

Code: AE07
Time: 3 Hours

Subject: NUMERICAL ANALYSIS & COMPUTER PROGRAMMING
Max. Marks: 100

JUNE 2011

NOTE: There are 9 Questions in all.

- Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

Q.1 Choose the correct or the best alternative in the following: (2×10)

- a. The error which is already present in the statement of the problem before its solution is called

(A) inherent error (B) round-off error
(C) truncation error (D) relative error

- b. 'C' language was developed by

(A) Donald Richie (B) Dennis Pierre
(C) Dennis Richie (D) Donald Pierre

c.

- (i) In secant method we replace the function $f(x)$ by a chord
(ii) In the Newton-Raphson method we replace the function $f(x)$ by a normal

Which of the above statements are correct?

(A) Both (i) & (ii) (B) (i) only
(C) (ii) only (D) None of these

- d. An iterative method has the rate of convergence p if p is the largest positive real number for which there exists a finite constant $c \neq 0$ such that $|\epsilon_{k+1}| \leq c|\epsilon_k|^p$. consider:

- (i) ϵ_{k+1} is the error at k^{th} iteration
(ii) c is called asymptotic error.
(iii) for Newton Raphson method $p = 2$
Which of the above statements are correct?

(A) (i) only (B) (ii) only
(C) (i) & (ii) (D) (ii) & (iii)

- e. In complete pivoting we interchange the

(A) Rows only (B) Columns only
(C) Both rows and columns (D) Neither the rows nor the columns

- f. Let A be a non singular matrix. Consider $(A - \lambda I)X = 0$
- (i) $\det(A - \lambda I) = 0$ is called the characteristic equation
 - (ii) λ is called an eigen value of A
 - (iii) the smallest eigen value in modulus is called spectral radius of A .
- Which of the above statements are correct?
- (A) (i), (ii) & (iii) (B) (i) & (ii)
 (C) (ii) & (iii) (D) (ii) only
- g. (i) The degree of an interpolating polynomial through $(n+1)$ points is less than or equal to n .
- (ii) In quadratic interpolation we approximate the function $f(x)$ by a polynomial of degree two.
- Which of the above statements are correct?
- (A) (i) only (B) (ii) only
 (C) Both (i) & (ii) (D) None of these
- h. Let $P(x)$ be the polynomial due to linear interpolation of the function $f(x)$. The deviation $E = f(x) - P(x)$ is called
- (A) truncation error (B) rounding error
 (C) absolute error (D) relative error
- i. Given the following values of $f(x) = \log x$
- | | | | |
|--------|---|---------|---------|
| x | : | 2.0 | 2.2 |
| $f(x)$ | : | 0.69315 | 0.78846 |
- The approximate value of $f'(2.0)$ using the method based on linear interpolation is
- (A) 0.74566 (B) 0.64755
 (C) 0.47655 (D) 0.57644
- j. (i) The simple trapezoidal rule is based on approximating $f(x)$ by a straight line
- (ii) The simple Simpson's $\frac{1}{3}$ rule is based on approximating $f(x)$ by a quadratic curve.
- In the above, which of the statements are correct?
- (A) (i) only (B) (ii) only
 (C) Both (i) & (ii) (D) None of these

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

- Q.2** a. Write a C program to evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Trapezoidal rule. (8)
- b. Construct a Newton Raphson iteration to the $\sqrt[p]{K}$ for a positive constant K .
 Hence, estimate $\sqrt[3]{11}$ by carrying out 3 iterations, starting from $x_0 = 2$. (8)

Q.3 a. Obtain a second degree polynomial approximation to $f(x) = e^x$ on $[0, 1]$ using the Taylor series expansion about $x = 0$. Use the expansion to approximate $f(0.5)$ and find a bound of the truncation error. (8)

b. Given equation $x^4 - 2x - 1 = 0$. Use secant method to find smallest positive root of the given equation correct to three decimal places. (8)

Q.4 a. Solve the system (8)

$$2x + y + z = 10$$

$$3x + 2y + 3z = 18$$

$$x + 4y + 9z = 16$$

by the Gauss-Jordan method.

b. Solve the following system of equations by Gauss Seidel iterative method (8)

$$9x + 4y + z = -17$$

$$x - 2y - 6z = 14$$

$$x + 6y = 4$$

(Perform three iterations)

Q.5 a. Consider the equations

$$x + y + z = 1$$

$$4x + 3y - z = 6$$

$$3x + 5y + 3z = 4$$

Use the decomposition method to find the solution of the equation. (8)

b. Give the application of union data structures. (8)

Q.6 a. For the following data, calculate the differences and obtain the backward difference polynomial. Interpolate at $x = 0.35$

$$x: \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5$$

$$f(x): \quad 1.40 \quad 1.56 \quad 1.76 \quad 2.00 \quad 2.28 \quad (8)$$

b. Using $\cos 0 = 1$ and $\cos(1.5) = .0707$, find an approximate value of $\cos 1$ by Lagrange's interpolation. Obtain a bound of the truncation error. (8)

Q.7 a. Obtain a quadratic approximation of $f(x) = x^{1/2}$ on $[0, 1]$ using the method of least squares. (8)

b. Evaluate the integral $I = \int_{-1}^1 (1 - x^2)^{3/2} \cos x dx$ using Gauss-Legendre three point formula. (4+4 = 8)

Q.8 a. Evaluate the integral $I = \int_0^1 \frac{dx}{1+x}$ using composite Trapezoidal rule with $n = 4$. Find an upper bound of the error. (8)

- b. From the following table of values of x and $f(x)$, obtain $f'(1.2)$

x	$f(x)$
1.0	2.7183
1.2	3.3201
1.4	4.0552
1.6	4.9530
1.8	6.0496
2.0	7.3891
2.2	9.0250

- Q.9** a. Given $\frac{dy}{dx} = y - x$ where $y(0) = 2$, find $y(0.1)$ and $y(0.2)$ using Runge kutta method of order 4. (Take $h = 0.1$). (8)
- b. Write a short note on partial and complete pivoting. (8)