Code: AC09/AT09 **Subject: NUMERICAL COMPUTI**
Time: 3 Hours **Max. Marks: 10**

JUNE 2011

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2 × 10)**

a. The error quantity which must be added to the true representation of the quantity in order that the result be exactly equal to the quantity we are seeking to generate is called

(A) truncation error **(B)** round-off error
(C) relative error **(D)** absolute error

b. The iterative method for finding a root of an equation which has linear rate of convergence will be

(A) Secant method **(B)** Regula-falsi method
(C) Newton-Rapson method **(D)** none of the above

c. In case of the direct method for obtaining the solution of the matrix equation Ax = b, on its completion the original matrix becomes an identity matrix when it is

(A) Gauss elimination method **(B)** Gauss-Jordan method
(C) Doolittle's method **(D)** Crout's method

d. Consider the following two statements
(i) The Gauss-Seidel iterative method converges for the solution of the matrix equation Ax = b for any initial starting vector $x^{(0)}$ if the matrix A is strictly diagonally dominant.
(ii) The rate of convergence of Gauss-Seidel scheme is twice that of Jacobi scheme.
Identify the correct statement(s).

(A) both the statements are true **(B)** only statement (i) is true
(C) only statement (ii) is true **(D)** both the statements are false

e. The eigenvalues of the following matrix are

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 8 & 2 & -1 \end{bmatrix}$$

(A) 1, -2, 8 **(B)** 1, -2, 2
(C) 1, 1, -1 **(D)** none of these

f. Given the following values of $f(x)$,

i	0	1	2
x_i	2.0	2.2	2.6
$f(x_i)$	0.69315	0.78846	0.95551

the approximate value of $f'(2.0)$ using linear interpolation will be

(A) 0.4825 **(B)** 0.5
(C) 0.5125 **(D)** none of the above

g. Identify the number of correct statements among the following:
(i) Trapezoidal rule is exact only for polynomials of degree less than or equal to one.
(ii) Simpson's rule is exact only for polynomials of degree less than or equal to two.
(iii) The Newton-Cotes formula which approximates $\int_a^b f(x)dx$ to be the area of the region with width $(b-a)$ and ordinates $f(a), f(b)$ is Simpson's rule.

(A) one **(B)** two
(C) three **(D)** none

h. The nth divided difference $f[x_0, x_1, x_2, ... x_n]$ can be written in terms of function values as

(A) $\sum_{i=0}^{n} f(x_i) \prod_{j=0}^{n} (x_i - x_j)$ **(B)** $\sum_{i=0}^{n} \frac{f(x_i)}{\prod_{j=0, j\neq i}^{n} (x_i - x_j)}$

(C) $\prod_{j=0}^{n} \left(\sum_{i=0, i\neq j}^{n} \frac{f(x_i)}{(x_i - x_j)} \right)$ **(D)** none of the above

i. The Legendre polynomials $P_n(x)$ with $P_0(x) = 1$ and $P_1(x) = x$ satisfy the recurrence relation

(A) $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$
(B) $(n+1)P_{n+1}(x) = (2n-1)xP_n(x) - nP_{n-1}(x)$
(C) $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) + nP_{n-1}(x)$
(D) $(n+1)P_{n+1}(x) = (2n-1)xP_n(x) + nP_{n-1}(x)$

j. Which of the method(s) is/are used for finding solutions of differential equations

(A) Gauss-Jacobi method **(B)** Gauss-Hermite method
(C) Euler method **(D)** None of these

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. A real root of the equation

$$f(x) = x^3 - 5x + 1 = 0$$

lies in the interval (0,1). Perform 4 iterations of the Secant method to obtain this root. **(8)**

b. Apply Newton-Raphson method to determine a root of the equation

$$f(x) = \cos x - xe^x = 0$$

upto 3 iterations whose initial approximation is taken as $x_0 = 1$. **(8)**

Q.3 a. Solve the following system of linear equations by using Gauss elimination method

$$2x_1 + x_2 + x_3 = 5$$
$$4x_1 - 6x_2 = -2$$
$$-2x_1 + 7x_2 + 2x_3 = 9$$

(8)

b. Using Gauss-Seidel iterative method, solve the following system of linear equations

$$20x_1 + 2x_2 + 6x_3 = 28$$
$$x_1 + 20x_2 + 9x_3 = -23$$
$$2x_1 - 7x_2 - 20x_3 = -57$$

by starting from the initial guess to the solution as (0,0,0), compute the approximate solutions up to 3 iterations. **(8)**

Q.4 a. Show that the matrix $\begin{bmatrix} 12 & 4 & -1 \\ 4 & 7 & 1 \\ -1 & 1 & 6 \end{bmatrix}$ is positive-definite. **(6)**

b. Find the largest eigenvalue and its corresponding eigenvector for the matrix $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ using Power method starting with the approximation as (1,0,0). **(10)**

Q.5 a. Consider the four point formula

$$f'(x_2) = \frac{1}{6h}[-2f(x_1) - 3f(x_2) + 6f(x_3) - f(x_4)] + TE + RE$$

where $x_j = x_0 + jh$, $j = 1,2,3,4$ and TE, RE are respectively the truncation error and round off error. Determine the form of TE and RE. **(7)**

b. Solve the following matrix equation using Cholesky decomposition method.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ -10 \end{bmatrix}$$ **(9)**

Q.6 a. Use Newton's divided difference formula to find the interpolating polynomial of degree 3 for the data given by:
$f(0)=1, \quad f(2)=2$
$f(1)=1, \quad f(4)=5$ **(10)**

b. Show that

(i) $\delta = \nabla(1-\nabla)^{-1/2}$ (ii) $\mu = \left[1 + \frac{\delta^2}{4}\right]^{1/2}$ **(6)**

Q.7 a. Find the least squares straight line fit for the following data given by **(8)**

x	0.2	0.4	0.6	0.8	1.0
$f(x)$	0.447	0.632	0.775	0.894	1.0

b. For the following data given in tabular form

x	0.4	0.6	0.8
$f(x)$	0.0256	0.1296	0.4096

find $f'(0.8)$ and $f''(0.8)$ using quadratic interpolation method. **(8)**

Q.8 a. Find the approximate value of

$$I = \int_0^1 \frac{dx}{1+x}$$

using (i) Trapezoidal rule (ii) Simpson's rule **(6)**

b. Evaluate the following integral given by

$$I = \int_1^2 \frac{2x\,dx}{1+x^4}$$

using Gauss-Legendre 1-point, 2-point and 3-point quadrature rules. **(10)**

Q.9 a. Using Taylor's series, find the solution of the differential equation, given by
$y^1 = y^2$, $y(0)=1$ at x = 0.01 **(8)**

b. For the equation given by $\frac{dy}{dx} = x - y$, $y(0)=1$, find the value of y when x = 0.1 with h = 0.1 using Runge-Kutta fourth order method. **(8)**