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MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 2

Friday 6 November 2009 (morning)

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

• Do not open this examination paper until instructed to do so.
• Answer all the questions.
• A graphic display calculator is required for this paper.
• Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
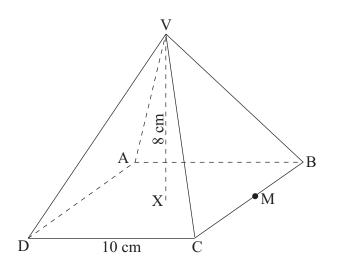
Please start each question on a new page. You are advised to show all working, where possible. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1.   *[Maximum mark: 16]*

   Part A

   The diagram below shows a square based right pyramid. ABCD is a square of side 10 cm. VX is the perpendicular height of 8 cm. M is the midpoint of BC.

diagram not to scale

   (a)   Write down the length of XM.   *[1 mark]*

   (b)   Calculate the length of VM.   *[2 marks]*

   (c)   Calculate the angle between VM and ABCD.   *[2 marks]*

(This question continues on the following page)

(Question 1 continued)

Part B

A path goes around a forest so that it forms the three sides of a triangle. The lengths of two sides are 550 m and 290 m. These two sides meet at an angle of 115°. A diagram is shown below.
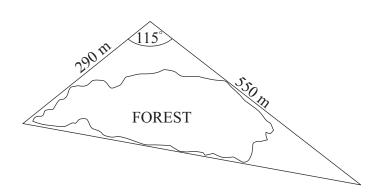
diagram not to scale

(a)   Calculate the length of the third side of the triangle. Give your answer correct to the nearest 10 m.   *[4 marks]*

(b)   Calculate the area enclosed by the path that goes around the forest.   *[3 marks]*

Inside the forest a second path forms the three sides of another triangle named ABC. Angle BAC is 53°, AC is 180 m and BC is 230 m.
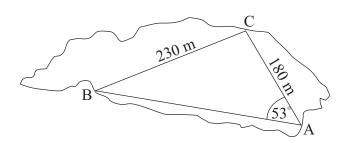
diagram not to scale
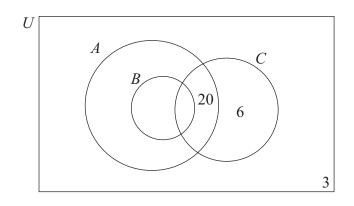

(c)   Calculate the size of angle ACB.   *[4 marks]*

2.   *[Maximum mark: 15]*

Part A

The Venn diagram below represents the students studying Mathematics (A), Further Mathematics (B) and Physics (C) in a school.

  50 students study Mathematics
  38 study Physics
  20 study Mathematics and Physics but not Further Mathematics
  10 study Further Mathematics but not Physics
  12 study Further Mathematics and Physics
  6 study Physics but not Mathematics
  3 study none of these three subjects.

(a)   Copy and complete the Venn diagram **on your answer paper**.   *[3 marks]*

(b)   Write down the number of students who study Mathematics but not Further Mathematics.   *[1 mark]*

(c)   Write down the total number of students in the school.   *[1 mark]*

(d)   Write down $n(B \cup C)$.   *[2 marks]*

(This question continues on the following page)

(Question 2 continued)

Part B

Three propositions are given as

p : It is snowing　　　　　q : The roads are open　　　　　r : We will go skiing

(a)　Write the following compound statement in symbolic form.

"It is snowing and the roads are not open."　　　　　*[2 marks]*

(b)　Write the following compound statement in words.

$$(\neg p \wedge q) \Rightarrow r$$　　　　　*[3 marks]*

An incomplete truth table for the compound proposition $(\neg p \wedge q) \Rightarrow r$ is given below.

(c)　Copy and complete the truth table **on your answer paper**.　　　　　*[3 marks]*

p	q	r	$\neg p$	$\neg p \wedge q$	$(\neg p \wedge q) \Rightarrow r$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

3. *[Maximum mark: 14]*

The height, $h(t)$, in centimetres, of a bicycle pedal above the ground at time, t, seconds is a cosine function of the form $h(t) = A\cos(bt) + C$, where (bt) is measured in degrees.

The graph of this function for $0 \leq t \leq 4.3$ is shown below.

(a)   Write down the maximum height of the pedal above the ground.   *[1 mark]*

(b)   Write down the minimum height of the pedal above the ground.   *[1 mark]*

(c)   Find the amplitude of the function.   *[2 marks]*

(d)   Hence or otherwise, find the value of A and of C.   *[2 marks]*

(e)   (i)   Write down the period of the function $h(t)$.

(ii)   Hence find the value of b.   *[3 marks]*

(f)   Use your graphic display calculator to determine the first value of t for which the height of the pedal above the ground is 30 cm.   *[3 marks]*

(g)   Calculate the number of times the pedal rotates in one minute.   *[2 marks]*

4. *[Maximum mark: 21]*

Part A

In a mountain region there appears to be a relationship between the number of trees growing in the region and the depth of snow in winter. A set of 10 areas was chosen, and in each area the number of trees was counted and the depth of snow measured. The results are given in the table below.

Number of trees (x)	Depth of snow in cm (y)
45	30
75	50
66	40
27	25
44	30
28	5
60	35
35	20
73	45
47	25

(a) Use your graphic display calculator to find

　　(i)　the mean number of trees;

　　(ii)　the standard deviation of the number of trees;

　　(iii)　the mean depth of snow;

　　(iv)　the standard deviation of the depth of snow. *[4 marks]*

The covariance, $S_{xy} = 188.5$.

(b) Write down the product-moment correlation coefficient, r. *[2 marks]*

(c) Write down the equation of the regression line of y on x. *[2 marks]*

(d) If the number of trees in an area is 55, estimate the depth of snow. *[2 marks]*

(e) (i) Use the equation of the regression line to estimate the depth of snow in an area with 100 trees.

　　(ii)　Decide whether the answer in (e)(i) is a valid estimate of the depth of snow in the area. Give a reason for your answer. *[3 marks]*

(This question continues on the following page)

(Question 4 continued)

Part B

In a study on 100 students there seemed to be a difference between males and females in their choice of favourite car colour. The results are given in the table below. A χ^2 test was conducted.

	Blue	Red	Green
Males	14	6	8
Females	31	24	17

(a) Write down the total number of male students.   *[1 mark]*

(b) Show that the expected frequency for males, whose favourite car colour is blue, is 12.6.   *[2 marks]*

The calculated value of χ^2 is 1.367.

(c) (i) Write down the null hypothesis for this test.

(ii) Write down the number of degrees of freedom.

(iii) Write down the critical value of χ^2 at the 5 % significance level.

(iv) Determine whether the null hypothesis should be accepted. Give a reason for your answer.   *[5 marks]*

5. *[Maximum mark: 24]*

Part A

(a) Sketch the graph of $y = 2^x$ for $-2 \le x \le 3$. Indicate clearly where the curve intersects the y-axis. *[3 marks]*

(b) Write down the equation of the asymptote of the graph of $y = 2^x$. *[2 marks]*

(c) On the same axes sketch the graph of $y = 3 + 2x - x^2$. Indicate clearly where this curve intersects the x and y axes. *[3 marks]*

(d) Using your graphic display calculator, solve the equation $3 + 2x - x^2 = 2^x$. *[2 marks]*

(e) Write down the maximum value of the function $f(x) = 3 + 2x - x^2$. *[1 mark]*

(f) Use Differential Calculus to verify that your answer to (e) is correct. *[5 marks]*

Part B

The curve $y = px^2 + qx - 4$ passes through the point $(2, -10)$.

(a) Use the above information to write down an equation in p and q. *[2 marks]*

The gradient of the curve $y = px^2 + qx - 4$ at the point $(2, -10)$ is 1.

(b) (i) Find $\dfrac{dy}{dx}$.

(ii) Hence, find a second equation in p and q. *[3 marks]*

(c) Solve the equations to find the value of p and of q. *[3 marks]*