ROLL NO. ___________________

Code: AC65 Subject: DISCRETE STRUCTURES

AMIETE – CS

Time: 3 Hours **DECEMBER 2012** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. Number of elements in the power set of set A ={ϕ} is

(A) 0 **(B)** 2
(C) 1 **(D)** 4

b. Number of relations that can be defined on set A = {1, 2, 3, 4} is

(A) 1024 **(B)** 16
(C) 65536 **(D)** 256

c. Number of ways in which the letters of the word BANANA can be distinctly arranged is

(A) 60 **(B)** 720
(C) 16 **(D)** 120

d. Which of the following is not a rational number

(A) Sum of two rational numbers **(B)** Product of two irrational numbers
(C) Sum of two recurring numbers **(D)** Difference of two rational numbers

e. Suppose the universe consists of all integers. Consider
$P(x): x \le 3$,
$r(x): x > 0$
What is the truth value of $\neg P(3) \vee r(0)$?

(A) TRUE **(B)** FALSE

f. Relation defined on a partially ordered set is not

(A) Anti-Symmetric **(B)** Symmetric
(C) Transitive **(D)** Reflexive

g. Which of the following is not true about composition of functions?

(A) Associative **(B)** Commutative
(C) Existence of Inverse **(D)** Existence of Identity

h. A group G is cyclic if

(A) G is a subgroup of a cyclic group
(B) G is of finite order
(C) G is isomorphic to a cyclic group
(D) None of these

i. The hamming distance between x = 111000111 and y = 000100111 is

(A) 3 **(B)** 4
(C) 5 **(D)** 6

j. Addition of two integers x and y is even then

(A) Both are odd **(B)** x is even and y is odd
(C) x is odd and y is even **(D)** x is odd and y is even multiple of x

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Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Using Venn diagrams, prove that, for any sets A, B, C.
$\overline{(A \cup B) \cap C} = (\overline{A} \cap \overline{B}) \cup \overline{C}$ **(8)**

b. Three students X, Y, Z write an examination. Their chances of passing are $1/2$, $1/3$ and $1/4$ respectively. Find the probability that
(i) all of them pass
(ii) at least one of them passes
(iii) at least two of them pass. **(8)**

Q.3 a. Read the following paragraph. Convert each statement into propositional expressions and show that the conclusion is true.
"Ram is a student and he is sincere. Every sincere student excels in the class. Shyam is also a sincere student. Therefore Ram and Shyam both excel in their class." **(8)**

b. Show that the Boolean expressions $(z' \vee x) \wedge ((x \wedge y) \vee z) \wedge (z' \vee y)$ and $x \wedge y$ are logically equivalent. **(8)**

Q.4 a. Prove that $R \to S$ is a valid conclusion from the premises $P \to (Q \to S), \sim R \vee P$ and Q. **(8)**

b. Prove that
$\{\forall x[P(x) \vee Q(x)]\} \wedge \{\exists x \neg P(x)\} \wedge \{\forall x [\neg Q(x) \vee R(x)]\} \wedge \{\forall x[S(x) \to \neg R(x)]\} \Rightarrow \exists x \neg S(x)$
(8)

Q.5 a. Using mathematical induction method, prove that $11^{n+2} + 12^{2n+1}$ is divisible by 133 for any $n \in N$. **(8)**

b. A sequence $\{a_n\}$ is defined recursively by $a_1 = 4$, $a_n = a_{n-1} + n$ for $n \geq 2$. Find a_n in explicit form. **(8)**

Q.6 a. Show that if a relation R defined on a set A is symmetric and transitive then R is not irreflexive. **(8)**

b. Let L is a distributive lattice. Show that if there exists an element *a* such that $a \wedge x = a \wedge y$ and $a \vee x = a \vee y$ then x = y. **(8)**

Q.7 a. Let *f* be a function defined on a real number *R* such that *f(x) = 2x – 1* then show that *f* is an invertible function and find inverse function of *f*. **(8)**

b. Define even permutation function. Show that composition of two even permutation function is even permutation function. **(8)**

Q.8 a. Let (G, *) be a group then for any two elements a and b of (G, *) prove that $(a * b)^{-1} = b^{-1} * a^{-1}$ **(8)**

b. Show that set *G = {1, i, –i, –1}* forms a cyclic group under the binary operation of multiplication. **(8)**

Q.9 Write short notes on **TWO** of the followings:

(i) Parity Check Matrix
(ii) Hamming distance
(iii) Ring z_n **(8+8)**