AMIETE – ET/CS/IT (OLD SCHEME)

Code: AE35/AC35/AT35 **JUNE 2011** **Subject: MATHEMATICS-**
Time: 3 Hours **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. Eliminating a and b from the $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ the partial differential equation is

(A) $2z = xp - yq$ **(B)** $2z = xp + yq$
(C) $2z = xq - yp$ **(D)** $2z = xq + yp$

b. Solution of $p\sqrt{x} + q\sqrt{y} = \sqrt{z}$ is

(A) $\sqrt{x} - \sqrt{y} = f\left(\sqrt{x} - \sqrt{z}\right)$ **(B)** $\sqrt{x} + \sqrt{y} = f\left(\sqrt{x} - \sqrt{z}\right)$
(C) $\sqrt{x} - \sqrt{y} = f\left(\sqrt{x} + \sqrt{z}\right)$ **(D)** $\sqrt{x} + \sqrt{y} = f\left(\sqrt{x} + \sqrt{z}\right)$

c. Residue of $\cot z$ at z = 0 is

(A) 1 **(B)** -1
(C) 2 **(D)** 0

d. The function w = log z is analytic everywhere except at the value of z when z is equal to

(A) -1 **(B)** 1
(C) 2 **(D)** 0

e. The value of integral $\oint_C \frac{dz}{z(z^2+4)}, C : |z| = 1$ is given by

(A) $-\frac{\pi i}{2}$ **(B)** $\frac{\pi i}{4}$
(C) $\frac{\pi i}{2}$ **(D)** $-\frac{\pi i}{4}$

f. If $f(x)=\begin{cases} kx^3, & 0<x<1 \\ 0, & \text{otherwise} \end{cases}$ is a probability density function. Then the value of k is

(A) 4 **(B)** 2
(C) 3 **(D)** 1

g. If X is a binomial variate with p = 1/5, for the experiment of 50 trials, then the standard deviation is equal to

(A) 6 **(B)** -8
(C) 8 **(D)** $2\sqrt{2}$

h. If $u\overline{F} = \nabla v$, where u, v are scalar fields and $\overline{F}$ is a vector field, the value of $\overline{F}.curl\overline{F}$ is equal to

(A) 6 **(B)** -8
(C) 8 **(D)** 0

i. The work done in moving a particle in the force field $\overline{F} = 3x^2\vec{i} + (2xz - y)\vec{j} + z\vec{k}$ along the curve $x^2 = 4y, 3x^3 = 8z$ from x = 0 to x = 2 is equal to

(A) 15 **(B)** 17
(C) 16 **(D)** 21

j. The probability mass function of a variate X is given below

X	0	1	2	3	4	5	6
P(X)	k	3k	5k	7k	9k	11k	13k

The value of k is

(A) 1/30 **(B)** 1/40
(C) 49 **(D)** 1/49

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y = a\sin(\pi x/l)$ from which it is released at time t = 0. Show that the displacement of any point at a distance x from one end at time t is given by $y(x,t) = a\sin(\pi x/l)\cos(\pi ct/l)$ **(8)**

b. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with boundary conditions u(x,0)=3sin(nπx), u(0,t)=0 and u(1,t) = 0, where 0<x<1, t>0. **(8)**

Q.3 a. X is a continuous random variable with probability density function given by $f(x)=\begin{cases} x^3 & 0 \le x \le 1 \\ (2-x)^3 & 1 \le x \le 2 \end{cases}$ find standard deviation and also the mean deviation about the mean. **(8)**

b. A target is to be destroyed in a bombing exercise. There is 75% chance that any one bomb will strike the target. Assuming that two direct hits are required to destroy the target completely. How many bomb must be dropped in order that the chance of destroying the target is $\ge 99\%$? **(8)**

Q.4 a. Solve the telephone equation $\frac{\partial^2 E}{\partial x^2} = LC\frac{\partial^2 E}{\partial t^2} + RC\frac{\partial E}{\partial t}$ when $E(0,t) = E_0 \sin qt,\ \ E = 0\, as\, x \to \infty$ assuming that $\frac{qL}{R}$ is large compared with unity. **(8)**

b. Show that the vector field defined by the vector function $\overline{v} = xyz(yz\hat{i} + xz\hat{j} + xy\hat{k})$ is conservative. **(8)**

Q.5 a. If $\overline{F} = 3y\vec{i} - xz\vec{j} + yz^2\vec{k}$ where S is the surface of the paraboloid $2z = x^2+y^2$ bounded by z = 2 evaluate $\iint_S (\nabla \times \overline{F}).dS$ using Stroke's theorem. **(8)**

b. If $\nabla.\overline{E} = 0,\ \nabla.\overline{H} = 0,\ \nabla \times \overline{E} = -\frac{\partial \overline{H}}{\partial t},\ \nabla \times \overline{H} = \frac{\partial \overline{E}}{\partial t}$, show that vector E and H satisfy the wave equation $\nabla^2 u = \frac{\partial^2 u}{\partial t^2}$ **(8)**

Q.6 a. Using the Green's theorem, show that $\oint_C \frac{\partial u}{\partial n} ds = \iint_R \nabla^2 u\, dx\, dy$ where n is the unit vector outwarad normal to C. **(8)**

b. Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin even though CR equations are satisfied thereof. **(8)**

Q.7 a. Evaluate the integral $\int_C \frac{e^z}{(z^2+\pi^2)^2} dz, C : |z| = 4$ **(8)**

b. Show that the function $f(z) = \overline{z}$ is continuous at the point z = 0, but not differentiable at z = 0. **(8)**

Q.8 a. Find Taylor's expansion of $f(z)=\dfrac{1}{(z+1)^2}$, about $z=-i$. (

b. Evaluate $\oint_C \dfrac{e^z}{\cos \pi z} dz, C:|z|=1$ **(8)**

Q.9 a. Using complex integration, compute $\int_0^{2\pi} e^{\cos\theta} \cos(\sin\theta) d\theta$ **(8)**

b. Show that under the mapping w = 1/z, all circles and straight lines in the z-plane are transformed to circles and straight lines in the w-plane. **(8)**