

DipLETE – ET/CS (OLD SCHEME)

Code: DE23/DC23
Time: 3 Hours

JUNE 2011

Subject: MATHEMATICS - II
Max. Marks: 100

NOTE: There are 9 Questions in all.

- Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. The modulus of the complex number $\cos \theta + i \sin \theta$ is

- (A) -1 (B) 1
(C) 2 (D) 0

b. The principal argument of 2 is

- (A) 0 (B) 1
(C) -1 (D) i

c. The scalar product of the vectors $\hat{i} + 3\hat{j} - 8\hat{k}$ and $-3\hat{i} - 5\hat{j} + 4\hat{k}$ is

- (A) 60 (B) -55
(C) -50 (D) 45

d. The adjoint of the square matrix $\begin{bmatrix} 7 & 4 \\ 3 & -6 \end{bmatrix}$ is

- (A) $\begin{bmatrix} -6 & -4 \\ -3 & 7 \end{bmatrix}$ (B) $\begin{bmatrix} -6 & 7 \\ -3 & -4 \end{bmatrix}$
(C) $\begin{bmatrix} -6 & -3 \\ -4 & 7 \end{bmatrix}$ (D) $\begin{bmatrix} -3 & -6 \\ 7 & -4 \end{bmatrix}$

e. The product of $\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 1 \\ 2 & 4 \end{bmatrix}$ is

- (A) $\begin{bmatrix} 9 & 6 \\ -6 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & 9 \\ 3 & 6 \end{bmatrix}$
(C) $\begin{bmatrix} -6 & 9 \\ 2 & 6 \end{bmatrix}$ (D) $\begin{bmatrix} 6 & 9 \\ -2 & 2 \end{bmatrix}$

f. The value of $\begin{vmatrix} 1 & 5 & 27 \\ 1 & 5 & 40 \\ 1 & 5 & 18 \end{vmatrix}$ is

- (A) 125
(C) 0

- (B) 200
(D) 225

g. The value $\begin{vmatrix} \sin 10^\circ & -\cos 10^\circ \\ \sin 80^\circ & \cos 80^\circ \end{vmatrix}$ is

- (A) 0
(C) 1

- (B) $\frac{1}{2}$
(D) -1

h. The solution of the differential equation $(D^2 + 4)y = 0$ is

- (A) $y = A \cos 2x + B \sin 2x$
(C) $y = (A_1 + A_2)\cos^2 x + (A_3 + A_4)\sin^2 x$

- (B) $y = e^x(A \cos^2 2x + B \sin^2 2x)$
(D) $y = (A_1 + A_2x)\cos 2x + (A_3 + A_4)\sin 2x$

i. The value $L^{-1} \left[\frac{s}{s^2 + a^2} \right]$ is

- (A) $\frac{1}{a} \cos at$

- (B) $\cosh at$

- (C) $\cos at$

- (D) $\frac{1}{a} \cosh at$

j. The function $f(x) = e^x$ has the Fourier expansion $e^x = \sum_1^\infty b_n \sin nx$ in the

interval $(0, \pi)$. Then $\sum_1^\infty (b_n)^2$ converges to

- (A) $\frac{1}{\pi}(e^\pi - 1)$

- (B) $\frac{1}{\pi}(e^\pi + 1)$

- (C) $\frac{1}{\pi}(e^{2\pi} - 1)$

- (D) $\frac{1}{\pi}(e^{2\pi} + 1)$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If $\frac{a+ib}{c+id} = x+iy$, prove that $x^2 + y^2 = \frac{a^2 + b^2}{c^2 + d^2}$ (8)

b. If $Z_1=2+7i$ and $Z_2=1-5i$, then verify that (i) $|Z_1 Z_2| = |Z_1| |Z_2|$

$$(ii) \left| \frac{Z_1}{Z_2} \right| = \frac{|Z_1|}{|Z_2|} \quad (8)$$

Q.3 a. If $2\cos\theta = x + \frac{1}{x}$
 $2\cos\phi = y + \frac{1}{y}$

Show that one of the value of

$$\frac{x^m}{y^n} + \frac{y^n}{x^m} \text{ is } 2\cos(m\theta - n\phi) \quad (8)$$

b. Prove using vectors:

If the diagonals of a parallelogram are equal, then it is a rectangle (8)

Q.4 a. Show that $|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$ (8)

b. Forces $P=2i+5j+6k$ and $Q=-i-2j-k$ act on a particle. Determine the work done when the particle is displaced from a point A with position vector $4i-3j-2k$ to a point B with position vector $6i+j-3k$. (8)

Q.5 a. Show that $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ (8)

b. Use Cramer's Rule to solve the equations

$$\begin{aligned} x+y+z &= -1 \\ x+2y+3z &= -4 \\ x+3y+4z &= -6 \end{aligned} \quad (8)$$

Q.6 a. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1} and hence prove that $A^2 - 4A - 5I = 0$ (8)

b. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$. Hence find A^{-1} (8)

Q.7 a. Find the Fourier series for $f(x) = \pi + x$ in $(-\pi, \pi)$. (8)

b. Find the Fourier series for $f(x) = x^2$ in $(-\pi, \pi)$. Hence derive value of

$$\sum \frac{(-1)^{n-1}}{n^2}. \quad (8)$$

Q.8 a. Find the Laplace transform of $\sin(\sqrt{t})$. (8)

b. Find the inverse Laplace transform of $\frac{s^2 + 1}{s^3 + 3s^2 + 2s}$. (8)

Q.9 a. Solve the differential equation, $(D^2 - 2D + 5)y = e^{2x} \sin x$; where $D = \frac{d}{dx}$. (8)

b. Use Laplace transform method to solve

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t$$

with

$$x(0) = 2$$

$$x'(0) = -1$$

(8)