

b. Find the equation of the tangent to the curve $x^2 + 2y = 8$ which is perpendicular to the line x - 2y + 1 = 0.

Ans. The given curve is $x^2 + 2y = 8$ ______(i) Differentiating w.r.t. 'x' we get $2x + 2 \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -x$ and the given line is x - 2y + 1 = 0Differentiating w.r.t. 'x' we get $1 - 2 \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{1}{2}$ Since the tangent is perpendicular to the line therefore, (slope of tangent) (slope of line) = -1 $\Rightarrow (-x) \left(\frac{1}{2}\right) = -1 \Rightarrow x = 2$ Now, we have to find y co-ordinate when x = 2 on putting x = 2 in (i), we get $(2)^2 + 2y = 8 \Rightarrow 2y = 8 - 4 = 4 \Rightarrow y = 2$ \therefore Equation of tangent to (i) at the point (2, 2) is,

$$y-2 = -2(x-2)$$

$$\Rightarrow y-2 = -2x + 4$$

$$\Rightarrow 2x + y = 6$$

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Ans.
Let
$$I = \int e^{-x} .\cos x dx$$
, then
 I II
 $= e^{-x} \sin x - \int -e^{-x} .\sin x dx$
 $= e^{-x} \sin x + \int e^{-x} .\sin x dx$
I II
 $= e^{-x} \sin x + e^{-x} (-\cos x) - \int (-e^{-x})(-\cos x) dx$
 $= e^{-x} \sin x - e^{-x} \cos x - \int e^{-x} \cos x dx$
or $I = e^{-x} \sin x - e^{-x} \cos x - I$
or $2I = e^{-x} (\sin x - \cos x)$
or $I = \frac{e^{-x}}{2} (\sin x - \cos x)$

b. Evaluate
$$\int_{1}^{2} \frac{1}{x(1+x^2)} dx$$

Ans.

Let
$$\frac{1}{x(1+x^2)} = \frac{A}{x} + \frac{Bx+C}{1+x^2}$$
, then
 $1 = A(1+x^2) + (B x + C) x$ ______(i)
Putting $x = 0$ is (i), we get $A = 1$. Comparing the coefficients of x^2 and x
we get,
 $A+B=0$ and $C=0 \Rightarrow B=-1$ and $C=0$ [$\because A=1$]
 $\therefore \frac{1}{x(1+x^2)} = \frac{1}{x} - \frac{x}{1+x^2}$
So, $\int_{1}^{2} \frac{1}{x(1+x^2)} dx = \int_{1}^{2} \frac{1}{x} dx - \frac{1}{2} \int_{1}^{2} \frac{2x}{1+x^2} dx$
 $= [logx]_{1}^{2} - \frac{1}{2} [log(1+x^2)]_{1}^{2}$
 $= (log2 - log1) - \frac{1}{2} [log5 - log2]$
 $= log2 - \frac{1}{2} log5 + \frac{1}{2} log5$

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StudentBounts.com a. Find the matrix A satisfying the equation $\begin{vmatrix} 2 & 1 \\ 3 & 3 \end{vmatrix} A \begin{vmatrix} 5 & 3 \\ 3 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ **O.4**

Ans.

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Let $|B| = \begin{vmatrix} 2 & 1 \\ 3 & 3 \end{vmatrix}$ and $|C| = \begin{vmatrix} 5 & 3 \\ 3 & 2 \end{vmatrix}$, then $|\mathbf{B}| = \begin{vmatrix} 2 & 1 \\ 3 & 3 \end{vmatrix} = 6 - 3 = 3 \neq 0$ and $|\mathbf{C}| = \begin{vmatrix} 5 & 3 \\ 3 & 2 \end{vmatrix} = 10 - 9 = 1 \neq 0$ So, B and C are invertible matrices. The given matrix equation is BAC = Ion pre- multiplying by B⁻¹ and post multiplying by C⁻¹, we get, \Rightarrow B⁻¹ (BAC) C⁻¹ = B⁻¹I C⁻¹ \Rightarrow (B⁻¹B) A (C C⁻¹) = B⁻¹.C⁻¹ $\Rightarrow I A I = B^{-1}.C^{-1}$ $\Rightarrow A = B^{-1}.C^{-1}$ Matrix of co- factors of B = $\begin{bmatrix} 3 & -3 \\ -1 & 2 \end{bmatrix}$ (i) Adjoint B = $\begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix}$ $B^{-1} = \frac{1}{|B|}$ Adj $B = \frac{1}{3}\begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix}$ (ii) Matrix of co-factors of C = $\begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$ Adjoint C = $\begin{vmatrix} 2 & -3 \\ -3 & 5 \end{vmatrix}$ $C^{-1} = \frac{1}{|C|}$ Adj $C = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$ (iii) on putting the value of B⁻¹ from (ii) and C⁻¹ from (iii) in (i), we get $\therefore A = B^{-1} \cdot C^{-1} = \frac{1}{3} \begin{bmatrix} 3 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$ $= \frac{1}{3} \begin{bmatrix} 6+3 & -9+5\\ -6-6 & 9+10 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 9 & -14\\ -12 & 19 \end{bmatrix}$ $=\begin{bmatrix} 3 & -14/3 \\ -4 & 19/3 \end{bmatrix}$

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Ans. Let
$$\Delta = \begin{vmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} = -2$$

 $\Delta x = \begin{vmatrix} 9 & -1 & 3 \\ 6 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix} = -2$
 $\Delta y = \begin{vmatrix} 2 & 9 & 3 \\ 1 & 6 & 1 \\ 1 & 2 & 1 \end{vmatrix} = -4$
 $\Delta z = \begin{vmatrix} 2 & -1 & 9 \\ 1 & 1 & 6 \\ 1 & -1 & 2 \end{vmatrix} = -4$
Now, $x = \frac{\Delta x}{\Delta} = \frac{-2}{-2} = 1$
 $y = \frac{\Delta y}{\Delta} = \frac{-4}{-2} = 2$
 $z = \frac{-6}{-2} = 3$
 $x = 1, y = 2, z = 3$

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Q.5 a. Solve $x^2 dy + y(x + y) dx = 0$

Ans. The given equation is
$$\frac{dy}{dx} + \frac{y(x+y)}{x^2} = 0$$
 _____ (i)

which is a homogeneous differential equation of first order, putting y = v xand $\frac{dy}{dx} = v + x \frac{dv}{dx}$, equation (i) yields $v + x \frac{dv}{dx} + \frac{vx(x + vx)}{x^2} = 0$ or $x \frac{dv}{dx} + v^2 + 2v = 0$

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or
$$\frac{dv}{v^2 + 2v} + \frac{dx}{x} = 0$$

 $\frac{1}{2}\left(\frac{1}{v} + \frac{1}{v+2}\right)dv + \frac{dx}{x} = 0$
on integrating the required solution is

$$\frac{1}{2} \left[logv - log(v+2) \right] + logx = logc$$
$$log\left(x \sqrt{\frac{v}{v+2}} \right) = logc$$
or
$$x \sqrt{\frac{y/x}{y/x+2}} = c, \text{ as } v = \frac{y}{x}$$
or
$$x \sqrt{\frac{y}{y+2x}} = c$$

$$x^2y = a(y + 2x)$$
 where $c^2 = a$

b. Solve
$$(1 + y^2)dx + xdy = tan^{-1} ydy$$

Ans.

$$(1+y^2)dx = (\tan^{-1}y - x) dy$$

$$\Rightarrow (1+y^2)\frac{dx}{dy} = \tan^{-1}y - x$$

$$\Rightarrow \frac{dx}{dy} = \frac{\tan^{-1}y}{1+y^2} - \frac{x}{1+y^2}$$

$$\Rightarrow \frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$$

$$\Rightarrow I. F. = e^{\int \frac{1dx}{1+y^2}}$$

$$= e^{\tan^{-1}y}$$

So solution of the given differential equation is,

$$= x e^{\tan^{-1} y} = \int \frac{\tan^{-1} y}{1+y^2} \cdot e^{\tan^{-1} y} \cdot dy + c$$

$$\Rightarrow x e^{\tan^{-1} y} = \int \frac{\tan^{-1} y \cdot e^{\tan^{-1} y} \cdot dy}{1 + y^2} + c$$

Put $tan^{-1}y = t$

$$\Rightarrow \frac{1}{1+y^2} dy = dt$$

$$\Rightarrow x e^{\tan^{-1}y} = \int te^t dt + c$$

$$\Rightarrow x e^{\tan^{-1}y} = t e^t - \int e^t dt + c$$

$$\Rightarrow x e^{\tan^{-1}y} = t e^t - e^t + c$$

$$\Rightarrow x e^{\tan^{-1}y} = e^t (t-1) + c$$

$$\Rightarrow x e^{\tan^{-1}y} = e^{\tan^{-1}y} (\tan^{-1}y + 1) + c$$

$$\Rightarrow (\tan^{-1}y - 1) + c e^{\tan^{-1}y} = x$$

Q.6 a. Prove that the coefficient of x^r in the expansion of $(1-4x)^{-1/2}$ is $\frac{(2r)!}{(r!)^2}$

We know that the general term of the expansion Ans.

$$(1 + x)^n$$
 is $T_{r+1} = \frac{n(n-1)(n-2)...(n+1-r).x^r}{r!}$

Putting
$$n = -\frac{1}{2}$$
 and $-4x$ for x, we get

$$T_{r+1} = \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\dots\left[-\left(\frac{2r-1}{2}\right)\right]}{r!} \quad (-1)^{r} \cdot (2^{2})^{r} \cdot x^{r}$$

$$= (-1)^{r} (-1)^{r} \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot \dots \cdot (2r-1)}{2^{r} \times r!} \cdot 2^{2r} \cdot x^{r}$$

$$=\frac{1.3.5.7....(2r-1)}{r!}2^{r}.x^{r}$$

Multiplying and dividing by 2.4.6...2r, we have

$$= \frac{1.2.3.4....(2r-1)^{2r}}{[2.4.6.8...2r]r!} 2^{r} \cdot x^{r} = \frac{2r!}{2^{r}(1.2.3.r)r!} 2^{r} \cdot x^{r}$$

$$= \frac{2r!}{r!r!}x^{r} = \frac{2r!}{(r!)^{2}}x^{r}$$

Hence, the coefficient of x^r is $\frac{2r!}{(r!)^2}$ <u>Proved</u>

b. Find three number in A.P. whose sum is 21 and their product is 315.

Ans. Let three numbers is A.P. be, a – d, a, a + d _____ (i) Their sum a - d + a + a + d = 21 \therefore 3a = 21 \Rightarrow a = 7 Now product of 3 numbers = (a - d) a (a + d) $(a - d) \cdot a (a + d) = 315$ Putting the value of a, we get (7-d) 7 (7+d) = 315 $\Rightarrow (49 - d^2) = \frac{315}{7} = 45$ \Rightarrow d² = 49-45 = 4 $\Rightarrow d = + 2$ <u>Case I-</u> When a = 7, d = 2 putting in (i) : Numbers are 7-2, 7, 7+2 i.e. 5, 7, 9 Case II When a = 7, d = -2 putting in (i) : Numbers 7+2, 7, 7-2 is 9, 7, 5 \therefore Required three numbers are 5, 7, 9, or 9, 7, 5

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Given that $A + B + C = \pi = 180^{\circ}$ then $2 A + 2B + 2C = 360^{\circ}$ or $2A + 2B = 360^{\circ} - 2C$ Taking tangent of both sides $\tan (2A + 2B) = \tan (360^{\circ} - 2C)$ $\frac{\tan 2A + \tan 2B}{\tan 2A + \tan 2B} = -\tan 2C$ $1 - \tan 2A \cdot \tan 2B$ **Cross-multiplying** \therefore tan2A + tan2B = -tan2C + tan2A.tan2B.tan2C Transpasing, $\tan 2A + \tan 2B + \tan 2C = \tan 2A \cdot \tan 2B \cdot \tan 2C$ Hence Proved.

b. Prove that, $\sin 10^{\circ} . \sin 50^{\circ} . \sin 60^{\circ} . \sin 70^{\circ} = \frac{\sqrt{3}}{16}$

Ans.

We have,

LHS =
$$\sin 10^{\circ} .\sin 50^{\circ} .\sqrt{3} / 2.\sin 70^{\circ}$$

= $\frac{\sqrt{3}}{2} \cdot \frac{1}{2} [2\sin 50^{\circ} .\sin 10^{\circ}] \sin 70^{\circ}$
= $\frac{\sqrt{3}}{4} [\cos(50^{\circ} - 10^{\circ}) - \cos(50^{\circ} + 10^{\circ})] \sin 70^{\circ}$
= $\frac{\sqrt{3}}{4} \frac{1}{2} [2\sin 70^{\circ} .\cos 40^{\circ} - 2\sin 70^{\circ} \cos 60^{\circ}]$
= $\frac{\sqrt{3}}{8} [\sin(70^{\circ} + 40^{\circ}) + \sin(70^{\circ} - 40^{\circ}) - 2\sin 70^{\circ} \frac{1}{2}]$
= $\frac{\sqrt{3}}{8} [\sin(110^{\circ} + \sin 30^{\circ} - \sin 70^{\circ}]$
= $\frac{\sqrt{3}}{8} [\sin(180^{\circ} - 70^{\circ}) + \frac{1}{2} - \sin 70^{\circ}]$
= $\frac{\sqrt{3}}{8} [\sin 70^{\circ} + \frac{1}{2} - \sin 70^{\circ}]$
= $\frac{\sqrt{3}}{8} [\sin 70^{\circ} + \frac{1}{2} - \sin 70^{\circ}]$
= $\frac{\sqrt{3}}{8} [\sin 70^{\circ} + \frac{1}{2} - \sin 70^{\circ}]$
Hence Proved

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- DECEME 2012 DECEME 2012 DECEME Composition DECEME DECEME 2012 a. Find the equation of the straight lines through the point (2,-1) and **Q.8** making an angle of 45° with the line 6x+5y-1 = 0.
- Equation of the straight line through (2,-1) and having slope m is Ans. y + 1 = m(x + 2)____(i)

(i) Makes an angle 45° with the line 6x + 5y - 1 = 0 (ii) Slope of (ii) in - $\frac{6}{5}$

$$\tan 45^{\circ} = \pm \frac{m + \frac{6}{5}}{1 - \frac{6m}{5}}$$
 or $1 = \pm \frac{5m + 6}{5 - 6m}$

taking + ve sign,

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$$5-6m = + (5m+6) \Rightarrow m = -\frac{1}{11}$$

Taking -ve sign.

 $5-6m = -(5m + 6) \implies m = 11$

Putting the values of m in (i) we get $y + 1 = -\frac{1}{11}(x - 2)$ and y + 1 = 11 (x - 2)

or x + 11y + 9 = 0 and 11x - y - 23 = 0 are the required eqn. of the & line.

Ans. x + 11y + 9 & 11x - y - 23 = 0

b. Find the equation of lines parallel to 3x - 4y - 5 = 0 at a unit distance from it.

Equation of any line parallel to 3x - 4y - 5 = 0 is 3x - 4y + k = 0Ans. Put x = 0 in 3x - 4y - 5 = 0, we get $y = \frac{-5}{4}$

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DECEMP 2012 Therefore, $\left(0, \frac{-5}{4}\right)$ is a point on the line 3x-4y-5 = 0 since the distance between the two lines is one unit therefore, the length of the perpendicular from $\left(0, \frac{-5}{4}\right)$ to 3x - 4y + k = 0 is 1. Hence $\frac{3 \times 0 - 4 \times \frac{-5}{4} + k}{\sqrt{3^2 + (-4)^2}} = \pm 1$ $\frac{5+k}{5} = \pm 1 \implies 5+k = 5 \text{ and } 5+k = -5$ \Rightarrow k = 0 and k = -10 Putting the value of k = 0 and k = -10 in (i) 3x - 4y = 0 and 3x - 4y - 10 = 0 are the required equation of the straight lines. $\begin{array}{c}
3x - 4y = 0 \\
3x - 4y - 10 = 0
\end{array}$

Q.9 a. Find the equation of the circle which passes through the points (3,-2),(-2,0) and having its centre on the line 2x-y-3=0

Ans. Let the equation of the circle be $x^{2} + y^{2} + 2gx + 2fy + c = 0$ (i) as (i) passes through (3, -2) $\therefore 9 + 4 + 6g - 4f + c = 0$ or 13 + 6g - 4f + c = 0or 6g - 4f + c = -13____(ii) also (i) passes through (-2, 0) $\therefore 4 + 0 - 4g - 0 + c = 0$ or $4g - c = \tilde{4}$ (iii) the centre (-g, -f) of (i) lies on 2x - y = 3-2g + f = 3 _____ (iv)

adding (ii) and (iii), we get

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10g - 4f = -9 _____ (v)

Solving (iv) and (v), we get,

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$$g = \frac{3}{2}, f =$$

Putting g in (iii), we get, C = 2

Substituting these values of g, f and c in (i) we get,

$$x^2 + y^2 + 3x + 12y + 2 = 0$$

which is the required equation of circle.

 $x^2 + y^2 + 3x + 12y + 2 = 0$

b. Find the vertex, focus directrix, latus-rectum and axis of parabola $3x^2 + 12x - 8y = 0$

The given equation is $3x^2 + 12x - 8y = 0$ Ans.

$$3(x^{2} + 4x - \frac{8}{3}y) = 0$$

or $x^{2} + 4x + 4 - 4 - \frac{8}{3}y = 0$
or $(x + 2)^{2} = \frac{8}{3}y + 4$
or $(x + 2)^{2} = \frac{8}{3}(y + \frac{3}{2})$ _____ (i)

Put x + 2 = X and y + $\frac{3}{2}$ = Y (shifting the on origin)

$$\therefore$$
 (i) reduces to $X^2 = \frac{8}{3}Y$ or $X^2 = 4(\frac{2}{3})Y$

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Comparing with	h standard equation we get the f	following table:	htty.com
Equation	$X^{2} = \frac{8}{3}Y = 4(\frac{2}{3}Y)$	$(x+2)^2 = \frac{8}{3}(y+\frac{3}{2})$	
Vertex	$(0,0) \Longrightarrow X = 0, Y = 0$	$\left(-2,\frac{3}{2}\right)[\text{As } X = x + 2 \implies x = -2$ $Y = y + \frac{3}{2} \implies y = -\frac{3}{2}]$	
Focus	$(0, \frac{2}{3}) \Longrightarrow X=0, Y=\frac{2}{3}$	$(-2, \frac{-5}{6}) [As X = x + 2 \implies x = -2$ $Y = y + \frac{3}{2} = \frac{2}{3} \implies y = -\frac{5}{6}]$	
Directrix	$Y + \frac{2}{3} = 0$	$Y + \frac{13}{6} = 0 [As y + \frac{3}{2} + \frac{2}{3} = 0$ $\Rightarrow y + \frac{13}{6} = 0$	
L.R.	$\frac{8}{3}$	$\frac{8}{3}$	
Axis	X = 0	x + 2 = 0	1

TEXTBOOKS

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