Q2 (a).

Determine power and energy of the following signals

(i) $x(t) = e^{j\omega_0 t} \quad -\infty < t < \infty$

(ii) **x(t) =A cos** (ωt)

(iii) x(n)=u(n)

Solution:

(i) $x(t) = e^{jw_0 t} \quad -\infty < t < \infty$

$$I = \int_{-T}^{T} |x(t)|^2 dt$$

$$|x(t)| = |e^{jw_0 t}| = 1$$

$$= \int_{-T}^{T} |x(t)|^2 dt = \int_{-T}^{T} 1 dt = 2T$$

$$E = \underset{T\to\infty}{lt} [I] = \infty$$

$$P = \underset{T\to\infty}{lt} [\frac{I}{2T}] = 1$$

Power is finite , it is a power signal

(ii) x(t) =A cos(wt)

$$I = \int_T |x(t)|^2 dt$$

$$= \int_T \cos^2 wt dt$$

$$= \int_T \frac{(1+\cos 2wt)}{2} dt = \int_T \frac{1}{2} dt + \frac{1}{2}\int_T \cos 2wt dt$$

$$= T/2$$

$$E = \underset{T\to\infty}{lt} [I] = \infty$$

$$P = \underset{T\to\infty}{lt} [\frac{I}{T}] = \frac{1}{2}$$

Power is finite , it is a power signal

(iii) x(n) = u(n)

$$I = \sum_{n=-N}^{N} |x(n)|^2$$

$$I = \sum_{n=-N}^{N} |1|^2 = [2N+1]$$

$$E = \underset{N\to\infty}{lt}\,[I] = \infty$$

$$P = \underset{N\to\infty}{lt}\,\frac{[I]}{2N+1} = 1$$

Power is finite , it is a power signal

Q2 (b) **Given x(t) as shown in Fig.3**

Sketch the following

(i) x(-2t)
(ii) x(t-3)
(iii) x(t)u(t)
(iv) x(-t+1)

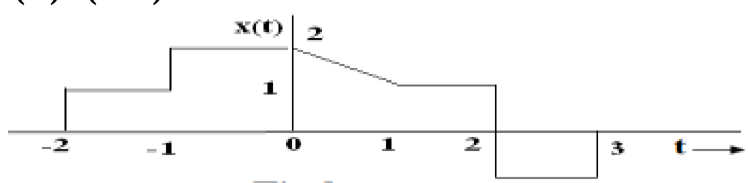
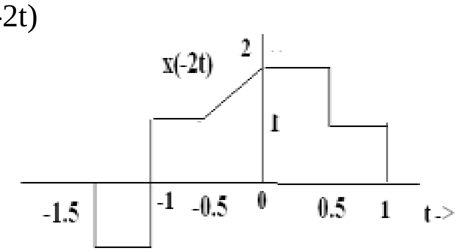


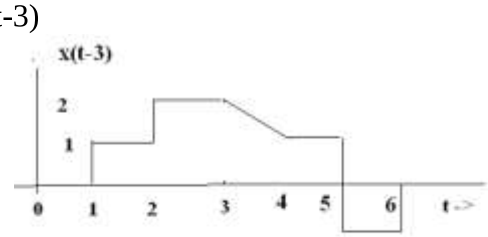
Fig.3

Solution:

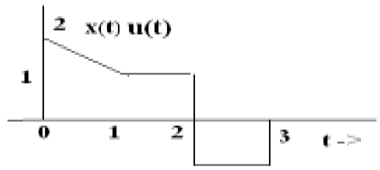
i) x(-2t)



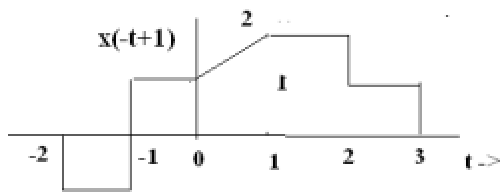
ii) x(t-3)



iii) x(t)u(t)



iv) x(-t+1)



Q3 (a) **Determine the Fourier's Series representation for signal;**

i) $x(n) = 1 + \sin(\frac{1}{12}\pi n + \pi/3)$

ii) $x(n) = 1 + 2\cos\left[\frac{\pi}{8}n + \pi/6\right]$

Solution:

(i) $x(n) = 1 + \sin(\frac{1}{12}\pi n + \pi/3)$

x(n) is periodic with period N = 24

Using Euler's formula

$$x(n) = 1 + \frac{1}{2j}\left[e^{j\left[\frac{\pi}{12}n+(\pi/3)\right]} + e^{-j\left[\frac{\pi}{12}n+(\pi/3)\right]}\right]$$

$$= \frac{1}{2j}e^{-j\frac{\pi}{3}}\underbrace{e^{-j(\pi/12)n}}_{k=-1} + \underbrace{e^{-j(0)n}}_{k=0} + \frac{1}{2j}\underbrace{e^{j\frac{\pi}{3}}e^{j[\pi/12]n}}_{k=1}$$

Comparing with DTFS equation

$$X(k) = \begin{cases} -\frac{1}{2j}e^{-jj(\pi/3)} & k = -1 \\ 1 & k = 0 \\ \frac{1}{2j}e^{jj(\pi/3)} & k = 1 \\ 0 & -11 \le k \le 12 \quad k \ne 0, \pm 1 \end{cases}$$

	(ii) $x(n) = 1 + 2\cos\left[\frac{\pi}{8}n + \pi/6\right]$ x(n) is periodic with period N = 16 Using Euler's formula $x(n) = 1 + \left[e^{j\left[\frac{\pi}{8}n+(\pi/6)\right]} + e^{-j\left[\frac{\pi}{8}n+(\pi/6)\right]}\right]$ $= e^{-j\frac{\pi}{8}}\underbrace{e^{-j(\pi/6)n}}_{k=-1} + \underbrace{e^{-j(0)n}}_{k=0} + \underbrace{e^{j\frac{\pi}{8}}e^{j[\pi/6]n}}_{k=1}$ Comparing with DTFS equation $X(k) = \begin{cases} e^{-jj(\pi/6)} & k=-1 \\ 1 & k=0 \\ e^{jj(\pi/6)} & k=1 \\ 0 & -7 \le k \le 8 \;\; k \ne 0, \pm 1 \end{cases}$
Q3 (b)	**State and prove the following Fourier series properties of continuous periodic signals.** **(i) Frequency shift property** **(ii) Scaling property** **Solution:** (i) **Frequency shift Property** Table 3.1. Page No. 206 of Text Book - I (ii**) Scaling property** If x(t) is a periodic signal then f(t)=x(at) is also periodic. If x(t) has fundamental period T then f(t) has fundamental period T/a $If \quad x(t) \leftrightarrow X[k] \;\; then$ $x(at) \leftrightarrow X[k]$ I.e Fourier series coefficients of x(t) and x(at) are identical Proof: since f9t) has fundamental period T/a

	$F[k]=\frac{a}{T}\int_{\frac{T}{a}} f(t)e^{-jkw_0t}dt$ $F[k]=\frac{a}{T}\int_{\frac{T}{a}} x(at)e^{-jkw_0t}dt$ Put p=at then t=p/a and dt=(1/a)dp $F[k]=\frac{a}{T}\int_{T} x(p)e^{-jkw_0p}dp\frac{1}{a}$ $F[k]=\frac{1}{T}\int_{T} x(p)e^{-jkw_0p}dp$ $\therefore F[k]=X[k]$ i.e $\begin{aligned}&If \quad x(t)\leftrightarrow X[k] \;\; then\\ &\quad\;\; x(at)\leftrightarrow X[k]\end{aligned}$
Q4 (a)	**State and prove Parseval's energy theorem for continuous aperiodic signals.** **Solution:** **Statement:** The energy may be found from the time signal x(t) or its spectrum $\lvert X(j\omega)\rvert$ i.e $E=\int_{-\infty}^{\infty}\lvert x(t)\rvert^2\,dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X(j\omega)\rvert^2 d\omega$ Proof: Energy of a signal x(t) is given by $E=\int_{-\infty}^{\infty}\lvert x(t)\rvert^2\,dt=\int_{-\infty}^{\infty}x(t)x^*(t)dt$ ---------(1) The Fourier transform and its inverse is $X(j\omega)=\int_{-\infty}^{\infty}x(t)e^{-j\omega t}dt$ $x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(j\omega)e^{j\omega t}d\omega$ Taking conjugate for the above equations $X^*(j\omega)=\int_{-\infty}^{\infty}x^*(t)e^{j\omega t}dt$ ------------(2) $x^*(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X^*(j\omega)e^{-j\omega t}d\omega$ Substitute x(t) in equation (1)

	$$E=\int_{-\infty}^{\infty}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty}X(j\omega)e^{j\omega t}d\omega\right]x^*(t)dt$$ $$E=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(j\omega)d\omega\int_{-\infty}^{\infty}x^*(t)e^{j\omega t}dt$$ Using equation (2) $$E=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(j\omega)X^*(j\omega)d\omega$$ $$E=\frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X(j\omega)\rvert^2 d\omega$$ This relation is called "Parsavel's theorem" or "Rayleigh's energy theorem".
Q4 (b)	**The transfer function of the system is given by:** $$H(j\omega)=\frac{j\omega}{(j\omega)^2+3(j\omega)+2}$$ **Find the system equation and also impulse response of the system.** **Solution:** $$H(jw)=\frac{jw}{(jw)^2+3(jw)+2}$$ $$\frac{Y(jw)}{X(jw)}=\frac{jw}{(jw)^2+3(jw)+2}$$ $$X(jw)(jw)=Y(jw)[(jw)^2+3(jw)+2]$$ *Taking* IFT $$\frac{d^2y(t)}{dt^2}+3\frac{dy(t)}{dt}+2y(t)=\frac{dx(t)}{dt}$$ $$H(jw)=\frac{jw}{(jw)^2+3(jw)+2}$$ Let m=jw $$H(jw)=\frac{m}{(m)^2+3(m)+2}=\frac{A}{m+2}+\frac{B}{m+1}$$ Solving A=2 and B=-1

	$H(jw)=\frac{2}{m+2}+\frac{-1}{m+1}$ $H(jw)=\frac{2}{jw+2}+\frac{-1}{jw+1}$ *Tahing IFT using* relation $e^{-at}u(t)\leftrightarrow\frac{1}{a+jw}$ $\therefore h(t)=2e^{-2t}u(t)-e^{-t}u(t)$
Q5 (a)	**State and prove the following properties of discrete time Fourier Transform.** **(i) Time shifting property** **(ii) Differentiation in frequency domain** **Solution:** **i) Time shifting property**: **Statements:** If $x(t)\xleftrightarrow{FT}X(j\omega)$ then $x(t-t_0)\xleftrightarrow{FT}X(j\omega)e^{-j\omega t_0}$ Shift in time domain will result in multiplying by an exponential in frequency domain **Proof.** $F\{x(t-t_0)\}=\int_{-\infty}^{\infty}x(t-t_0)e^{-j\omega t}dt$ Let $t-t_0=\tau$ $t=\tau+t_0$ and $dt=d\tau$ $=\int_{-\infty}^{\infty}x(\tau)e^{-j\omega(\tau+t_0)}d\tau$ $=\int_{-\infty}^{\infty}x(\tau)e^{-j\omega\tau}d\tau\ e^{-j\omega t_0}$ **ii) Differentiation in time domain property:** *If* $x(t)\xleftrightarrow{FT}X(j\omega)$ *Then* $\frac{dx(t)}{dt}\leftrightarrow(jw)X(j\omega)$ Differentiating a signal in time domain is same as multiplying their spectrum

	in frequency domain **Proof:** Inverse FT $x(t) = \frac{1}{2\pi}\int X(jw)e^{jwt}$ Differentiating with respect to t $\frac{dx(t)}{dt} = \frac{1}{2\pi}\int (jw)X(jw)e^{jwt}$ From the above equation we have $\frac{dx(t)}{dt} \leftrightarrow (jw)X(j\omega)$
Q5 (b)	**Consider a discrete time LTI System with impulse response.** **h [n] = α^n u [n] where $\lvert\alpha\rvert$<1. Use Fourier Transform to determine the response to the input x[n] = β^n u [n] with $\lvert\beta\rvert$ <1** **Solution:** Example 5.13, Page no. 385 of Text Book - I
Q6 (a)	**Determine the Nyquist rate for the following signals** **i) x(t)=1+cos(200πt)+4sin(400πt)** **ii) x(t)=2cos(600πt) cos(800πt)** **Solution:** i) x(t)= 1+cos(200πt)+4sin(400πt) f_1=100 Hz and f_2=200 Hz $f_{Nyq}=2fm_{(max)}$=2x200=400Hz ii) x(t)= 2cos(600πt) cos(800πt) =[cos1400 πt)+cos(200 πt)] f_1=700 Hz and f_2=100 Hz $f_{Nyq}=2fm_{(max)}$=2x700=1400Hz
Q6 (b)	**With diagrams explain sampling of discrete time signals.** **Solution:** Sampling theorem. Statement: Let m(t) is a message signal band limited to f_mHz, if this signal is

	sampled at a rate $f_s \geq 2f_m$ then we can reconstruct the message signals from the sampled value with minimum distortion. i.e $f_s \geq 2f_m$ where fs is sampling frequency and fm is maximum message frequency Let m(t)=message signal $m(t) \leftrightarrow M(f)$ $\delta_T(t) = \sum_n \delta(t - nT)$ is periodic delta function with Fourier series $\delta_T(f) = \frac{1}{T}\delta(f - nf_s)$ Sampled signal S(t)=m(t)∂_T(t) Multiplication in time domain is same as convolution in frequency domain $\therefore S(f) = M(f) * \delta_T(f)$ $= M(f) * \left[\frac{1}{T}\sum \delta_T(f - nf_s)\right]$ Convolving any function with delta function yield the same function $\therefore S(f) = \frac{1}{T}\sum_n M(f - nf_s)$ Spectrum of sampled signal is periodic with period fs.
Q6 (c)	**Find the frequency response and impulse response of the system with input** **x(t)=e^{-2t} u(t) and output y(t)=e^{-3t} u(t).** **Solution:** Applying FT for input and output signal $x(t) = e^{-2t}u(t)$ $F\{x(t)\} = X(jw) = \frac{1}{2 + jw}$ $y(t) = e^{-3t}u(t)$ $F\{y(t)\} = Y(jw) = \frac{1}{3 + jw}$ *Frequencyre response* $H(jw) = \frac{Y(jw)}{X(jw)} = \frac{2 + jw}{3 + jw} = 1 - \frac{1}{3 + jw}$ *Taking IFT* $h(t) = \delta(t) - e^{-3t}u(t)$

Q7 (a)	**Find, the output y(t) of the system described by the differential equatio... $\frac{dy(t)}{dt} + 5y(t) = x(t)$ by Laplace Transform method. Assume that the input x(t)= $3e^{-2t}u(t)$ and initial condition is $y(0^+) = -2$.** **Solution:** $\frac{dy(t)}{dt} + 5y(t) = x(t) = 3e^{-2t}u(t), \quad y(0^+) = -2$ Taking Laplace transform $sY(s) - y(0^+) + 5Y(s) = \frac{3}{s+2}$ $Y(s) = \frac{3}{(s+2)(s+5)} + \frac{-2}{(s+5)}$ $= \frac{A}{(s+2)} + \frac{B}{(s+5)} - \frac{2}{(s+5)}$ $A = 1 \quad B = -1$ $Y(s) = \frac{1}{(s+2)} + \frac{-1}{(s+5)} - \frac{2}{(s+5)}$ $Y(s) = \frac{1}{(s+2)} - \frac{3}{(s+5)}$ *TakingInverse LT* $y(t) = e^{-2t}u(t) - 3e^{-5t}u(t)$
Q7 (b)	Find x(t) from $X(S) = \frac{1}{(1+s)^2}$ Using convolution property **Solution:** $X(S) = \frac{1}{(1+s)^2} = \left[\frac{1}{(1+s)}\right]\left[\frac{1}{(1+s)}\right]$ $e^{-t}u(t) \leftrightarrow \frac{1}{(1+s)}$ Convolution property of LT is

	$x(t)=x_1(t)*x_2(t)\leftrightarrow X_1(w)X_2(w)$ $\therefore\ x(t)=\int_{-\infty}^{\infty}e^{-\tau}u(\tau)e^{-(t-\tau)}u(t-\tau)d\tau$ $=te^{-t},\quad for\quad t>0$ $\therefore x(t)=te^{-t}u(t)$
Q7 (c)	**Find the inverse Laplace transform of** $X(s)=\frac{2}{s^2+4s+8}$ **Solution:** $X(s)=\frac{2}{s^2+4s+8}$ $=\frac{2}{(s+2)^2+4}$ Using the relation $\sin(at)u(t)\leftrightarrow\frac{a}{s^2+a^2}$ $e^{-bt}\sin(at)u(t)\leftrightarrow\frac{a}{(s+b)^2+a^2}$ *ILT* $x(t)=e^{-2t}\sin(2t)u(t)$
Q8 (a) **(i)**	**Find the Z-transform of the following sequence and find the ROC** **(i)** $x[n]=\left[\frac{1}{3}\right]^{n-2}\sin\Omega_0(n-2)\,u[n-2]$ **(ii)** $x[n]=5\left(\frac{1}{2}\right)^n u[n]-2(3)^n u[-n-1]$ **Solution:** $x[n]=\left[\frac{1}{3}\right]^n\sin\Omega_0 n\,u[n]$ $\sin\Omega_0 n\,u[n]\leftrightarrow\frac{z^{-1}\sin\Omega_0}{1-2z^{-1}\cos\Omega_0+z^{-2}}$ ROC $\lvert z\rvert>1$ Using scaling property

	$\left[\frac{1}{3}\right]^n \sin\Omega_0 n\, u[n] \leftrightarrow \frac{\{1/3\}z^{-1}\sin\Omega_0}{1-(2/3)z^{-1}\cos\Omega_0+\frac{1}{9}z^{-2}} \quad \text{ROC } \lvert z\rvert > \frac{1}{3}$ Using shifting property $\left[\frac{1}{3}\right]^{n-2} \sin\Omega_0 (n-2)\, u[n-2] \leftrightarrow \left[\frac{\{1/3\}z^{-1}\sin\Omega_0}{1-(2/3)z^{-1}\cos\Omega_0+\frac{1}{9}z^{-2}}\right] Z^{-2}$ $\left[\frac{1}{3}\right]^{n-2} \sin\Omega_0 (n-2)\, u[n-2] \leftrightarrow \left[\frac{\{1/3\}z^{-3}\sin\Omega_0}{1-(2/3)z^{-1}\cos\Omega_0+\frac{1}{9}z^{-2}}\right] \quad \text{ROC } \lvert z\rvert > \frac{1}{3}$
(ii)	$x[n]=5\left(\frac{1}{2}\right)^n u[n]-2(3)^n u[-n-1]$ $X(z)=5\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n z^{-n}-2\sum_{n=-\infty}^{-1}3^n z^{-n}$ $X(z)=5\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n z^{-n}-2\sum_{n=1}^{\infty}3^{-n} z^{n}$ $X(z)=5\sum_{n=0}^{\infty}\left(\frac{z^{-1}}{2}\right)^n -2\sum_{n=1}^{\infty}(3^{-1}z)^{n}$ $X(z)=5.\left[\frac{z}{z-\frac{1}{2}}\right]+2\left[\frac{z}{z-3}\right]$ ROC: $\lvert z\rvert<3$ and $\lvert z\rvert>1/2$, Roc : $(1/2)<\lvert z\rvert<3$
Q8(b) (i)	**State and prove** **(i) Initial value theorem of z-transform** **(ii) Time Expansion property of z-transform** **Solution:** 1) Initial value theorem: Statement: If x(n) is causal and $x[n]\leftrightarrow X(z)$ $then\ x(0)=\underset{z\to\infty}{lt}\ X[Z]$ Proof: By definition

	$X(z) = \sum_{\eta=-\infty}^{\infty} x[n] z^{-n}$ For causal x(n) $X(z) = \sum_{\eta=0}^{\infty} x[n] z^{-n}$ $X(z) = x(0) + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3}$............ Taking limit $z \to \infty$ on both side $\underset{z\to\infty}{lt} X(z) = \underset{z\to\infty}{lt} [x(0) + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3}............]$ $\therefore \underset{z\to\infty}{lt} X(z) = x(0)$
Q8 (b) (ii)	Page no. 769 to 770 of Text Book – I
Q9 (a)	**Define the following terms with refers to probability theory** **(i) Sample space** **(ii) Event** **(iii) Mutually exclusive event** **(iv) Conditional probability** **(v) Joint probability** **(vi) Power spectral density** **Solution:** **Sample space**: Set consists of all possible outcome of an experiment **Event:** Event is a subset of a sample space **Mutually exclusive event**: If two events are mutually exclusive then there is no common element between them. **Conditional probability**:

	Probability of an event depends on some other event P(A/B)-probability of event A after the event B is over. **Joint Probability**: P(AB)=P(A)P(B/A) if A and B are statistically independent then, P(AB)=P(A)P(B) **The power spectral density**: PSD, describes how the power (or variance) of a time series is distributed with frequency. Mathematically, it is defined as the Fourier Transform of the autocorrelation sequence of the time series
Q9 (b)	**What is wide sense stationary process mention its properties.** **Solution:** A random process X(t) is called wide sense stationary if it satisfies 1. Mean of the process is constant 2. autocorrelation function is independent of time 3. variance of the process is constant
Q9 (c)	**Write short notes on:** **(i) Gaussian processes** **(ii) Ergodic processes** **Solution:** **(i)** Gaussian processes - Page no. 54 to 58 of Text Book - II **(ii)** Ergodic Processes - Page no. 41 to 42 of Text Book – II

TEXT BOOKS

1. Signals and Systems, A.V. Oppenheim and A.S. Willsky with S. H. Nawab, Second Edition, PHI Private limited, 2006

2. Communication Systems, Simon Haykin, 4th Edition, Wiley Student Edition, 7th Reprint 2007