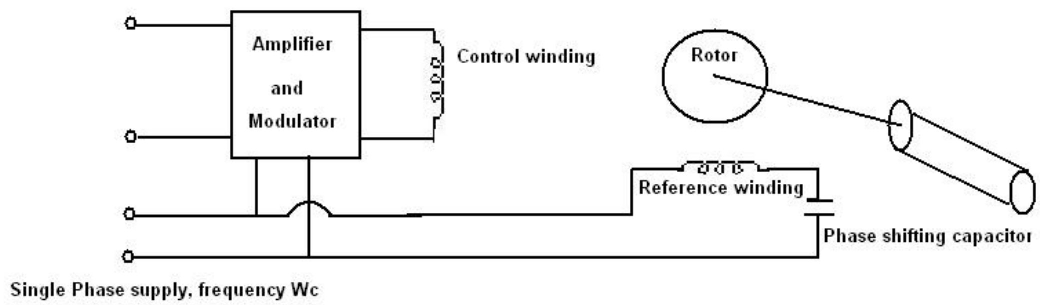
**Q2 (a)** Describe a two phase a.c. servomotor and derive its transfer function.

**Ans 2(a)**

Working of AC Servomotor:

The symbolic representation of an AC servomotor as a control system is shown in figure. The reference winding is excited by a constant voltage source with a frequency n the range 50 to 100Hz. By using frequency of 400Hz or higher, the system can be made less susceptible to low frequency noise. Due to this feature, ac drives are extensively used in aircraft and missile control system in which the noise and disturbance often create problems.

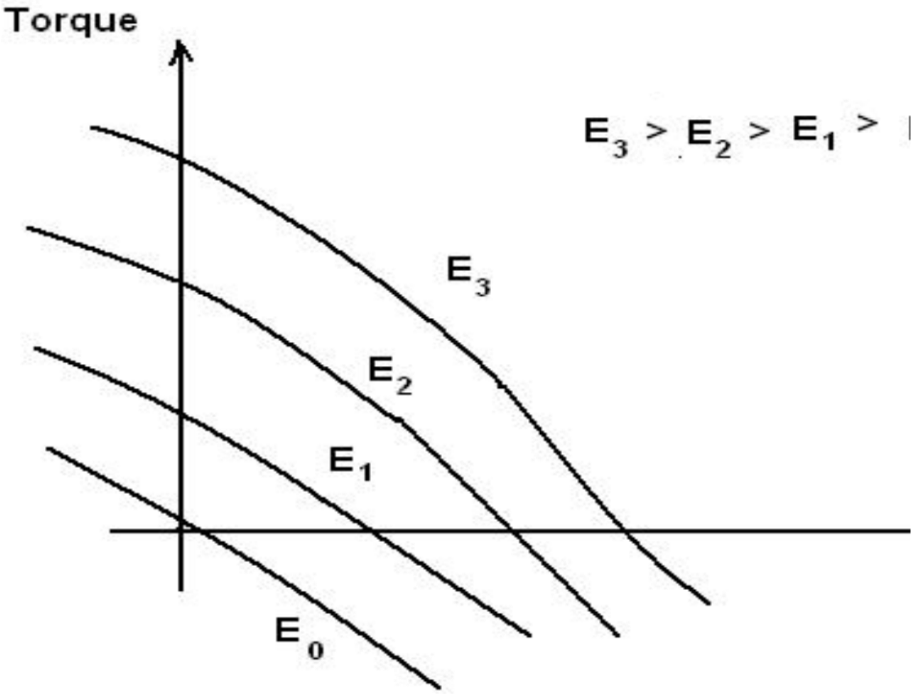



Symbolic Representation of AC servo motor

The control winding is excited by the modulated control signal and this voltage is of variable magnitude and polarity. The control signal of the servo loop (or the system) dictates the magnitude and polarity of this voltage.

The control phase voltage is supplied from a servo amplifier and it has a variable magnitude and polarity (+ or – $90^o$ phase angle w.r.to the reference phase). The direction of rotation of the motor reverses as the polarity of the control phase signal changes sign.

It can be proved that using symmetrical components that the starting torque of a servomotor under unbalanced operation is proportional to E, the rms value of the sinusoidal control voltage e(t). A family of torque –speed characteristics curves with variable rms control voltage is shown in figure. All these curves have negative slope.




Servo Characteristics

Note that the curve for zero control voltage goes though the origin and the motor develops a decelerating torque.

From the torque speed characteristics shown above we can write

$$T = -\ k_n \frac{d\theta}{dt} + k_c e_c \qquad (1)$$

Where
T= Torque
$k_n$= a positive constant = -ve of the slope of the torque-speed curve
$k_c$ = a positive constant = torque per unit control voltage at zero speed
$\theta$ = angular displacement

Further, for motor we have

$$T = J\frac{d^2\theta}{dt^2} + f\frac{d\theta}{dt} \quad (2)$$

Where J= moment of inertia of motor and load reffered to motor shaft
f= viscous friction coefficient of the motor and load to the motor shaft

form eqs.(1) and (2) we have

$$J\frac{d^2\theta}{dt} + (f + k_n)\frac{d\theta}{dt} = k_c e_c \quad (3)$$

Taking the laplace transform on both sides, putting initial conditions zero and simplifying we get

$$\frac{\theta(s)}{E_c(s)} = \frac{k_c}{Js^2 + (f + k_n)s} = \frac{k_m}{s(\tau_m s + 1)} \quad (4)$$

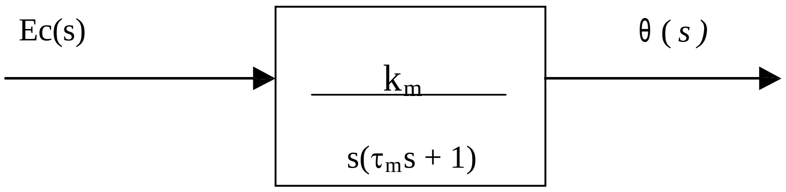
Where

$k_m = k_c/(f + k_n)$ = motor gain constant

If the moment of inertia J is small. Then $\tau_m$ is small and for the frequency range of relevance to ac servometer $|\tau_m s| << 1$, then from eq (4) we can write the transfer function as

$$\frac{\theta(s)}{E_c(s)} = \frac{k_m}{s} \quad (5)$$

It means that ac servometer works as an integrator. Following figure gives the simplified block diagram of an ac servometer.



**b.** Write the dynamic equation in respect of the mechanical system given in Fig.4.

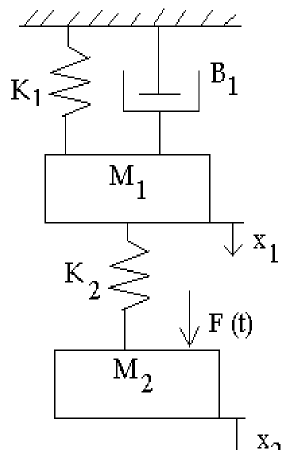
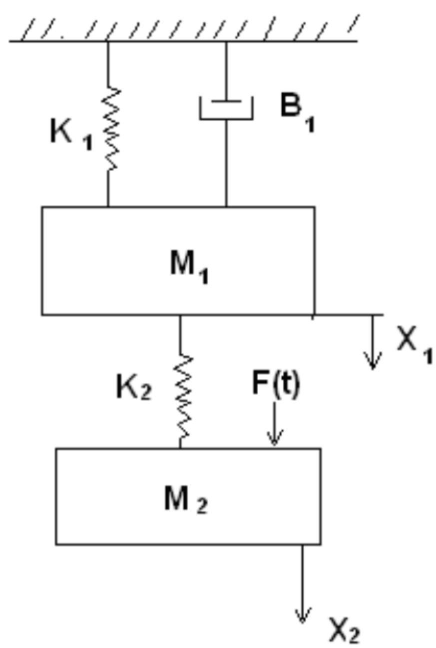



Fig.4

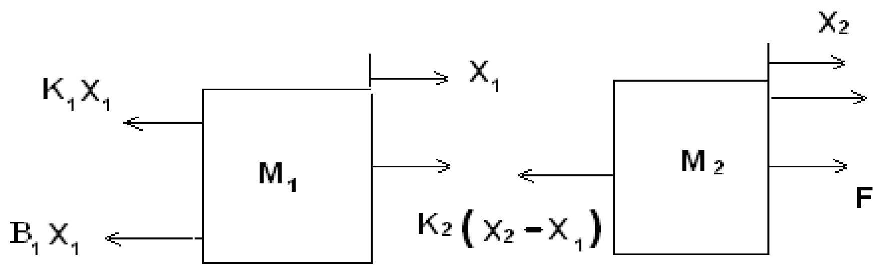
**Ans2(b)**




| | |
|---|---|
| $K_1$ $K_2$ | *Spring constants* |
| $M_1$ $M_2$ | *inertial constants* |
| $X_1$ $X_2$ | *displacements* |
| F(t) | *force* |
| $B_1$ | *viscous friction damping coefficient* |

From figure, a force F(t) is applied to mass $M_2$.

Free body diagrams for these two masses are




From these, the following differential equations describing the dynamics of the system.

$F(t) - k_2(X_2 \; X_1)$ :
$= M \quad 2 \quad X$

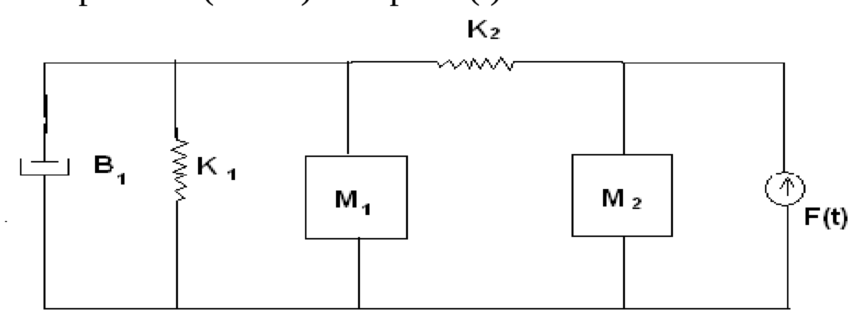
$k_2(X_2 \; X_1) - k_1 X_1 \; - \; B_1 X_1 = M_1 \; X_1$

From above we can write down

$M_2 \ddot{X}_2 + k_2 (X_2 \; X_1) = F(t)$

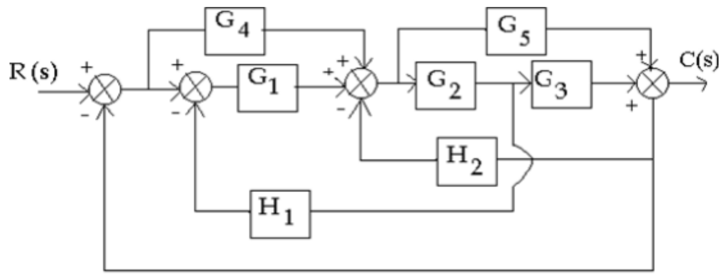
$M_1 \; X + k_1 \; \dot{X}_1 + B_1 X_1 - k_2 (X_2 - X_1) = 0;$

These two are simultaneous second order linear differential equations. Manipulation of these equations results in a single differential equations relating the response X(or X1) to input F(t).

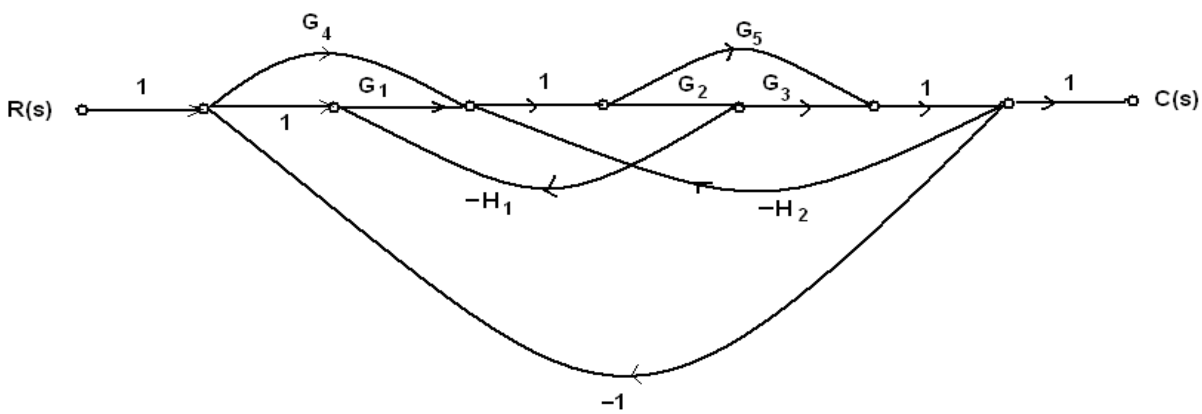



Mechanical network for the system

**Q3.** Determine the transfer function C(s)/R(s) for the block diagram shown in Fig.3 by first drawing its signal flow graph and then using the Mason's gain formula.




**Ans 3.** SIGNAL FLOW GRAPH OF THE BLOCK DIAGRAM:




Mason's gain formula
Overall system gain is given by

$$T = \frac{1}{\Delta}\Sigma_K \; P_k \, \Delta_K$$

$P_k$ –gain of $k^{th}$ forward path

$\Delta$ = det of the graph
= 1 – sum of loop gains of all individual loops + (sum of gain products of all possible combinations of two non-touching loops) – (sum of gain products of all possible combinations of three non touching loops)+ ……

$\Delta_K$ = the value of $\Delta$ for that part of the graph not touching the kth forward path.

There are four path gains

$P_1 = G_1G_2G_3$
$P_2 = G_4G_2G_3$
$P_3 = G_1G_5$
$P_4 = G_4G_5$

Individual loop gains are

$P_{11} = -G_1G_2H_1$
$P_{21} = -G_5H_2$
$P_{31} = -G_2G_3H_2$
$P_{41} = -G_4G_5$

$P_{51} = -G_1G_2G_3$

$P_{61} = -G_4G_2G_3$

$P_{71} = -G_1G_5$

There are no non touching loops

$\Delta = 1-(-G_1G_2H_1-G_5H_2-G_2G_3H_2-G_4G_5-G_1G_2G_3-G_4G_2G_3-G_1G_5)$

$$T = \frac{G_1G_5P_{71}\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4}{\Delta}$$

$$= \frac{G_1G_2G_3 + G_4G_2G_3 + G_1G_5 + G_4G_5}{1 + G_1G_2H_1 + G_5H_2 + G_2G_3H_2 + G_4G_5 + G_1G_2G_3 + G_4G_2G_3 + G_1G_5}$$

**Q4 a.** What are the main features of stepper motor which are responsible for its wide spread use?

**Ans 4 (a)**

1. The rotor rotates through definite known angle.
2. Control of the stepper motor is simple because neither a position (or) a speed neither sensor, nor the output response to follow the input command.
3. Since the nature of command is in the form of pulses, stepping motors are compatible with modern digital equivalent.
4. Positional error is non- cumulative in stepper motor.
5. As the motor speed is proportional to the rate of command pulses, it can be used for speed control.

**b.** A servo system is represented by the signal flow graph shown in Fig.5. The nominal values of the parameters are K1 =1, K2 =5 and K3 =5.Determine the overall transfer function $Y(s)/R(s)$ and its sensitivity to changes in $K_1$ under steady dc conditions, i.e., s = 0.

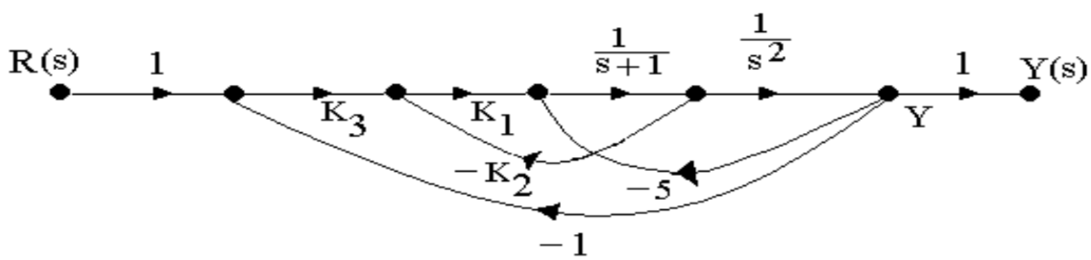



Fig. 5

**Ans.**

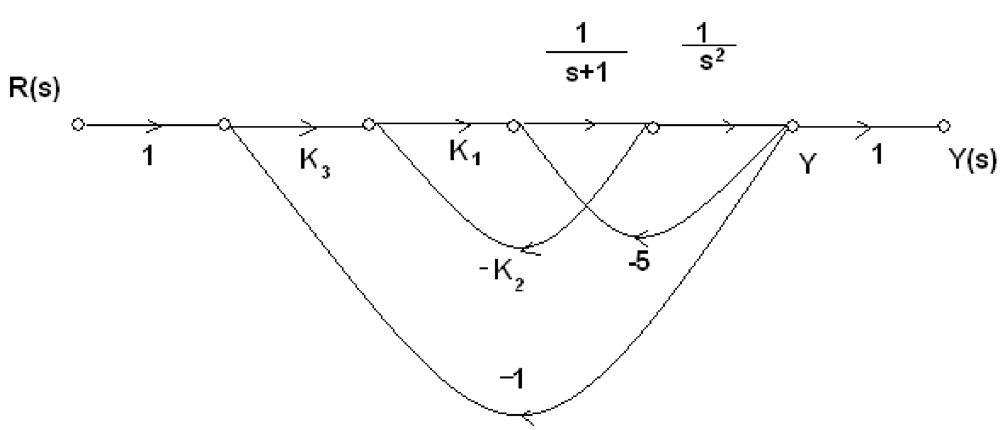



Fig: Signal flow graph

$\frac{Y(s)}{R(s)} = M(s) = P_1\Delta_1 / \Delta$

One forward path with gain $P1 = \frac{k_3 k_4}{s^2(s+1)}$

Three feedback loops with gains

$P_{11} = \frac{5}{s^2(s+1)}$

$P_{31} = -\frac{k_3 k_1}{s^2(s+1)}$

$P_{21} = -\frac{k_1 k_2}{s^2(s+1)}$

$\Delta_1 = 1$ since all loops are touching

$$M(s) = \frac{5k_1}{s^2(s+1+5k_1)+5k_1+5}$$

$$S_{k_1}^M = \frac{\partial M}{\partial k_1} * \frac{k_1}{M} = \frac{s^2(s+1+5k_1)+5-5k_1 s^2}{s^2(s+1+5k_1)+5k_1+5}$$

$$\left|S_{k_1}^M(jw)\right|_{w=0} = \frac{5}{5k_1+5} = 0.5$$

**Q.5 a.** For the system shown in the block diagram of Fig.7 determine the values of gain K1 and velocity feedback constant K2 so that the maximum overshoot with a unit step input is 0.25 and the time to reach the first peak is 0.8 sec. Thus obtain the rise time and settling time for 5% tolerance band.

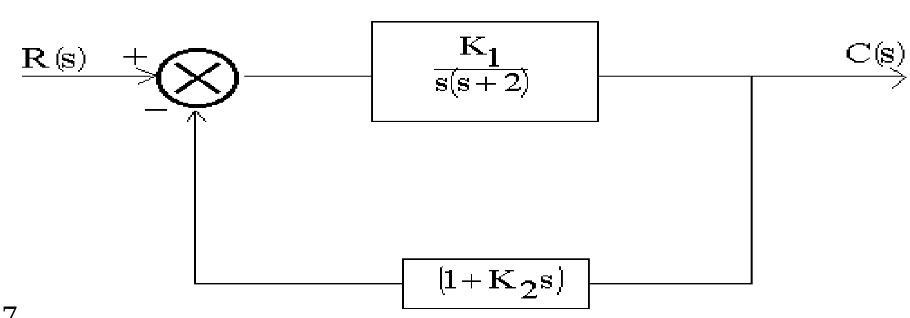



Fig.7

**Ans5.a** $M(s) = \dfrac{k_1/(s(s+2)}{1+k_1(1+k_2s)/(s(s+2)}$

$= \dfrac{k_1}{s^2 + 2s + k_1 + k_1 k_2 s}$

$= \dfrac{k_1}{s^2 + 2 + (k_1 k_2)s + k_1}$

peak over shoot = 0.25

$= e^{-\xi} \pi(1-\xi 2)^{1/2}$

= 0.403

$t_p = 0.8$ sec= $\dfrac{\pi}{w_n(1-\xi^2)^{1/2}}$

$w_n = 4.29$ rad/sec

$k_1 = w_n^2 = 18.4$

$(2+ k_1 k_2) = 2\xi w_n = 3.457$

$k_1 k_2 = 1.457$ $k_2=0.079$

$t_r = \dfrac{\pi - \tan^{-1}(1-\xi^2)^{1/2}/\xi}{w_n(1-\xi^2)^{1/2}} = 0.505$ sec

$t_s = 3/\xi w_n = 1.735$ sec

**b.** Obtain the unit-impulse response of a unity feedback control system whose open loop transfer function is $G(s)= \dfrac{2s+1}{s^2}$

**Ans.** $G(s)= \dfrac{2s+1}{S^2}$

$M(s) = \dfrac{G(s)}{1+G(s)} = \dfrac{2s+1}{s^2 + 2s +1}$

$$C(s) = \frac{2s + 1}{s^2 + 2s + 1}$$

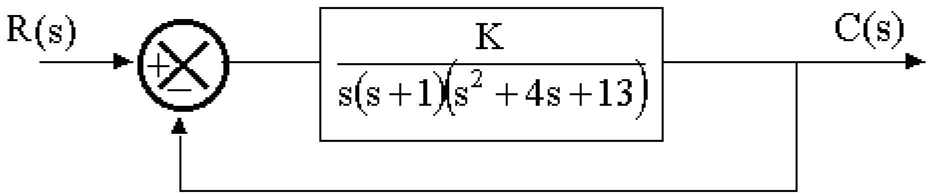
$$= \frac{2s + 1}{(s+1)^2} = \frac{2s}{(s+1)^2} + \frac{1}{(s+1)^2}$$

$$= 2[(s+1)/(s+1)^2 - 1/(s+2)^2] + 1/(s+1)^2$$

$$C(t) = 2[e^{-t} - te^{-t}] + te^{-t}$$

$$= (2-t)\, e^{-t}$$

**Q6** Sketch the root loci for the system shown in Fig.6 below.




**Ans.**

$$G(s) = \frac{k}{s(s+1)(s^2+4s+13)}$$

Poles $s=0, -1, -2\pm j3$

$n=4,\ m=0$

angle of asymptotes = $\pm 180(2q+1) / (n-m) = \pm 45, \pm 135$

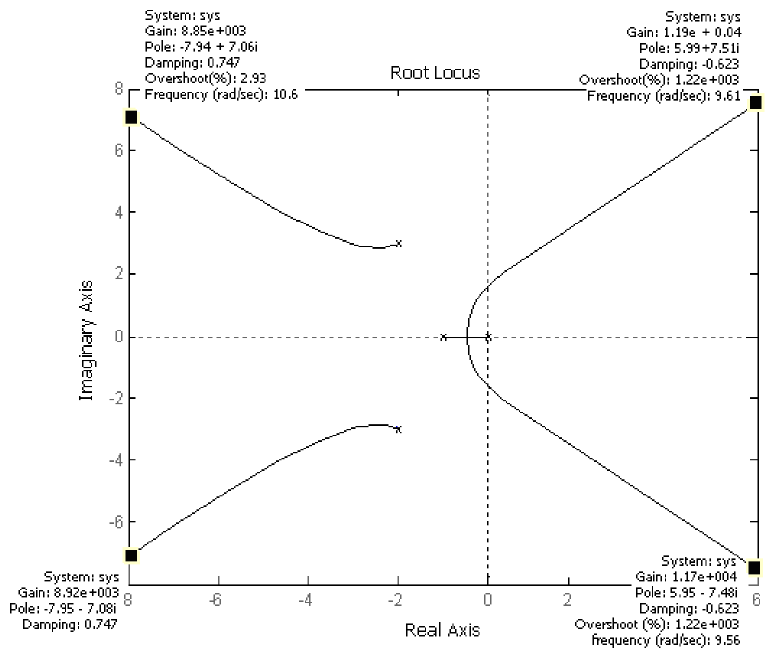
centroid = $\frac{-2-2-1}{4} = -1.25$

$$M(s) = \frac{4}{(s^2+s)(s^2+4s+13)+k}$$

The Characteristic equation is given as $s^4+4s^3+13s^2+s^3+4s^2+13s+k = 0$

$k = -(s^4+5s^3+17s^2+13s)$
$dk/ds = -(4s^3+15s^2+34s+13) = 0$
so, s= -0.4664, -1.6418±j2.067
so, the break away point is -0.4664

Angle of departures
For complex poles
$Ø1= 90, Ø2 = 180 – \tan^{-1}(3/1), Ø3 = 180 – \tan^{-1}(3/2)$
So, at A
$= 180^o – (Ø1+ Ø2+ Ø3)$
$= -142.12^o$




**Q7 a.** State and explain the Nyquist stability criterion.

**Ans 7(a):**

If the contour of the open-loop transfer function G(s) H(s) corresponding to the Nyquist contour in the s-plane encircles the point (-1+j0) in the counter clockwise direction as many times as the number of right half s-plane poles of G(s)H(s),the closed-loop system is stable.

In the commonly occurring case of the open-loop stable system, the closed-loop system is stable if the counter of G(s)H(s) does not encircle (-1 +j0) point, i.e., the net encirclement is zero.

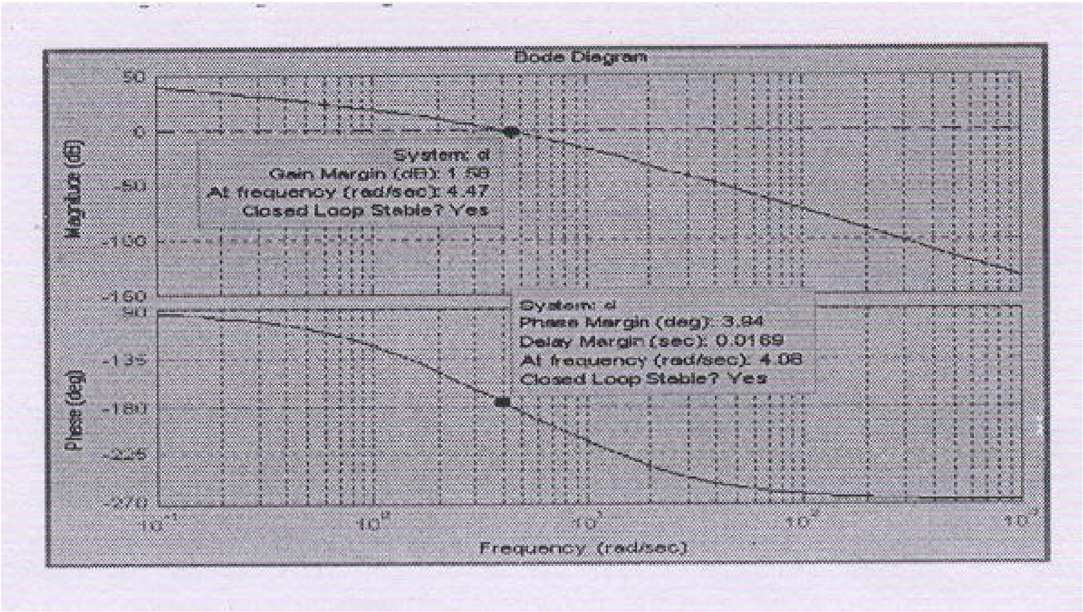
**(b)** The open loop transfer function of a unit feedback control system is

$$G(s)H(s) = \frac{10s}{s(1+0.5s)(1+0.1s)}$$

Sketch the bode plot of the system and determine the following:

i. Gain margin
ii. Phase margin

**Ans:**




From the bode plot of G(jw)H(jw) ; we find that wg = 4.08 rad/sec:
PM =3.9 deg; GM = 1.6 dB

**Q8(a)** Consider the control system shown in Fig 7 in which a proportional compensator is employed. A specification on the control system is that the steady-state error must be less than two per cent for constant inputs. Find Kc that satisfies this specification.

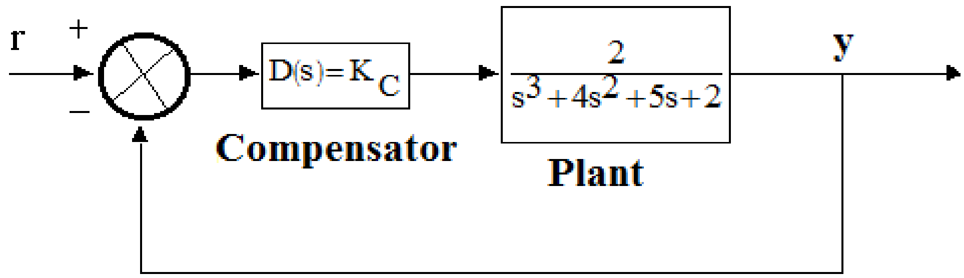



**Fig.7**

**Ans 8(a)**

| | | | |
|---|---|---|---|
| $s^3$ | 1 | 5 | 0 |
| $s^2$ | 4 | $2+2K_C$ | 0 |
| $s^1$ | $(18-2K_C)/4$ | 0 | 0 |
| $s^0$ | $2+2K_C$ | 0 | |
| | | | |

The system is stable for $K_c < 9$

$K_p = \lim_{s \to 0} D(s)G(s) =$ $e_{ss} = 1/(1+Kc)$; $e_{ss} = 0.1$ (10 %) is the minimum possible value for steady state error. Therefore $e_{ss}$ less than 2 % is not possible with proportional compensator.

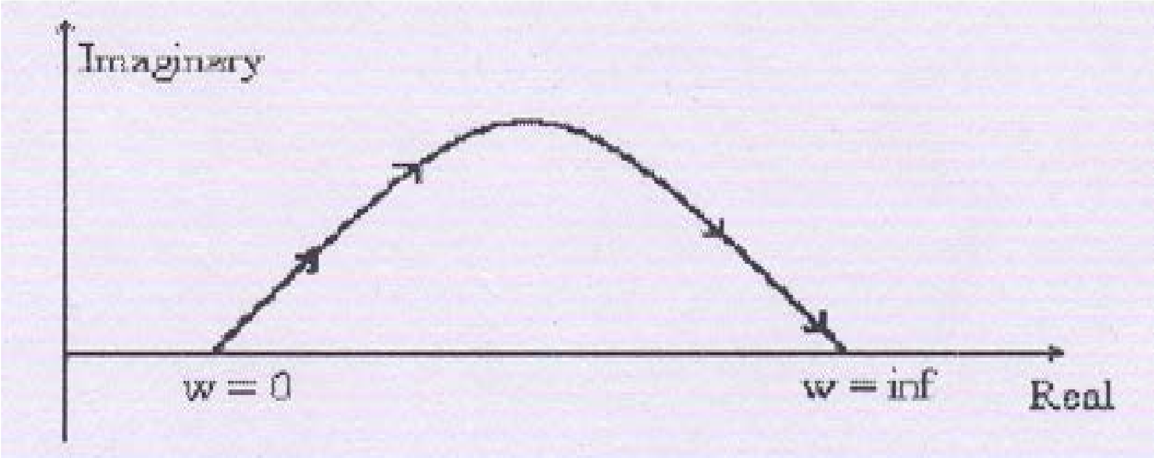
**b.** Discuss phase lead compensator.

**Ans 8(b)** Where $T_1= R_1C_1$ and $T_2=R_2/(R_1+R_2)T_1$

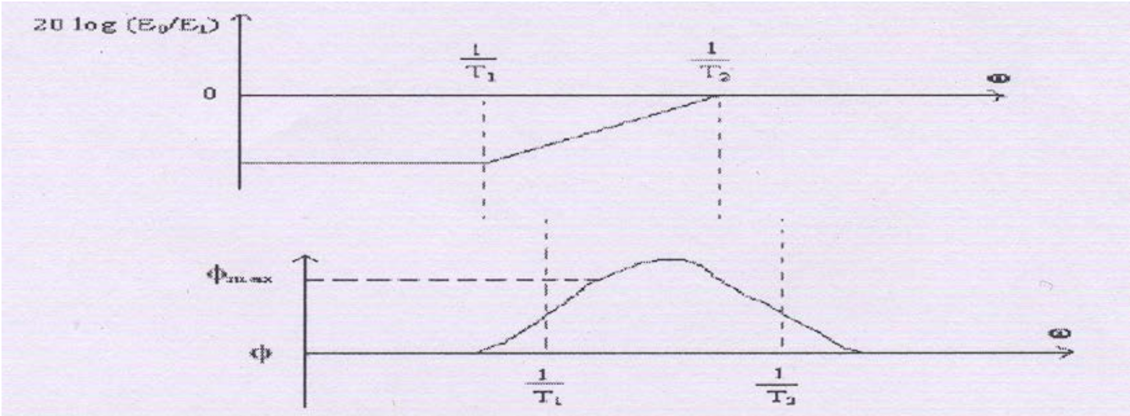
Obviously $T_1>T_2$, For getting the frequency response of the network, but s = jw i.e.,

$$\left|\frac{E_o}{E_i}\right| = \frac{T_2}{T_1}\sqrt{\frac{1+w^2T_1^2}{1+w^2T_2^2}}$$

And phase $\phi = \tan^{-1}wT_1 - \tan^{-1}wT_2$
Let us consider the polar plot for this transfer function as shown in figure below. We can observe that at low frequencies, the magnitude is reduced being $T_2/T_1$ at w=0.




Next, let us consider the bode plot the transfer function as shown in figures below.




We observe here that phase φ is always positive. From magnitude plot we observe that transfer function that zero db magnitude at w= $1/T_2$.
We can put dφ /dw = 0 to get maximum value of φ which occurs at some frequency $w_m$

i.e, $w_m = 1 / (\sqrt{T_1 / T_2})$

$\varphi_{max} = \tan^{-1}[T_1/T_2]^{1/2} - \tan^{-1}[T_2/T_1]^{1/2}$

In this network we have an attenuation of T2/T1 therefore; we can use an amplification of T1/T2 to nullify the effect of attenuation in the phase network.

**Q9 b.** By using Cayley-Hamilton technique find f (A) = $A^{10}$ for A= $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$.

**Ans 9b.**

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

State Transition matrix φ (t) is given by $e^{At} = I + At + \frac{1}{2!}A^2t^2 + \frac{1}{3!}A^3t^3 + \ldots\ldots$

Put 'A' collect terms

$$e^{At} = \begin{bmatrix} 1+t+0.5t\verb|^|\mathbf{2}+\ldots.. & 0 \\ t+t\verb|^|2 & 1+t+0.5t\verb|^|2 \end{bmatrix}$$

$$= \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}$$

$$\varphi(t) = \begin{bmatrix} e\verb|^| t & 0 \\ te\verb|^| t & e\verb|^| t \end{bmatrix}$$

Time response of the system is

$$X(t) = \varphi(t)[X_o + \int_0^t \varphi L - \tau)Budz]$$

Now with u=1,

$$\phi(-z)Bu = \begin{bmatrix} e\verb|^|\text{-}z & 0 \\ -\tau e\verb|^|\text{-}z & e\verb|^|\text{-}z \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} e^{-z} \\ e^{-z}(1-\tau \end{bmatrix}$$

$$\int_0^t \varphi(-\tau)B\upsilon dz = \begin{bmatrix} \int_0^t e^{-z}\,dz \\ \int_0^t e^{-z}(1-\tau dz \end{bmatrix}$$

$$= \begin{bmatrix} 1-e^{-t} \\ -te^{-t} \end{bmatrix}$$

$$X = \begin{bmatrix} e^{t} & 0 \\ te^{t} & e^{t} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1-e^{-t} \\ [\ ]te^{-t} \end{bmatrix}$$

$$= \begin{bmatrix} 2e^{-t}-1 \\ 2e^{--t} \end{bmatrix}$$

**<u>TextBook</u>**

**Control Systems Engineering, I.J. Nagrath and M. Gopal, 5th Edition (2007 New Age International Pvt. Ltd.)**