ROLL NO. _____

Code: DE55 / DC55 Subject: ENGINEERING MATHEMAT

DipIETE – ET/CS

Time: 3 Hours **DECEMBER 2012** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. The value of the $\lim_{x\to 0}\dfrac{\tan x-\sin x}{x^3}$ is

(A) 0 **(B)** $\dfrac{1}{2}$

(C) 2 **(D)** None of these

b. If $f(x)=\dfrac{\tan x+\sec x-1}{\tan x-\sec x+1}$, then $f'(x)$ is equal to

(A) sec x(tan x–sec x) **(B)** sec x(sec x–tan x)
(C) sec x(sec x+tan x) **(D)** None of these

c. If $z=4+i\sqrt{7}$, then value of $z^3-4z^2-9z+91$ is equal to

(A) 0 **(B)** 1
(C) –1 **(D)** 2

d. If $-\pi<\arg(z)<-\dfrac{\pi}{2}$, then $\arg\bar{z}-\arg(-\bar{z})$ is

(A) π **(B)** $-\pi$

(C) $\dfrac{\pi}{2}$ **(D)** $-\dfrac{\pi}{4}$

e. If $\vec{a}+\vec{b}+\vec{c}=0$, and $|\vec{a}|=3,\ |\vec{b}|=5,\ |\vec{c}|=7$, then the angle between $\vec{a}$ and $\vec{b}$ is equal to

(A) 0 **(B)** 30°
(C) 60° **(D)** 90°

f. If $I=\int_0^{\pi/2}\cos^n x\sin^n x\,dx=\lambda\int_0^{\pi/2}\sin^n x\,dx$ then λ equal to

(A) 2^{-n+1} **(B)** 2^{n-1}
(C) 2^{-n} **(D)** 2^{-1}

g. The solution of $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=0$ is

(A) $y=(C_1+C_2x)e^{-2x}$ **(B)** $y=(C_1+C_2x)e^{2x}$
(C) $y=(C_1+C_2)e^{-2x}$ **(D)** $y=(C_1+C_2)e^{2x}$

h. The period of the function of $|\sin x|$ is equal to

(A) 2π **(B)** π
(C) 3π **(D)** 4π

i. L{t sinh at} is equal to

(A) $\frac{2as}{(s^2-a^2)^2}$ **(B)** $\frac{as}{s^2+a^2}$
(C) $\frac{as}{s^2-a^2}$ **(D)** $\frac{2as}{(s^2+a^2)^2}$

j. $L^{-1}\left\{\frac{4s-3}{s^2+9}\right\}$ is equal

(A) 3 cos t+4 sin 3t **(B)** 3cos 4t–4sin 3t
(C) 4 cos 3t+3 sin t **(D)** 4 cos 3t–sin 3t

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Evaluate $\lim_{x\to 0}\frac{(1+x)^{\frac{1}{x}}-e}{x}$ in the form of indeterminate. **(8)**

b. Use Taylor's theorem, expand e^x in power of (x+3). **(8)**

Q.3 a. Show that the length of one loop of the curve $3ay^2=x(x-a)^2$ is $\frac{4a}{\sqrt{3}}$ **(8)**

b. Find the length of the arc from $\theta=0$ to $\theta=2\pi$ of the curve $x=a(\cos\theta+\theta\sin\theta)$, $y=a(\sin\theta-\theta\cos\theta)$ **(8)**

Q.4 a. Prove that, $(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n=2^{n+1}.\cos^n\frac{\theta}{2}.\cos\frac{n\theta}{2}$, where n is an integer **(8)**

b. The impedance of each of the following circuit at frequency of 50 cycles. Calculate (i) A resistance of 20 ohms in series with an inductance of 0.1 H. (ii) A resistance of 50 ohms in series with a capacitance of $40\mu F$. If the terminal voltage is 230 volts, find the value of current in each case and phase of each current relative to applied voltage. **(8)**

Q.5 a. A rigid body is rotating at the rate 2.5 radians per second about an axis AB, where A and B are the points (1,–2,1) and (3,–4,2). Find the velocity of the point P at (5,–1,–1) of the body. **(8)**

b. A force of 78 gram act at the point (2,3,5), the direction cosines of its line of acting being as (2,2,1). Find the magnitude of its moment about the line joining the origin to the point (12,3,4). **(8)**

Q.6 a. Solve the differential equation $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=x\sin x$ **(8)**

b. A condenser of capacity C is discharge through the inductance L and a resistance R in series and the charge q at any time t satisfies the equation

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{c}=0$$

Given that L=0.25 henery, R=250 ohms, C=2×10^{-6} farad and that when t=0, the charge q is 0.002 coulombs and the current $\frac{dq}{dt}$=0, obtain the value of q in terms of t. **(8)**

Q.7 a. Find the fourier series representing f(x)= x,0<x< 2π **(8)**

b. Obtain half range sine for e^x in 0<x<1 **(8)**

Q.8 a. Find the Laplace transform of sin 2t.cos 3t+cos(at+5). **(8)**

b. Find the Laplace transform of t^2 sin 2t. **(8)**

Q.9 a. Find $L^{-1}\left[\frac{3s+7}{s^2-2s-3}\right]$ **(8)**

b. Using Laplace transform, solve the equation

$$\frac{d^2x}{dt^2}+9x=\cos 2t;\ \text{if } x'(0)=1,\ x\left(\frac{\pi}{2}\right)=-1$$ **(8)**