AMIETE – CS (NEW SCHEME) – Code: AC65

Subject: DISCRETE STRUCTURES

Time: 3 Hours **Max. Marks: 100**

JUNE 2011

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions, answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. If R is a relation "Less Than" on A = {1, 2, 3, 4} then which of the following is not a partial order relation?

(A) {(1, 1), (2, 2), (3, 3), (4, 4), (3, 4)}
(B) {(1, 1), (2, 2), (3, 3), (4, 4), (3, 4), (4, 3)}
(C) {(1, 1), (2, 2), (3, 3), (4, 4), (3, 4), (1, 2)}
(D) {(1, 1), (2, 2), (3, 3), (4, 4), (1, 4), (4, 3), (1, 3)}

b. Number of reflexive relations that can be defined on a set having 4 elements is

(A) 16 **(B)** 10
(C) 12 **(D)** None of these

c. Composition of functions follows the following rules:

(A) Commutative but not Associative
(B) Commutative and Associative
(C) Not Commutative and Associative
(D) None of the above

d. The n^{th} term of the sequence 5, 13, 25, 41,….. is given by the recursive formula

(A) $f(n) = 4n + f(n-1)$ **(B)** $f(n) = 4n + f(n-1);\ f(0) = 1$
(C) $f(n) = 4n + f(n-1);\ f(1) = 1$ **(D)** $f(n+1) = 4n + f(n-1);\ f(0) = 1$

e. Which one is the contra positive of $q \to p$?

(A) $p \to q$ **(B)** $\neg p \to \neg q$
(C) $\neg q \to \neg p$ **(D)** None of these

f. Ramesh and Suresh took an examination and probability of their passing is [illegible] and 0.27 respectively. What is the probability that both will not fail in th[e] examination?

(**A**) 0.324 (**B**) 0.0324
(**C**) 0.97 (**D**) 0.39

g. If f: $G_1 \rightarrow G_2$ be a homomorphism from group G_1 to G_2 then identity element in group G_2 is given by

(**A**) $f(e_1)$, where e_1 is identity element in G_1.
(**B**) f(a), where a is any element in G_1.
(**C**) There is no relationship between elements of the two sets.
(**D**) $f(e_1)$, where e_1 is inverse of identity element in G_2.

h. Statement $p \rightarrow q \leftrightarrow \neg p \vee q$ is

(**A**) A tautology (**B**) A contingency
(**C**) A contradiction (**D**) None of the above

i. The inference "If you repent, you will go to heaven. You have repented. So you will go to heaven" is derived using

(**A**) Modus Ponens (**B**) Hypothetical Syllogism
(**C**) Modus Tollens (**D**) Theory of Attachment

j. An (m, m+1) encoding function e: $B^m \rightarrow B^{m+1}$ is even parity check code generator. The code for data 110 is then

(**A**) 1111 (**B**) 0110
(**C**) 1001 (**D**) 1100

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Prove or disprove the followings:

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $(A \cap B)^C = A^C \cup B^C$ **(4×2)**

b. An equipment has two components A and B attached in sequence. The probability that they function is 0.95 and 0.93 respectively. What is probability that the equipment will work? **(8)**

Q.3 a. Show the equivalence of the following statements:
(i) $((p \rightarrow q) \wedge p) \rightarrow q \leftrightarrow T$
(ii) $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow p \rightarrow r$ **(4×2)**

b. Convert the following statement into proposition using standard symbols
(i) If I appear in the examination then I will pass.
(ii) I will not pass the examination unless I appear in the examination.
(iii) If it is Sunday then I will go to temple. Today is Sunday.
(iv) Mohan will be late or he will not come to office today. **(2×4)**

Q.4 a. Prove that sum of a rational number and an irrational number is irrational. **(8)**

b. Verify the validity of the arguments: "X will pay Rs 1000 to Y if India defeats Sri Lanka in the Test. India won the Test against Sri Lanka. Therefore X paid Rs. 1000 to Y." **(8)**

Q.5 a. Solve the recurrence equation $f(n) = f(n - 1) + 5n$; $f(0) =1$. **(8)**

b. Use Mathematical Induction to prove that $10^{2n-1}+1$ is divisible by 11 for all natural number n. **(8)**

Q.6 a. Let R be a relation defined on set of Integer Z as for any two integer x and y, xRy iff $x*y = 0$. Show that R is not a transitive relation. **(8)**

b. Define lattice. Show that (R, ≤) is a lattice where R is set of real numbers and ≤ (less than equal to) is partial order relation. **(8)**

Q.7 a. Define an invertible function. Given a function $f: Z\rightarrow Z$ as $f(x) = 5x - 2$, determine whether the function is invertible on the set of integers Z. **(8)**

b. What is total function? Show that addition of two integers is a computable function. **(8)**

Q.8 a. Show that a subset H of group (G, *) is subgroup of G if and only if $a*b^{-1}$ is in H for all elements a, b of H. **(8)**

b. Define generator of a group. Show that the set $\{1, w, w^2\}$ forms a cyclic group under binary operation of multiplication of number, where 1, w and w^2 are cube roots of unity. **(8)**

Q.9 a. Define Group Codes. Show that (3, 6) encoding function $e: B^3\rightarrow B^6$ defined by
e(000) = 000000, e(001) = 001100, e(010) = 010011, e(011) = 011111,
e(100) = 100101, e(101) = 101001, e(110) = 110110, e(111) = 111010
is a group code. **(8)**

b. Show that identities of both the binary operators in a ring (R, +, *) with unity are unique. **(8)**