson distribution if 2P(x=1)=P(x=2) then the variance is equal to

The value of k is

(A) 4 **(B)** 0
(C) –1 **(D)** 2

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. A tightly stretched string with fixed end points x=0 and x=1 is initially in a position given by $y = y_0 \sin^3(\pi x / \ell)$. If it is released from rest from this position, find the displacement y(x,t) **(8)**

b. An infinitely long plate uniform is bounded by two parallel edge and an end at right angles to them. The breadth is π; this end is maintained at a temperature u_0 at all points and other edge at zero temperature. Determine the temperature at any point of the plate in the steady state. **(8)**

Q.3 a. X is a continuous random variable with probability density function given by $f(x) = \begin{cases} kx, 0 \le x < 2 \\ 2k, 2 \le x \le 4 \\ -kx + 6k, 4 \le x \le 6 \end{cases}$

find (i) K (ii) Expression of X **(8)**

b. If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals more than two will get a bad reaction. **(8)**

Q.4 a. A transmission line 100 km long is initially under steady-state conditions with potential 1300 volts at the sending end (x=0) and 1200 volts at the receiving end (x=1000). The terminal end of the line is suddenly grounded, but the potential at the source is kept at 1300 volts. Assuming the inductance and leakage to be negligible, find the potential v(x,t). **(8)**

b. If r is the distance of a point (x,y,x) from the origin, prove that $\text{curl}\left(\overline{K} \times \text{grad}\frac{1}{r}\right) + \text{grad}\left(\overline{K}.\text{grad}\frac{1}{r}\right) = 0, \overline{K}$ is the unit vector in the direction of OZ. **(8)**

Q.5 a. Evaluate $\int_c (x+y)dx - x^2dy + (y+z)dz$, where $C : x^2 = 4y, z = x, 0 \le x \le 2$ **(8)**

b. Determine $\overline{F} = (y^2 \cos x + z^3)\overline{I} + (2y \sin x - 4)\overline{J} + (3xz^2 + 2)\overline{K}$ is a conservative field? If so find the scalar potential. Also compute the work done in moving the particle from (1,1,–1) to (π/2,–1,2) **(8)**

Q.6 a. Evaluate $\int (a^2x^2 + b^2y^2 + c^2z^2)^{-1/2} dS$, where S is the surface of Ellipsoid $ax^2+by^2+cz^2=1$ **(8)**

b. Use the Divergence theorem to evaluate $\iint_S (\overline{v}\overline{n})dA$, where $\overline{v} = x^2z\hat{i} + y\hat{j} - xz^2\hat{k}$, and S is the boundary of the region bounded by the paraboloid $z=x^2+y^2$ and the plane z=4y. **(8)**

Q.7 a. Show that the function $v(x,y)=e^x \sin y$ is harmonic. Find its conjugate harmonic function u(x,y) and the corresponding analytic function f(z) **(8)**

b. Find the bilinear transformation which maps the points z, 1, i , –1 onto then points w=i,0,–i, find the image of $|z| < 1$. **(8)**

Q.8 a. Evaluate the integral $\int_c (x + y^2 - ixy)dz$, where $C : z = z(t) = \begin{cases} t - 2i & 1 \le t \le 2 \\ 2 - i(4 - t), & 2 \le t \le 3 \end{cases}$ **(8)**

b. Show that the function f(z)=Ln[z/z-1] is analytic in the region $|z| > 1$, obtain the Laurent series expansion about z=0 valid in the region. **(8)**

Q.9 a. Using complex integration, compute $\int_0^{2\pi} \frac{\cos 2\theta}{1 - 2a\cos} d\theta \frac{2\pi a^2}{1 - a^2} (a^2 < 1)$ **(8)**

b. Find the image in the w-plane of the disk $|z - 1| \le 1$, under the mapping=1/z. **(8)**