AMIETE - ET/CS/IT

Time: 3 Hours

DECEMBER 2012

Max. Marks:

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

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- Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

Q.1 Choose the correct or the best alternative in the following:

 (2×10)

- a. The function $f(z) = 2xy + i(x^2 y^2)$ is every where
 - (A) continuous and analytic
- (B) continuous but not analytic
- (C) analytic but not continuous
- (D) None of these
- b. The value of the integral $\int_{C} \frac{dz}{z-a}$ over circle C: |z-a| = r is
 - (A) 0

 $(\mathbf{B}) \pi$

(C) $2\pi i$

- (**D**) $4\pi i$
- c. The residue of $f(z) = \tan z$ at $z = \frac{\pi}{2}$ is
 - (A) -1

(B) +1

(C) -2

- **(D)** +2
- d. If $\vec{R} = xi + yj + zk$ and \vec{A} is a constant vector, then $\nabla(\vec{A} \cdot \vec{R})$ is equal to
 - $(\mathbf{A}) \ \vec{\mathbf{A}} + \vec{\mathbf{R}}$

 $(\mathbf{B}) \vec{A} - \vec{R}$

 $(\mathbf{C})\ \vec{R}$

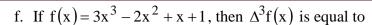
- **(D)** \vec{A}
- e. For any closed surface S, $\int_{S} [x(y-z)i + y(z-x)j + z(x-y)k] \cdot ds$ is equal to
 - $(\mathbf{A}) 0$

(B) π

(C) 2π

(**D**) None of these

Subject: ENGINEERING MATHEN $\Delta^3 f(x)$ is equal to



(A) 3

(C) 12

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g. If $x^2 + 2x - ay^2$ is harmonic, then a is

(A) 3

(B) 2

(C) 1

(D) 0

h. The probability that a leap year will have 53 Mondays is

(A) $\frac{1}{7}$

(B) $\frac{2}{7}$

(C) $\frac{3}{7}$

(D) None of these

i. If a random variable has a Poisson distribution such that P(1) = P(2), then mean of the distribution is

(A) 2

(B) 3

(C) 4

(**D**) None of these

j. If
$$f(x)=k(x+1)$$
, $-1 < x < 1$
=0, elsewhere

represents a probability density function, then K is equal to

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) 1

Answer any FIVE Questions out of EIGHT Questions. Each question carries 16 marks.

Q.2 a. Determine the analytic function f(z) = u + iv, if $u - v = \frac{\cos x + \sin x - e^{-y}}{2(\cos x - \cosh y)}$ and $f(\pi/2) = 0$.

b. Evaluate $\int_{C} \frac{z^2 - z + 1}{z - 1} dx$, where C is the circle

(i) |z| = 1

(ii) $|z| = \frac{1}{2}$

(8)

Q.3 a. Derive Cauchy-Riemann equations in polar form. **(8)**

b. Expand in Laurent's series the function $\frac{1}{Z^2 - 4Z + 3}$ for 1 < |Z| < 3. **(8)**

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Q.4 a. Find a unit vector normal to the surface $x^3 + y^3 + 3xyz = 3$ at the (1,2,-1)

b. Show that
$$\nabla \cdot (\overrightarrow{A} \times \overrightarrow{B}) = \overrightarrow{B} \cdot (\nabla \times \overrightarrow{A}) - \overrightarrow{A} \cdot (\nabla \times \overrightarrow{B})$$
 (8)

- Q.5 a. Use Green's theorem to evaluate $\int_{C} \left[\left(2x^2 y^2 \right) dx + \left(x^2 + y^2 \right) dy \right]$ where C is the boundary in the xy plane of the area enclosed by the x-axis and the semi-circle $x^2 + y^2 = 1$ in the upper half of xy plane. (8)
 - b. Verify Divergence theorem for $\vec{F} = (x^2 yz)i + (y^2 zx)j + (z^2 xy)k$ taken over the rectangular parallelopiped $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$. (8)
- Q.6 a. Using Lagrange's interpolation formula, find the values of y when x = 10, from the following table:
 x: 5 6 9 11
 y: 12 13 14 16

b. Prove that:
 (i)
$$\mu^2 = 1 + \left(\frac{1}{4}\right)\delta^2$$

 (ii) $\Delta = \left(\frac{1}{2}\right)\delta^2 + \delta\sqrt{1 + \left(\frac{1}{4}\right)\delta^2}$

- Q.7 a. Use Charpits method to solve $z = p^2x + q^2y$ (8)
 - b. Find the differential equation of all planes which are at a constant distance 'a' from the origin.(8)
- Q.8 a. State and prove Baye's theorem. (1+7)
 - b. A and B throw alternately with a pair of dice. The one who throws 9 first wins. If A starts the game, compare their chances of winning. (8)
- Q.9 a. The diameter of an electric cable is assumed to be a continuous variable with p.d.f. (probability density function) f(x) = 6x(1-x), $0 \le x \le 1$. Verify that the above is a p.d.f. Also find the mean and the variance. (2+3+3)
 - b. Out of 800 families with three children each, how many would you expect to have
 - (i) at least one boy
 - (ii) all three boys
 - (iii) one boy and two girls or 2 boys and one girl.

 Assume equal probabilities for boys and girls.

 (8)