Code: AE56/AC56/AT56 Subject: ENGINEERING MATHEMATICS - II

AMIETE – ET/CS/IT

Time: 3 Hours **DECEMBER 2012** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. The function $f(z) = 2xy + i(x^2 - y^2)$ is every where

(A) continuous and analytic **(B)** continuous but not analytic
(C) analytic but not continuous **(D)** None of these

b. The value of the integral $\int_C \frac{dz}{z-a}$ over circle $C : |z-a| = r$ is

(A) 0 **(B)** πi
(C) $2\pi i$ **(D)** $4\pi i$

c. The residue of $f(z) = \tan z$ at $z = \frac{\pi}{2}$ is

(A) -1 **(B)** $+1$
(C) -2 **(D)** $+2$

d. If $\vec{R} = xi + yj + zk$ and $\vec{A}$ is a constant vector, then $\nabla(\vec{A} \cdot \vec{R})$ is equal to

(A) $\vec{A} + \vec{R}$ **(B)** $\vec{A} - \vec{R}$
(C) $\vec{R}$ **(D)** $\vec{A}$

e. For any closed surface S, $\int_s [x(y-z)i + y(z-x)j + z(x-y)k] \cdot \vec{ds}$ is equal to

(A) 0 **(B)** π
(C) 2π **(D)** None of these

f. If $f(x) = 3x^3 - 2x^2 + x + 1$, then $\Delta^3 f(x)$ is equal to

(A) 3 **(B)** 6
(C) 12 **(D)** 18

g. If $x^2 + 2x - ay^2$ is harmonic, then a is

(A) 3 **(B)** 2
(C) 1 **(D)** 0

h. The probability that a leap year will have 53 Mondays is

(A) $\frac{1}{7}$ **(B)** $\frac{2}{7}$
(C) $\frac{3}{7}$ **(D)** None of these

i. If a random variable has a Poisson distribution such that P(1) = P(2), then mean of the distribution is

(A) 2 **(B)** 3
(C) 4 **(D)** None of these

j. If $f(x)=k(x+1), \quad -1<x<1$
$\quad\quad =0 \quad , \quad$ elsewhere

represents a probability density function, then K is equal to

(A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$
(C) $\frac{1}{2}$ **(D)** 1

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Determine the analytic function $f(z) = u + iv$, if $u - v = \frac{\cos x + \sin x - e^{-y}}{2(\cos x - \cosh y)}$ and $f(\pi/2) = 0$. **(8)**

b. Evaluate $\int_C \frac{z^2 - z + 1}{z - 1} dx$, where C is the circle

(i) $|z| = 1$ (ii) $|z| = 1/2$ **(8)**

Q.3 a. Derive Cauchy-Riemann equations in polar form. **(8)**

b. Expand in Laurent's series the function $\frac{1}{Z^2 - 4Z + 3}$ for $1 < |Z| < 3$. **(8)**

Q.4 a. Find a unit vector normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point $(1,2,-1)$ **(8)**

b. Show that $\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$ **(8)**

Q.5 a. Use Green's theorem to evaluate $\int_C \left[(2x^2 - y^2)dx + (x^2 + y^2)dy\right]$ where C is the boundary in the xy plane of the area enclosed by the x-axis and the semi-circle $x^2 + y^2 = 1$ in the upper half of xy plane. **(8)**

b. Verify Divergence theorem for $\vec{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ taken over the rectangular parallelopiped $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$. **(8)**

Q.6 a. Using Lagrange's interpolation formula, find the values of y when x = 10, from the following table: **(8)**

x :	5	6	9	11
y :	12	13	14	16

b. Prove that: **(8)**

(i) $\mu^2 = 1 + \left(\frac{1}{4}\right)\delta^2$ (ii) $\Delta = \left(\frac{1}{2}\right)\delta^2 + \delta\sqrt{1 + \left(\frac{1}{4}\right)\delta^2}$

Q.7 a. Use Charpits method to solve $z = p^2x + q^2y$ **(8)**

b. Find the differential equation of all planes which are at a constant distance 'a' from the origin. **(8)**

Q.8 a. State and prove Baye's theorem. **(1+7)**

b. A and B throw alternately with a pair of dice. The one who throws 9 first wins. If A starts the game, compare their chances of winning. **(8)**

Q.9 a. The diameter of an electric cable is assumed to be a continuous variable with p.d.f. (probability density function) $f(x) = 6x(1-x), 0 \le x \le 1$. Verify that the above is a p.d.f. Also find the mean and the variance. **(2+3+3)**

b. Out of 800 families with three children each, how many would you expect to have

(i) at least one boy
(ii) all three boys
(iii) one boy and two girls or 2 boys and one girl.

Assume equal probabilities for boys and girls. **(8)**