Subject: ENGINEERING MATHEMATICS - I

Time: 3 Hours **JUNE 2011** **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions, answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. If in a determinant the corresponding elements of two rows (or columns) are proportional to each other, then the value of the determinant is

(A) unity **(B)** zero
(C) infinity **(D)** none of the above

b. In case of matrix multiplication of two matrix A and B, if AB = 0 (where '0' stands for null matrix), it means that

(A) either A = 0 or B = 0
(B) both of them '0'
(C) does not necessary that either A = 0 or B = 0
(D) none of the above

c. If $u = F(x-y, y-z, z-x)$, then

(A) $\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ **(B)** $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$
(C) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} - \frac{\partial u}{\partial z} = 0$ **(D)** $\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} - \frac{\partial u}{\partial z} = 0$

d. The Newton-Raphson method to find a root of the equation f(x) fails when

(A) For a particular value of $x = x_0$ (say), $f(x_0)$ becomes zero.
(B) For a particular value of $x = x_0$ (say), $f(x_0)$ becomes unity.
(C) For a particular value of $x = x_0$ (say), $f'(x_0)$ becomes zero.
(where $f'(x)$ in the first derivative of f w.r.t. x)
(D) For a particular value of $x = x_0$ (say), $f(x_0)$ becomes equal to $f'(x_0)$.

e. If $\dfrac{\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}}{N}$ is a function of x, alone say f(x), then integrating factor is

(A) $e^{\int x dx}$ **(B)** $e^{\int y dy}$

(C) $e^{\int f(x) dx}$ **(D)** $e^{\int f(y) dy}$

f. The maximum value of $\left(3x^4 - 2x^3 - 6x^2 + 6x + 1\right)$ in the interval (0, 2) is

(A) 1 **(B)** 21

(C) $1/2$ **(D)** None of the above

g. Value of $\int_0^1 dx \int_0^x e^{y/x} dy$ is

(A) $\frac{1}{2}(e-1)$ **(B)** $\frac{1}{2}(1-e)$

(C) 1 **(D)** None of the above

h. The value of $J_{1/2}(x)$ is

(A) $\sqrt{\left(\frac{2}{\pi x}\right)\sin x}$ **(B)** $\sqrt{\left(\frac{2}{\pi x}\right)\cos x}$

(C) $\sqrt{\left(\frac{2}{\pi x}\right)}\sin x$ **(D)** $\sqrt{\left(\frac{2}{\pi x}\right)}\cos x$

i. Value of $\left|\overline{-\tfrac{1}{2}}\right.$ is

(A) $\sqrt{\pi}$ **(B)** $-\sqrt{\pi}$

(C) $2\sqrt{\pi}$ **(D)** $-2\sqrt{\pi}$

j. A matrix ‘A’ is said to be idempotent matrix if

(A) $A^T A = I$

(B) $A^2 = A$

(C) $A^K = A$, K is any positive integer value

(D) $A = A^T$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If $u = \sin^{-1}\left[\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right]$. Prove that

$$x^2\frac{\partial^2 u}{dx^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{dy^2} = -\frac{\sin u\cos 2u}{4\cos^3 u}$$ **(8)**

b. Find the shortest distance between the line $y = 10 - 2x$, and the ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$. **(8)**

Q.3 a. Using the transformation $x + y = u, y = uv$, show that $\iint [xy(1 - x - y)]^{1/2}\,dxdy = \dfrac{2\pi}{105}$, integration being taken over the area of the triangle bounded by the lines x = 0, y = 0, x+y = 1. **(8)**

b. A rectangular box, open at the top is to have a volume of 32 c.c. Find the dimension of the box requiring least for material for its construction. **(8)**

Q.4 a. Find the eigen values and eigen vectors of the matrix

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$ **(8)**

b. Test the consistency of the following system of equations and solve them if possible:

$$3x + 3y + 2z = 1$$
$$x + 2y = 4$$
$$10y + 3z = -2$$ **(8)**

Q.5 a. Find by the method of Regula Falsi a root of the equation $x^3 + x^2 - 3x - 3 = 0$ lying between 1 an d 2. **(8)**

b. Perform three iterations of the Gauss-Seidel method for solving the system of equations

$$\begin{bmatrix} 4 & 0 & 2 \\ 0 & 5 & 2 \\ 5 & 4 & 10 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ 11 \end{bmatrix}$$

Take the components of the approximate initial vector as $x_i^{(0)} = \frac{b_i}{a_{ii}}, i = 1,2,$

Compare with the exact solution $x = [1,-1,1]^T$. **(8)**

Q.6 a. Solve $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$ **(8)**

b. Solve $\frac{dy}{dx} = \frac{y+x-2}{y-x-4}$ **(8)**

Q.7 a. Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 2x^2 + 3e^{-x}$ **(8)**

b. Find the general solution of the equation $y'' + 3y' + 2y = 2e^x$, using method of variation of parameters. **(8)**

Q.8 a. (i) If
$$f(x) = 0 \quad -1 < x \le 0$$
$$= x \quad 0 < x < 1$$

Show that $f(x) = \frac{1}{4}P_0(x) + \frac{1}{2}P_1(x) + \frac{5}{16}P_2(x) - \frac{3}{32}P_4(x) + ...$ **(4)**

(ii) Prove that $\int J_3(x)dx + J_2(x) + \frac{2}{x}J_1(x) = 0$ **(4)**

b. Prove that $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$ **(8)**

Q.9 a. Using Beta and Gamma functions show that for any positive integer 'm'

(i) $\int_0^{\pi/2} \sin^{2m-1}(\theta)\,d\theta = \frac{(2m-2)(2m-4)......2}{(2m-1)(2m-3)......3}$ **(4)**

(ii) $\int_0^{\pi/2} \sin^{2m}(\theta)\,d\theta = \frac{(2m-1)(2m-3)......1}{(2m)(2m-2)......2} \cdot \frac{\pi}{2}$ **(4)**

b. Prove that

(i) $\beta\left(m, \frac{1}{2}\right) = 2^{2m-1}\beta(m,m)$ **(4)**

(ii) $\Gamma(m)\,\Gamma(m + 1/2) = \frac{\sqrt{\pi}}{2^{2m-1}}\,\Gamma(2m)$ **(4)**