Code: DE01 / DC01 **JUNE 2011** **Subject: MATHEMATICS - I**
Time: 3 Hours **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2 ×10)**

a. How many terms are there in the sequence
3, 6, 9, 12 ....., 111?

(A) 34 **(B)** 36
(C) 37 **(D)** 33

b. If $\sin A = \frac{3}{5}$ and $\cos B = \frac{9}{41}, 0 < A < \frac{\pi}{2}, 0 < B < \frac{\pi}{2}$, find the value of $\sin(A + B)$

(A) $-\frac{133}{205}$ **(B)** $\frac{187}{205}$
(C) $\frac{156}{205}$ **(D)** $\frac{-84}{205}$

c. The area of triangle whose vertices are (6, 3), $(-3,5)$ & $(4,-2)$ is :

(A) 24.5 sq. unit **(B)** 24 sq. unit
(C) 25.5 sq. unit **(D)** 25 sq unit

d. Evaluate $\lim_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1}$

(A) 2 **(B)** $\frac{1}{2}$
(C) 4 **(D)** $\frac{1}{4}$

e. If $y = \tan^{-1}\left(\frac{1+\tan x}{1-\tan x}\right)$ then $\frac{dy}{dx}$ is :

(A) 1 **(B)** -1

(C) 0 **(D)** $\frac{1}{2}$

f. Evaluate $\int \frac{1}{16+9x^2}dx$

(A) $\frac{1}{6}\tan^{-1}\left(\frac{3x}{4}\right)+C$ **(B)** $\frac{1}{12}\tan^{-1}\left(\frac{3x}{4}\right)+C$

(C) $\tan^{-1}\left(\frac{3x}{4}\right)+C$ **(D)** $-\frac{1}{12}\tan^{-1}\left(\frac{3x}{4}\right)+C$

g. Evaluate $\int_0^1 xe^x dx$

(A) 0 **(B)** -1

(C) 2 **(D)** 1

h. If $\frac{dy}{dx} = x\log x$ then the value of y will be:

(A) $\frac{x^2}{2}\log x+\frac{1}{2}\left(\frac{x^2}{2}\right)+C$ **(B)** $\frac{x^2}{2}\log x-\frac{1}{2}\left(\frac{x^2}{2}\right)+C$

(C) $\frac{x}{2}\log x+\frac{1}{2}\left(\frac{x^2}{2}\right)+C$ **(D)** $\frac{x^2}{2}\log x-\frac{x^2}{2}+C$

i. From a class of 32 students, 4 are to be chosen for a competition. In how many ways can this be done?

(A) 35960 **(B)** 35900

(C) 35940 **(D)** 35980

j. Find the equation of the line which makes intercepts -4 & 5 on the axes.

(A) $5x+4y-20=0$ **(B)** $5x+4y+20=0$

(C) $5x-4y+20=0$ **(D)** $-5x+4y+20=0$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Find three numbers in G.P. whose sum is 13 and the sum of whose squares is 91. **(8)**

b. If x is numerically so small that x^2 and higher power of x may be neglected then prove that $\frac{(1-2x)^{2/3}(4+5x)^{3/2}}{\sqrt{1-x}} \simeq 8+\frac{25x}{3}$ **(8)**

Q.3 a. Prove that:

$$\frac{\sin A-\sin 3A+\sin 5A-\sin 7A}{\cos A-\cos 3A-\cos 5A+\cos 7A}=\cot 2A$$ **(8)**

b. In any triangle ABC, prove that:

$$(b-c)\cot\frac{A}{2}+(c-a)\cot\frac{B}{2}+(a-b)\cot\frac{C}{2}=0$$ **(8)**

Q.4 a. Prove by the principle of mathematical induction that for all $n \in N$:

$1+4+7+.....+(3n-2)= \frac{1}{2}n(3n-1)$ **(8)**

b. If p be the length of perpendicular from the origin to the line whose intercepts on the axes are a & b respectively then show that $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$ **(8)**

Q.5 a. Find the equation of circle which passes through the points $(5,-8),(2,-9)$ & (2, 1). Find also the co-ordinates of its centre & radius. **(8)**

b. Find the equation of the parabola whose focus is $(1,-1)$ and whose vertex is $(2,1)$. Also find its axis. **(8)**

Q.6 a. Differentiate $y=a^x$ w.r.t. 'x' from first principle. **(8)**

b. If $y=\log\sqrt{\frac{a+b\sin x}{a-b\sin x}}$, then find $\frac{'dy'}{dx}$ **(8)**

Q.7 a. Find all the points of maxima minima and the corresponding maximum and minimum values of the function:

$f(x)=-x^3+12x^2-5$ **(8)**

b. Evaluate $\int\frac{\log x}{x^2}dx$ **(8)**

Q.8 a. Evaluate $\int_0^{\pi/4}\log(1+\tan x)dx$ **(8)**

b. Evaluate $\int\frac{x-1}{x^3+1}dx$ **(8)**

Q.9 a. Find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$ **(8)**

b. Solve the differential equation

$(x+y+1)\frac{dy}{dx}=1$ **(8)**