Subject: SIGNALS AND SYSTEMS

Time: 3 Hours **JUNE 2011** **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. A continuous time system is described by y(t)=log x(t). Then the system is

(A) Time Invariant and Linear **(B)** Time variant and Linear
(C) Time Invariant and Nonlinear **(D)** None of these

b. Energy signals are the signals with

(A) $0 < E < \infty, P = 0$ **(B)** $0 < E < \infty, P = \infty$
(C) $0 < P < \infty, E = \infty$ **(D)** $0 < P < \infty, E = 0$

c. The signal $x[n] = \cos\left(\frac{n\pi}{12}\right) + \sin\left(\frac{n\pi}{18}\right)$ is periodic with a period equal to

(A) 15 **(B)** 27
(C) 72 **(D)** None of these

d. The impulse response of the system is given by $h(n)=(1/2)^n u[n]$. Then step response the system is

(A) $2\left[1-\left(\frac{1}{2}\right)^{n+1}\right]u[n]$ **(B)** $2\left[1-\left(\frac{1}{2}\right)^{n-1}\right]u[n]$

(C) $2\left[1-\left(\frac{1}{2}\right)^{n}\right]u[n]$ **(D)** $\left[1-\left(\frac{1}{2}\right)^{n-1}\right]u[n]$

e. The Fourier transform of the signal $x(t) = e^{-a|t|}$ is

(A) $X(j\omega) = \frac{a}{a^2 + \omega^2}$ **(B)** $X(j\omega) = \frac{2a}{a^2 + \omega^2}$

(C) $X(j\omega) = \frac{2}{\ldots}$ **(D)** $X(j\omega) = \frac{1}{\ldots}$

f. Laplace transform of $\frac{dx(t)}{dt}$ is

(A) $\frac{1}{s}X(s)$ **(B)** sX(s)
(C) $s^2X(s)$ **(D)** $X(s)/s^2$

g. Inverse z-transform of X[z]=[1/Z]

(A) 1 **(B)** $\delta(n)$
(C) $\delta(n-1)$ **(D)** $\delta(n+1)$

h. ROC of the z-transform of U(-n) sequence is

(A) $|z|<1$ **(B)** $|z|>1$
(C) Real part of z>0 **(D)** $|z|=0$

i. The Fourier transform of the function sgn(t) given in the Fig. 1

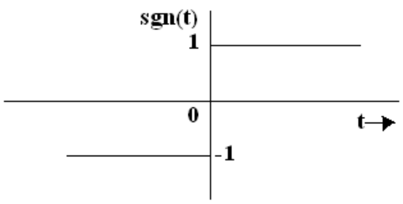

Fig. 1

(A) $\frac{-2}{j\omega}$ **(B)** $\frac{4}{j\omega}$
(C) $\frac{2}{j\omega}$ **(D)** $\frac{1}{j\omega}+1$

j. A random process X(t) is called wide sense stationary if its

(A) The mean of the process is constant
(B) Second order moment is constant
(C) Autocorrelation function is dependent of time
(D) All of the above

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. For each of the following systems determine whether the system is Linear, Casual, Stable, Time-invariant and Memory less
(i) $y(n)=e^{x(n)}$ (ii) y(t)=log(x(t)) **(8)**

b. Represent the following signals using basic signals (Fig. 2) **(4)**

(i) (ii)

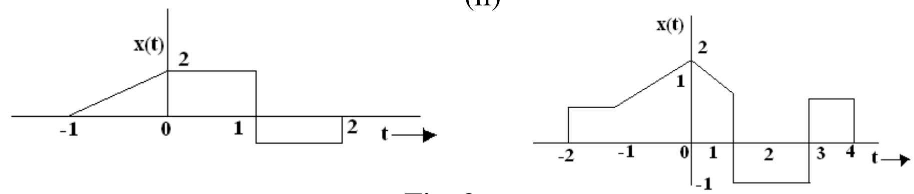

Fig. 2

c. Find the response of a system to an arbitrary input $x(n)=2^n u(n)$ given the impulse response $h(n)=3^n u(n)$. **(4)**

Q.3 a. Find the Fourier series representation of the signal x(t), as shown in Fig. 3. **(6)**

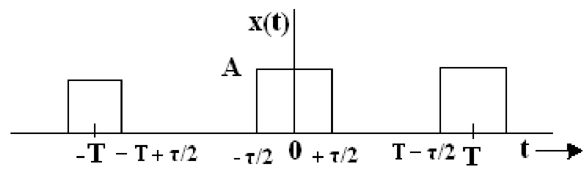

Fig. 3

b. Determine the Fourier series representation for signal

(i) $x(t)=1+\sin(2\pi t-5)+2\sin(6\pi t)$ (ii) $x(n) = 2\cos\left[\frac{\pi}{3}n + \phi\right]$ **(6)**

c. State and prove parsevel's power theorem for discrete signal. **(4)**

Q.4 a. Determine the Fourier transform of the signal $x(n)= a^{|n|}, -1 < a < 1$ **(6)**

b. For the system equation 3y(n)-4y(n-1)+y(n-2)=3x(n) find the transfer function and the impulse response. **(6)**

c. Derive the Fourier transform $x(e^{j\omega})$ of x(n)=u(n). **(4)**

Q.5 a. State and prove the following properties of continuous signal Fourier transform. (i) Time reverse property (ii) Convolution property. **(6)**

b. Find the FT of the signal x(t) as shown in Fig. 4. **(6)**

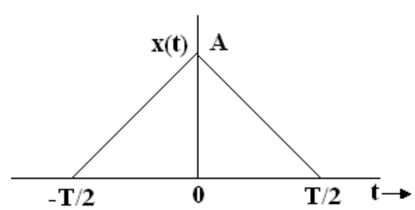

Fig. 4

c. Find the inverse Fourier Transform of $X(j\omega) = \dfrac{\omega^2 - 4j\omega - 6}{(-\omega^2 + 3j\omega + 2)(4 + j\omega)}$ **(4)**

Q.6 a. Determine the Nyquist rate for the following signals:
(i) $x(t)= \cos(640\pi t)+\sin(840\pi t)$ (ii) $x(t)= \cos(640\pi t)+\cos(840\pi t)$ **(6)**

b. State and prove sampling theorem for Band limited. **(6)**

c. Determine the differential equation for the following system with transfer function: (i) $H(j\omega)=\frac{(j\omega)}{(2+j\omega)}$ (ii) $H(j\omega)=\frac{(4+j\omega)}{(1+j^2\omega^2+j\omega)}$ **(4)**

Q.7 a. Find the laplace transform of the following signals: (i) $X(t)=e^{-4(t-3)}u(t-3)$
(ii) $X(t)=e^{-5t}\cos(3t)u(t)$ **(8)**

b. Find the Inverse Laplace transform of the following X(s) **(4)**

$$X(s)=\frac{3}{(S^2+10S+34)}$$

c. State and prove Time shifting property in Laplace transform. **(4)**

Q.8 a. State and prove the following properties of Z-transform of: (i) Convolution property (ii) Scaling property (iii) Time Reversal **(8)**

b. Find the inverse Z-transform of

$$X(z)=\frac{z^3+z^{-3}}{(z-1)(z-2)(z-3)} \quad \text{with ROC } |z|>3$$ **(8)**

Q.9 a. The random variable X is expresses as its density function

$$f_X(x)=\begin{cases}1/e^x & x>0\\ 0 & \text{otherwise}\end{cases}.$$

Find expected values E[x]. **(4)**

b. Write a note on Gaussian noise **(4)**

c. Define the following terms with refers to probability theory

(i) Wide sense stationary process
(ii) Power spectral density
(iii) Conditional probability
(iv) Covariance function. **(8)**