

Code: AE01/AC01/AT01
Time: 3 Hours

JUNE 2011

Subject: MATHEMATICS
Max. Marks: 100

NOTE: There are 9 Questions in all.

- Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

Q.1 Choose the correct or the best alternative in the following: (2 × 10)

- a. The value of $\lim_{(x,y) \rightarrow (0,0)} \frac{x+y}{x-y}$ is
 (A) 0 (B) 1
 (C) -1 (D) limit does not exist
- b. If $z = \log(x^2 + xy + y^2)$, then the value of $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y}$ is
 (A) 0 (B) 1
 (C) 2 (D) 4
- c. If $u = f(x-y, y-z, z-x)$, then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ is
 (A) x (B) y
 (C) z (D) 0
- d. The value of integral $\int_0^1 \int_0^{\sqrt{1-y^2}} x^3 y dx dy$ is
 (A) $\frac{1}{24}$ (B) $\frac{1}{12}$
 (C) $\frac{1}{6}$ (D) $\frac{1}{4}$
- e. The solution of the differential equation $y \frac{dy}{dx} + x = 0$ is
 (A) $x^2 - y^2 = c^2$ (B) $x^2 + y^2 = c^2$
 (C) $x^2 y^2 = c^2$ (D) None of these.

f. The solution of the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = e^{3x}$ is

(A) $y = (c_1 + c_2x)e^{3x} + xe^{3x}$

(B) $y = c_1e^x + c_2e^{-3x} + xe^{3x}$

(C) $y = c_1e^x + c_2e^{3x} + \frac{xe^{3x}}{2}$

(D) $y = c_1e^{-x} + c_2e^{3x} + \frac{x}{2}e^{3x}$

g. The rank of the matrix $\begin{bmatrix} 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 5 & 6 & 7 & 8 \\ 10 & 11 & 12 & 13 \end{bmatrix}$ is

(A) 4

(B) 3

(C) 2

(D) 1

h. If two eigen values of $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ are 2 and 3, then third eigen value is

(A) 1

(B) -1

(C) 4

(D) 6

i. The value of $\frac{d}{dx}(J_0(x))$ is

(A) $J_1(x)$

(B) $-J_1(x)$

(C) $xJ_1(x)$

(D) $-xJ_1(x)$

j. The value of $\int_{-1}^1 P_2(x)P_3(x)dx$ is

(A) 0

(B) 1

(C) -1

(D) none of these

**Answer any FIVE Questions out of EIGHT Questions.
Each Question carries 16 marks.**

Q.2 a. If z is a homogeneous function of x, y of degree n , prove using Euler's

theorem that $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = n(n-1)z$. (8)

b. If $x^x y^y z^z = c$, show that at $x=y=z$, $\frac{\partial^2 z}{\partial x \partial y} = -(x \log(ex))^{-1}$ (8)

- Q.3** a. Expand $\sin(xy)$ in power of $(x-1)$ and $\left(y - \frac{\pi}{2}\right)$, upto the second terms
- b. Change the order of integration and evaluate the integral
- $$\int_0^3 \int_1^{\sqrt{4-y}} (x+y) dx dy \quad (8)$$
- Q.4** a. Discuss the maximum and minimum values of $\sin x \sin y \sin(x+y)$ (8)
- b. Solve the differential equation $(1+x+y+xy)^2 \frac{dy}{dx} = 1$ (8)
- Q.5** a. Solve the differential equation $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = \sin x + xe^{3x}$ (8)
- b. Use method of variation of parameters to solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \log x$. (8)
- Q.6** a. Solve the differential equation $x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^2 + 2 \log x$. (8)
- b. Use elementary row transformations to find the inverse of $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ (8)
- Q.7** a. For what values of k , the equation $x+y+z=1$, $2x+y+4z=k$ and $4x+y+10z=k^2$ have a solution and solve them completely in each case. (8)
- b. Find the eigen values and eigen vectors of $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ (8)
- Q.8** a. Define a unitary matrix and show that $\begin{bmatrix} \alpha + i\gamma & -\beta + i\delta \\ \beta + i\delta & \alpha - i\gamma \end{bmatrix}$ is unitary matrix if $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1$. (8)
- b. Solve in series the equation $(1-x^2) \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} + y = 0$. (8)

Q.9 a. Show that $J_n(x) = \frac{x}{2n} [J_{n-1}(x) + J_{n+1}(x)]$

b. Show that $\int_{-1}^{+1} x P_n(x) P_{n-1}(x) dx = \frac{2n}{4n^2 - 1}$ (8)