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MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 1

Monday 11 November 2013 (afternoon)

1 hour 30 minutes

Candidate session number

0	0								

Examination code

8	8	1	3	–	7	4	0	1

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **Mathematical Studies SL** *information booklet* is required for this paper.
- Answer all questions.
- Write your answers in the boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is *[90 marks]*.

24EP01

Please **do not** write on this page.

Answers written on this page
will not be marked.

Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Write your answers in the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. A survey was carried out on a road to determine the number of passengers in each car (excluding the driver). The table shows the results of the survey.

Number of passengers	0	1	2	3	4
Number of cars	37	23	36	15	9

(a) State whether the data is discrete or continuous. *[1]*

(b) Write down the mode. *[1]*

(c) Use your graphic display calculator to find

    (i) the mean number of passengers per car;

    (ii) the median number of passengers per car;

    (iii) the standard deviation. *[4]*

Working:

Answers:

(a) ....

(b) ....

(c) (i) ....

    (ii) ....

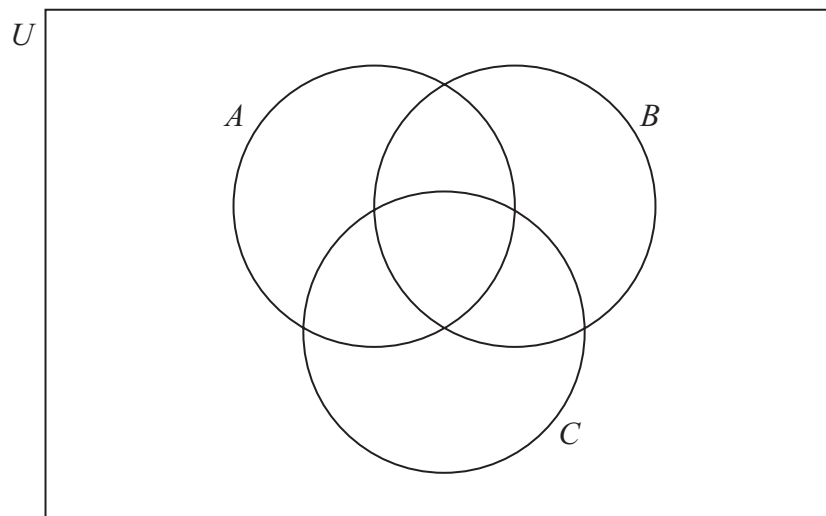   (iii) ....

24EP03

Turn over

2. U is the set of **positive** integers less than or equal to 10.
A, B and C are subsets of U.

$$A = \{\text{even integers}\}$$
$$B = \{\text{multiples of 3}\}$$
$$C = \{6, 7, 8, 9\}$$

(a) List the elements of A. *[1]*

(b) List the elements of B. *[1]*

(c) Complete the Venn diagram with **all** the elements of U. *[4]*

Working:

Answers:

(a) . . . .

(b) . . . .

3. Consider the propositions

p : *I have a bowl of soup.*
q : *I have an ice cream.*

(a) Write down, in words, the compound proposition $\neg p \Rightarrow q$.   *[2]*

(b) Complete the truth table.

p	q	$\neg p$	$\neg p \Rightarrow q$
T	T		
T	F		
F	T		
F	F		

[2]

(c) Write down, in symbolic form, the converse of $\neg p \Rightarrow q$.   *[2]*

Working:

Answers:

(a) . . . .

. . . .

. . . .

. . . .

. . . .

(c) . . . .

Turn over

4.　240 cars were tested to see how far they travelled on 10 litres of fuel.　The graph shows the cumulative frequency distribution of the results.
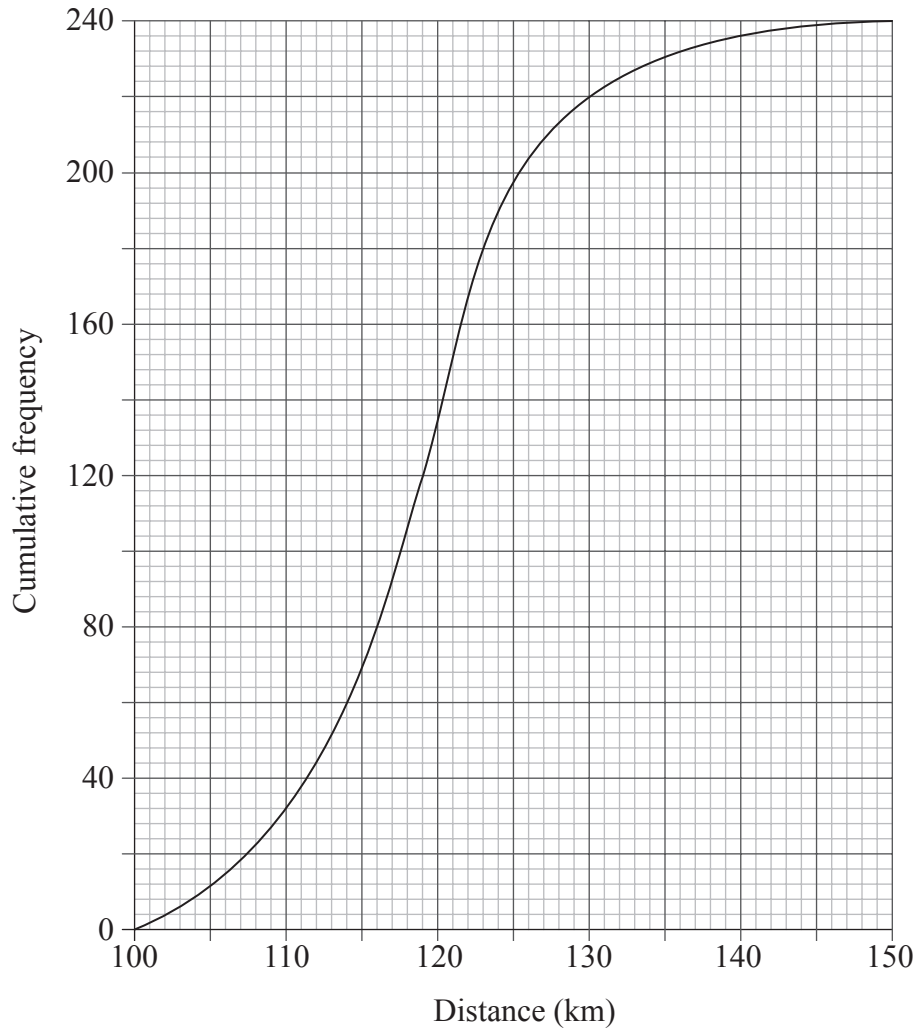
(a)　Find the median distance travelled by the cars.　　　　　　　　　　　　　*[2]*

(b)　Calculate the interquartile range of the distance travelled by the cars.　　　　*[2]*

(c)　Find the number of cars that travelled more than 130 km.　　　　　　　　*[2]*

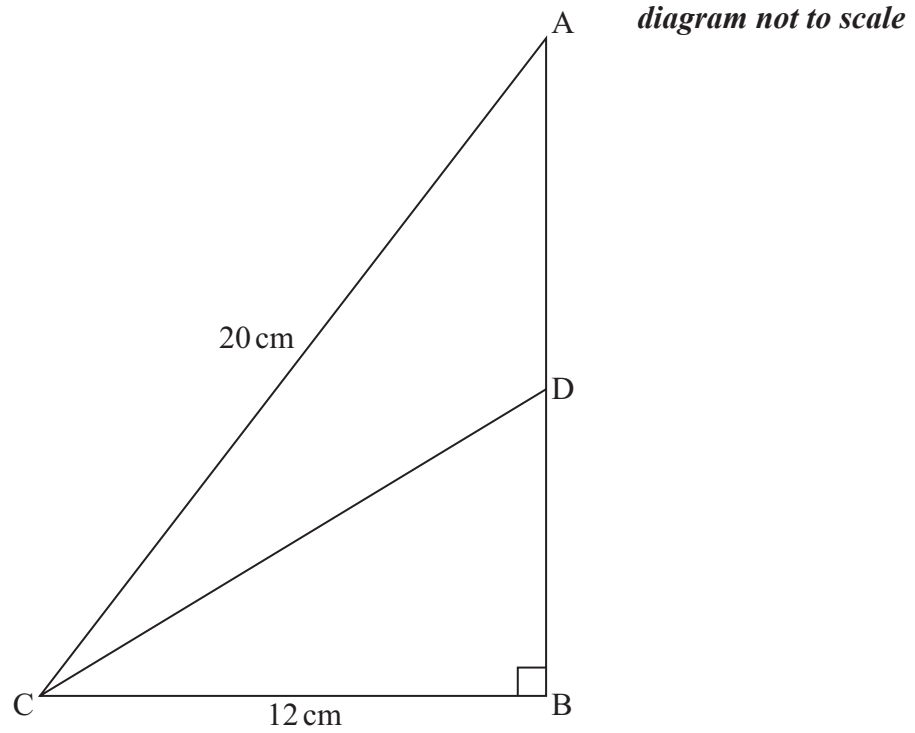*(This question continues on the following page)*

24EP06

(Question 4 continued)

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

Turn over

5. In triangle ABC, $AC = 20\,cm$, $BC = 12\,cm$ and $A\hat{B}C = 90°$.

diagram not to scale

(a)   Find the length of AB.   *[2]*

D is the point on AB such that $\tan(D\hat{C}B) = 0.6$.

(b)   Find the length of DB.   *[2]*

(c)   Find the area of triangle ADC.   *[2]*

(This question continues on the following page)

(Question 5 continued)

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

6. The first term, u_1, of an arithmetic sequence is 145. The fifth term, u_5, of the sequence is 113.

(a) Find the common difference of the sequence. *[2]*

The nth term, u_n, of the sequence is -7.

(b) Find the value of n. *[2]*

(c) Find S_{20}, the sum of the first twenty terms of the sequence. *[2]*
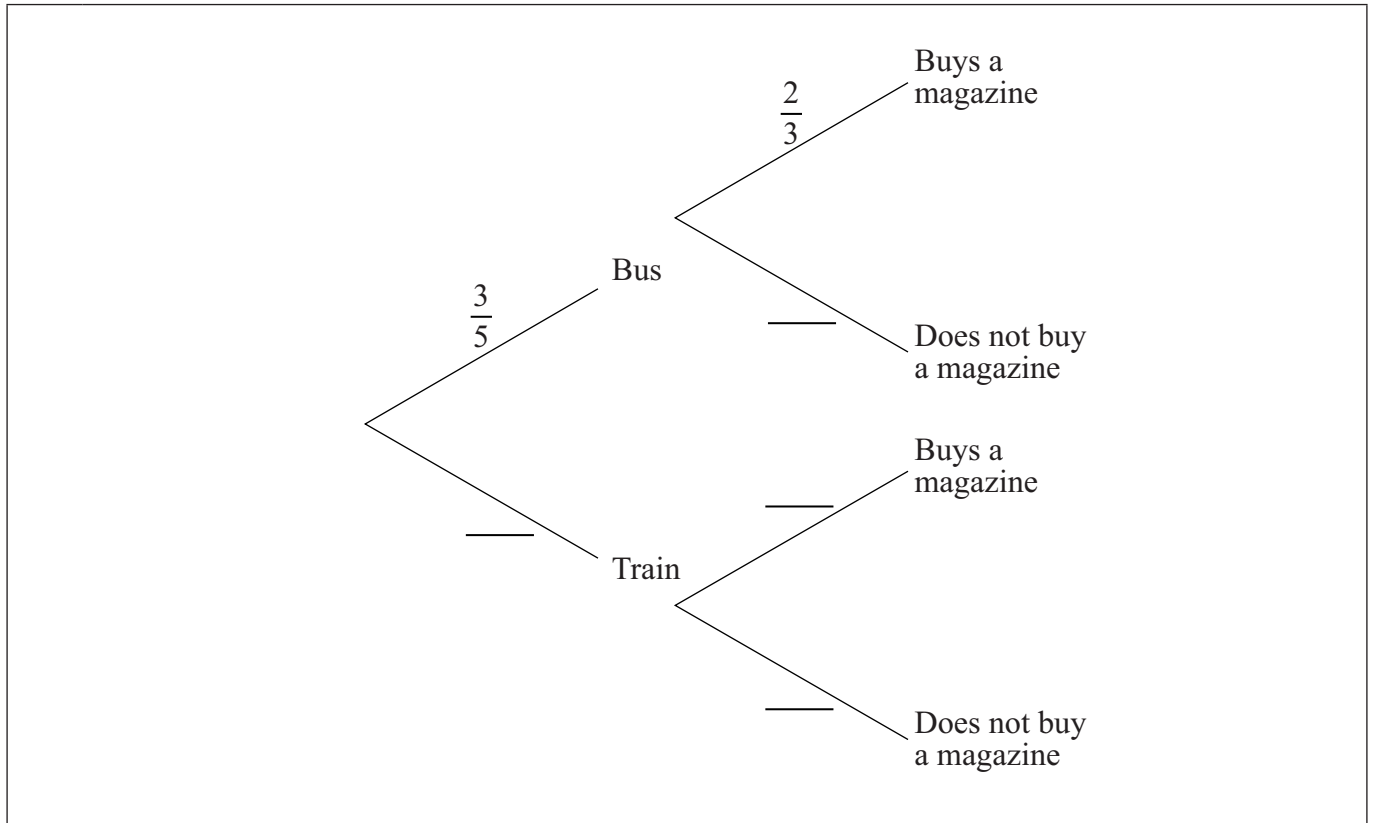
Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

7. Ramzi travels to work each day, either by bus or by train. The probability that he travels by bus is $\frac{3}{5}$. If he travels by bus, the probability that he buys a magazine is $\frac{2}{3}$. If he travels by train, the probability that he buys a magazine is $\frac{3}{4}$.

(a) Complete the tree diagram. *[3]*

(b) Find the probability that Ramzi buys a magazine when he travels to work. *[3]*

Working:

Answers:

(b) . . . .

Turn over

8. 180 spectators at a swimming championship were asked which, of four swimming styles, was the one they preferred to watch.

The results of their responses are shown in the table.

Swimming style	Male	Female
Freestyle	20	15
Butterfly	20	30
Backstroke	10	35
Breaststroke	10	40

A χ^2 test was conducted at the 5% significance level.

(a) Write down the null hypothesis for this test. *[1]*

(b) Write down the number of degrees of freedom. *[1]*

(c) Write down the value of χ^2_{calc}. *[2]*

The critical value, at the 5% significance level, is 7.815.

(d) State, giving a reason, the conclusion to the test. *[2]*

(This question continues on the following page)

(Question 8 continued)

Working:

Answers:

(a) . . . .

. . . .

. . . .

(b) . . . .

(c) . . . .

(d) . . . .

. . . .

. . . .

9. (a) Expand the expression $x(2x^3 - 1)$. *[2]*

(b) Differentiate $f(x) = x(2x^3 - 1)$. *[2]*

(c) Find the *x*-coordinate of the local minimum of the curve $y = f(x)$. *[2]*

Working:

Answers:

(a) . . . .

(b) . . . .

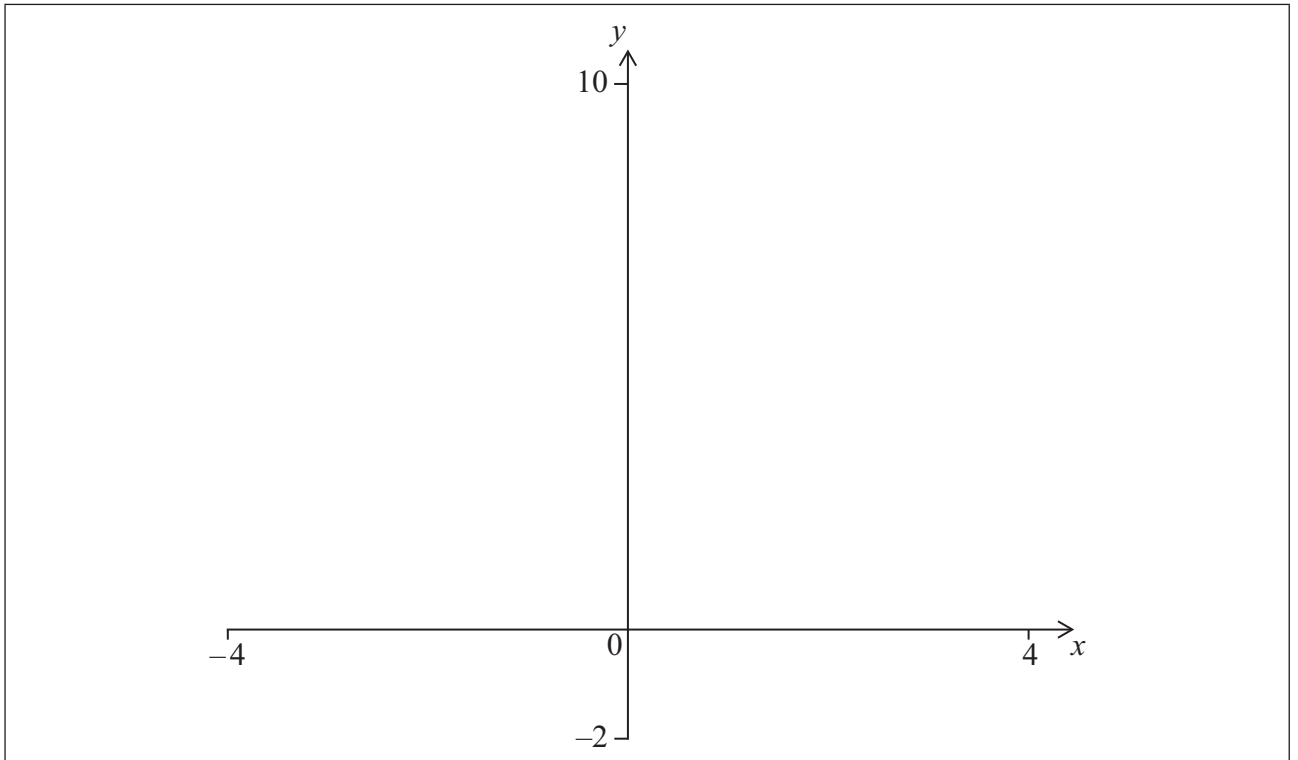(c) . . . .

10. Consider the two functions, f and g, where

$$f(x) = \frac{5}{x^2 + 1}$$

$$g(x) = (x-2)^2$$

(a) Sketch the graphs of $y = f(x)$ and $y = g(x)$ on the axes below. Indicate clearly the points where each graph intersects the y-axis. *[4]*

(b) Use your graphic display calculator to solve $f(x) = g(x)$. *[2]*

Working:

Answers:

(b) ....

Turn over

11. 512 competitors enter round 1 of a tennis tournament, in which each competitor plays a match against one other competitor.

The winning competitor progresses to the next round (round 2); the losing competitor leaves the tournament.

The tournament continues in this manner until there is a winner.

(a) Find the number of competitors who play in round 6 of the tournament. *[3]*

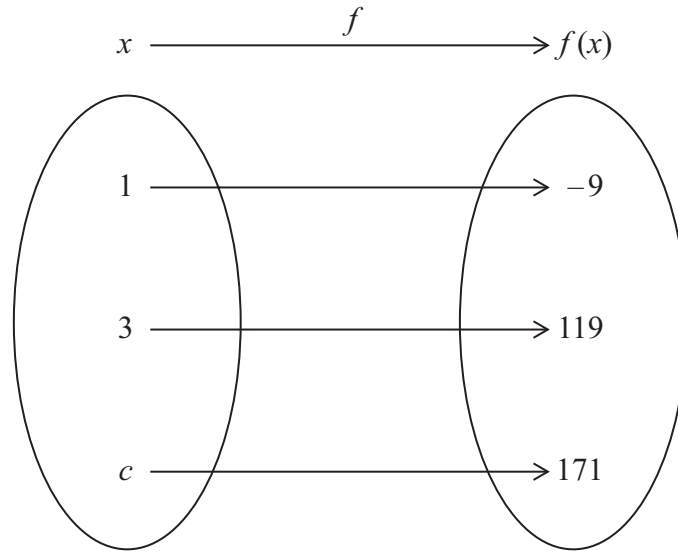(b) Find the total number of matches played in the tournament. *[3]*

Working:

Answers:

(a) ....

(b) ....

12. A quadratic function $f : x \mapsto ax^2 + b$, where a and $b \in \mathbb{R}$ and $x \geq 0$, is represented by the mapping diagram.

$$x \xrightarrow{\hspace{1cm} f \hspace{1cm}} f(x)$$

$$
\begin{array}{ccc}
1 & \longrightarrow & -9 \\
3 & \longrightarrow & 119 \\
c & \longrightarrow & 171
\end{array}
$$

(a)   Using the mapping diagram, write down two equations in terms of a and b.   *[2]*

(b)   Solve the equations to find the value of

(i) a;

(ii) b.   *[2]*

(c)   Find the value of c.   *[2]*

Working:

Answers:

(a)   . . . .

. . . .

(b)   (i)   . . . .

(ii)   . . . .

(c)   . . . .

13. The table shows the **monthly** repayments for every $10 000 borrowed at the nominal annual interest rates and for the loan terms shown.

Loan term (years)	Monthly repayment in $, for every $10 000		
	Nominal annual interest rate		
	3 %	5 %	7 %
5	179.69	188.72	198.02
10	96.57	106.07	116.11
15	69.06	79.08	89.89
20	55.46	66.00	77.53
25	47.43	58.46	70.68

Zachary borrows $80 000 at a nominal annual interest rate of 5 %, to be repaid over a 20 year term.

(a)   **Using the table**, calculate the **total interest** that Zachary pays.   *[3]*

Finley borrows a sum of money at a nominal annual interest rate of 7 % over a 10 year term. His monthly repayment must not be more than $500.

(b)   **Using the table**, calculate the maximum amount that Finley can borrow. Give your answer correct **to the nearest $100**.   *[3]*

(This question continues on the following page)
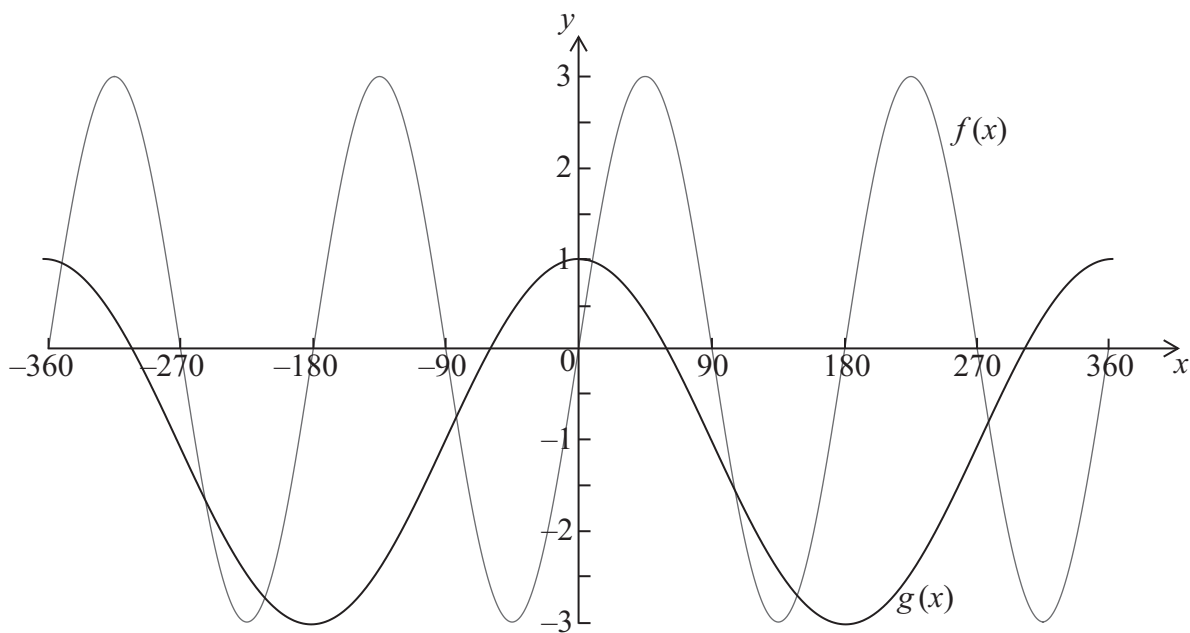
(Question 13 continued)

Working:

Answers:

(a) . . . .

(b) . . . .

Turn over

14. The diagram shows the graph of a cosine function, $g(x) = a\cos bx + c$ for $-360° \le x \le 360°$, and a sine function, $f(x)$.

(a) Write down

    (i) the amplitude of $f(x)$;

    (ii) the period of $f(x)$. *[2]*

(b) Write down the value of

    (i) a;

    (ii) b;

    (iii) c. *[3]*

(c) Write down the number of solutions of $f(x) = g(x)$ in the domain $-180° \le x \le 360°$. *[1]*

(This question continues on the following page)

(Question 14 continued)

Working:

Answers:

(a) (i)   . . . .

     (ii)   . . . .

(b) (i)   . . . .

     (ii)   . . . .

     (iii) . . . .

(c)   . . . .

15. A computer virus spreads according to the exponential model

$$N = 200 \times (1.9)^{0.85t}, \ t \geq 0$$

where N is the number of computers infected, and t is the time, in hours, after the initial infection.

(a) Calculate the number of computers infected after 6 hours. *[2]*

(b) Calculate the time for the number of infected computers to be greater than $1\,000\,000$. Give your answer correct to the nearest hour. *[4]*

Working:

Answers:

(a) . . . .

(b) . . . .