INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
BACHILLERATO INTERNACIONAL

MARKSCHEME

November 2001

MATHEMATICAL STUDIES

Standard Level

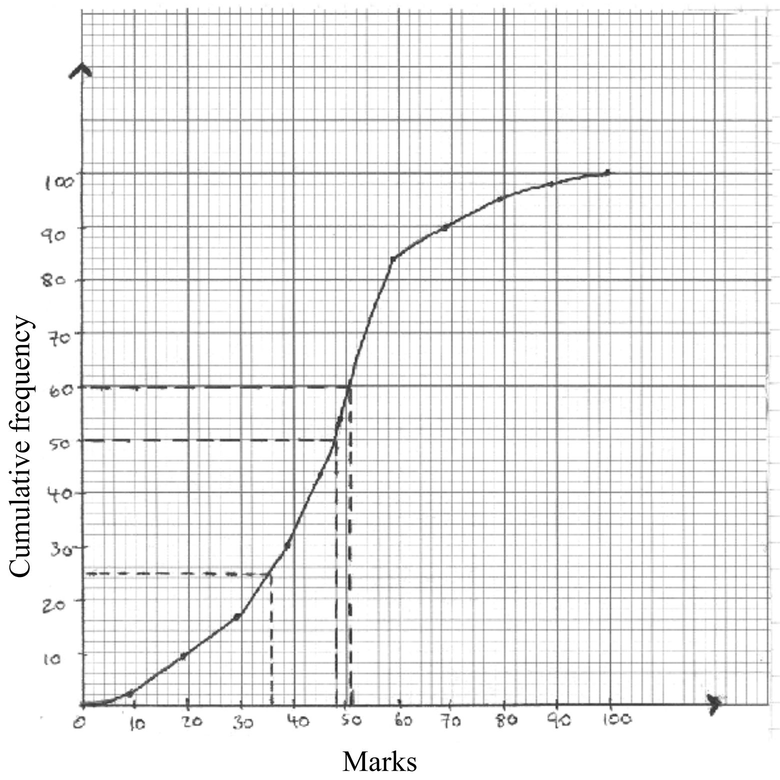
Paper 2

23 pages

1. (i) (a) $s = 17, \quad t = 90$ ***(A1)(A1)***

[2 marks]

(b)



(M1)(M2)

Note:	Award ***(M1)*** for both axes with correct scales and correctly labelled. Award ***(M2)*** for 8, 9, 10 points plotted correctly, ***(M1)*** for 5, 6, 7 points plotted correctly, ***(M0)*** for 4 or less. Accept a polygon or a curve.

[3 marks]

(c) (i) Median mark $= 48\,(\pm 1)$ ***(A1)***

(ii) Lower quartile $= 36\,(\pm 1)$ ***(A1)***

(iii) Pass mark if 40 % pass $= 51\,(\pm 1)$ ***(M1)(A1)***

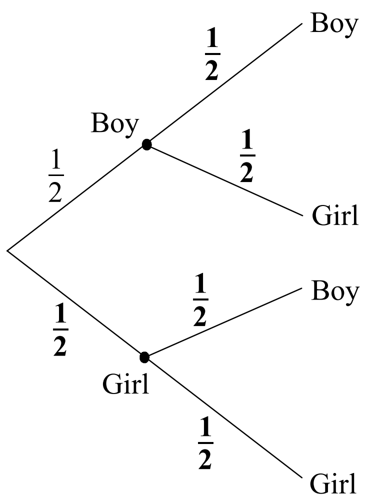
Note:	Follow through with candidate's own graph. Award ***(M0)(A1)*** **ft** if candidate correctly finds the grade (44) where **40 % fail**.

[4 marks]

continued…

Question 1 continued

(ii) (a)



(A2)

[2 marks]

Note:	Award ***(A2)*** for 5 or 4 correct probabilities, ***(A1)*** for 3 or 2, ***(A0)*** for 1 or 0.

(b)

Note to examiners marking in Spanish

The question has been changed in Spanish to ask about 'girls' (niñas) instead of 'boys' (niños). This is due to translation problems.

(b) (i) P(2 girls)
(ii) P(2 girls/first child is a girl)
(iii) P(2 girls/girl in family)

(i) P(2 boys) $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ ***(A1)***

(ii) P(2 boys | first child is boy) $= \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$ ***(A1)***

(iii) P(2 boys | boy in family) $= \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$ ***(A1)***

Note:	Answers can be obtained logically without using conditional probability. Award full marks for correct answers.

[3 marks]

Total [14 marks]

2. (i) (a) $n(\text{MTV} \cap \text{BBC}) = 11$ ***(A1)***

(b) $n(\text{MTV} \cup \text{BBC}) = 74$ ***(A1)***

(c) $n(\text{CNN} \cap \text{BBC} \cap \text{MTV}') = 2$ ***(A1)***

(d) $n\big((\text{MTV} \cup \text{CNN}) \cap \text{BBC}'\big) = 77$ ***(A1)***

[4 marks]

(ii) (a) (i) *If you do not watch the music TV channel,* ***(C1)***
then you do not like music. ***(C1)***

(ii) *If you like music,* ***(C1)***
then you watch the music TV channel. ***(C1)***

[4 marks]

(b)

				(i)	(ii)	(iii)	(iv)
p	q	$\neg p$	$\neg q$	$p \Rightarrow q$	$\neg p \Rightarrow \neg q$	$p \vee \neg q$	$\neg p \wedge q$
T	T	F	F	**T**	**T**	**T**	**F**
T	F	F	T	**F**	**T**	**T**	**F**
F	T	T	F	**T**	**F**	**F**	**T**
F	F	T	T	**T**	**T**	**T**	**F**

(A4)

Note: Award ***(A1)*** for each correct bold column.
(ft) with errors in (ii) which are same as in (i).

[4 marks]

(c) $(\neg p \Rightarrow \neg q)$ and $(p \vee \neg q)$ are logically equivalent. ***(C1)***

Note: Follow through with candidate's answers to part (b) (i), (ii), (iii) and (iv).
This may mean there are no equivalent statements.

[1 mark]

Total [13 marks]

3. (i) (a) (i) GM = 4 cm *(A1)*

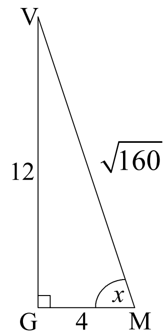
(ii) $\text{VM}^2 = 4^2 + 12^2$
$= 16 + 144$
$= 160$
$\text{VM} = \sqrt{160} = 12.6$ cm (3 s.f.) *(A1)*

[2 marks]

(b) (i) SA = area of square base + 4 (area of triangular face)
$= 8 \times 8 + 4 \times \frac{1}{2} \times 8 \times \sqrt{160}$ *(M1)*
$= 64 + 202.4$
$= 266\ \text{cm}^2$ (3 s.f.) *(A1)*

Note: Using $\text{VM} = 12.6$ gives same final answer to 3 significant figures.

(ii)



$\tan x = \frac{12}{4} = 3$ *(M1)*

$x = 71.6^\circ$ (or 1.25 radians) *(A1)*

OR

$\sin x = \frac{12}{\sqrt{160}}$ *(M1)*

$\Rightarrow x = 71.6^\circ$ (or 1.25 radians) *(A1)*

OR

$\cos x = \frac{4}{\sqrt{160}}$ *(M1)*

$\Rightarrow x = 71.6^\circ$ (or 1.25 radians) *(A1)*

OR

$\sin x = \frac{12}{12.6}$ *(M1)*

$\Rightarrow x = 72.2^\circ$ (or 1.26 radians) *(A1)*

OR

$\cos x = \frac{4}{12.6}$ *(M1)*

$\Rightarrow x = 71.5^\circ$ (or 1.25 radians) *(A1)*

[4 marks]

continued…

Question 3 continued

(ii) (a)

	Amount	End of month + Interest
January	600	604.50
February	1904.50	1918.78

end January: $600 \times 1.0075 = 604.50$ ***(M1)***
begin February: $604.50 + 1300 = 1904.50$ ***(M1)***
end February: $1904.50 \times 1.0075 = 1918.78$ ***(M1)(AG)***

[3 marks]

(b) March amount $= 1918.78 + 230$
$= 2148.78$

end of March $= 2148.78 \times 1.0075$
$= 2164.90$ ***(M1)***

April amount $= 2164.90 + 710$
$= 2874.90$

end of April $= 2874.90 \times 1.0075$
$= 2896.46$ ***(A1)***

[2 marks]

(c) 2896.46×1.0075^8 ***(M1)***
$= 3074.88$ ***(A1)***

[2 marks]

(d) $3074.88 \times 1.035^n = 3300$
$n = 1 \quad 3074.88 \times 1.035 = 3182.50$
$n = 2 \quad 3074.88 \times 1.035^2 = 3293.89$
$n = 3 \quad 3074.88 \times 1.035^3 = 3409.17$ ***(M1)***
Hence after 3 years. ***(A1)***

OR

$3074.88 \times 1.035^n = 3300$
$\Rightarrow n = 3$, that is, after 3 years. ***(G2)***

Note: Candidates may also use logarithms to solve this.

[2 marks]

Total [15 marks]

4. (a) $A = (5+2x)(7-2x)$ ***(M1)***

$= 35 - 10x + 14x - 4x^2$

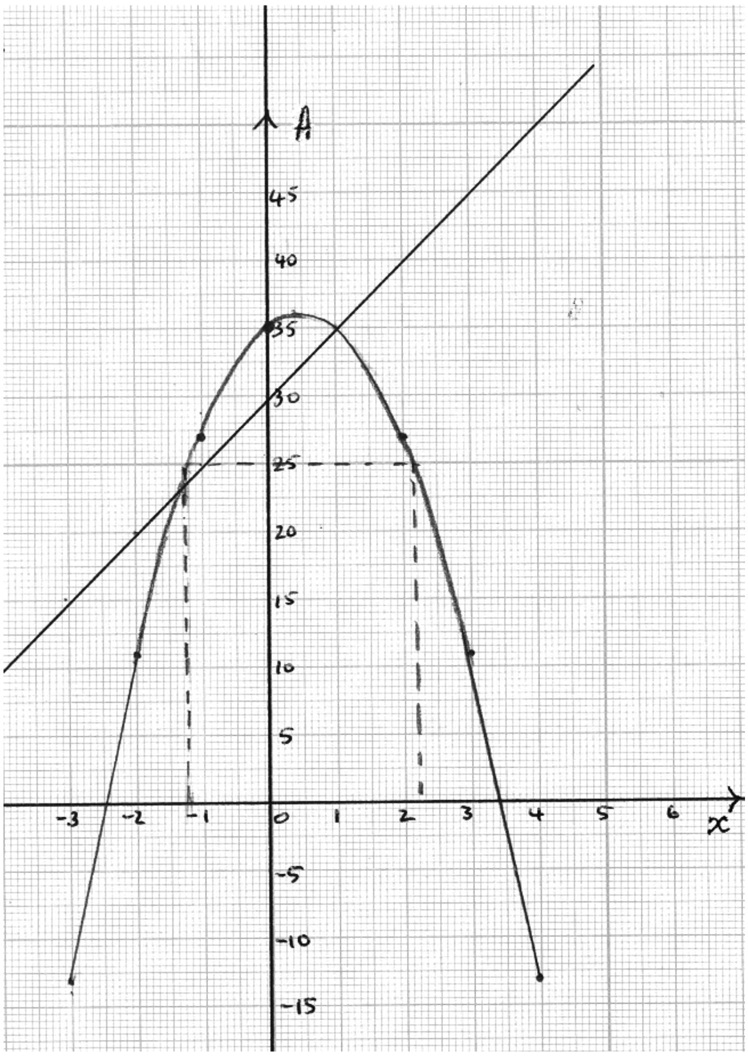
$= 35 + 4x - 4x^2$ ***(AG)***

[1 mark]

(b) (i) $p = 11, q = 35, r = 27, s = -13$ ***(A2)***

Note: Award ***(A2)*** for all four correct, ***(A1)*** for two or three correct.

(ii)



(A4)

Notes: Award ***(A1)*** for axes with correct scales and labelling.
Award ***(A2)*** for 6, 7 or 8 points correctly plotted, ***(A1)*** for 3, 4, or 5 points, ***(A0)*** for 2 or fewer.
Award ***(A1)*** for a smooth curve through reasonably correct points.

[6 marks]

continued…

Question 4 continued

(c) (i) Axis of symmetry is $x = \frac{1}{2}$ ***(A1)***

(ii) $A = 27 \Rightarrow x = -1$ or $x = 2$ ***(A1)***

Note:	Award ***(A1)*** for **one** correct value of *x*.

(iii) $x = -1$, rectangle is $(5-2)\times(7+2)$ ***(M1)***

i.e. 3×9 ***(A1)***

OR

$x = 2$, rectangle is $(5+4)\times(7-4)$ ***(M1)***

i.e. 9×3 ***(A1)***

Notes:	Award ***(A2)*** for the correct answer. Follow through with answers for *x* from the candidate's graph.

[4 marks]

(d) (i) Line on graph. ***(A1)***

(ii) From graph solutions are $x = 1$ and $x = -1.3$ (± 0.1) ***(A2)***

(Follow through with candidate's graph of parabola and straight line.)

OR

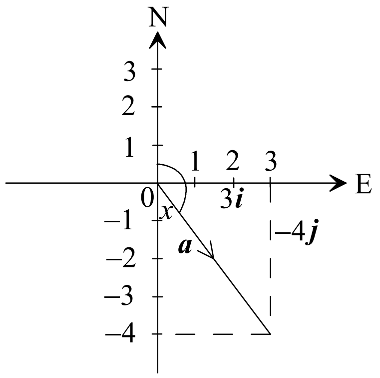
Factorizing gives $(x-1)(4x+5) = 0$ ***(M1)***

$\Rightarrow x = 1$ or $x = -1.25$ ***(A1)***

[3 marks]

Total [14 marks]

5. (i) (a) (i)



(A1)(A1)

(ii) (a) $|\boldsymbol{a}| = \sqrt{3^2 + (-4)^2}$

$= \sqrt{9+16}$ ***(A1)***

$= 5$

(b) $\tan x = \dfrac{3}{4}$

$\Rightarrow x = 36.9^\circ$

Therefore, the angle between $\boldsymbol{a}$ and $\boldsymbol{j}$ is

$180^\circ - 36.9^\circ = 143.1^\circ$

$= 143^\circ$ ***(A1)***

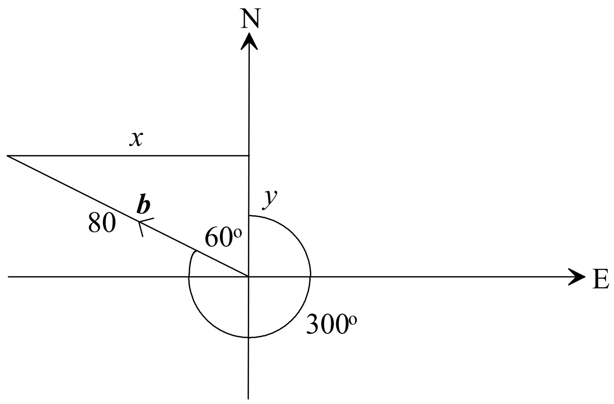
OR

$90^\circ + \arctan\left(\dfrac{4}{3}\right) = 90^\circ + 53.1^\circ = 143.1^\circ$

Therefore the angle is 143°. ***(A1)***

[4 marks]

(b)



$\sin 60^\circ = \dfrac{x}{80}$

$\Rightarrow x = 80 \sin 60^\circ$

$= 69.3$ ***(M1)***

$\cos 60^\circ = \dfrac{y}{80}$

$\Rightarrow y = 80 \cos 60^\circ$

$= 40$ ***(M1)***

Therefore, $\boldsymbol{b} = -69.3\boldsymbol{i} + 40\boldsymbol{j}$ $\left(\text{or } -40\sqrt{3}\boldsymbol{i} + 40\boldsymbol{j}\right)$ ***(A1)***

[3 marks]

continued…

Question 5 continued

(ii) (a) $PR^2 = 7.8^2 + 11.1^2 - 2 \times 7.8 \times 11.1 \times \cos 102^\circ$ ***(M1)***

$= 60.84 + 123.21 - (^-36.00)$

$= 220.05$

$PR = 14.8 \text{ m } \left(\text{or } \sqrt{220.05}\right)$ ***(A1)***

[2 marks]

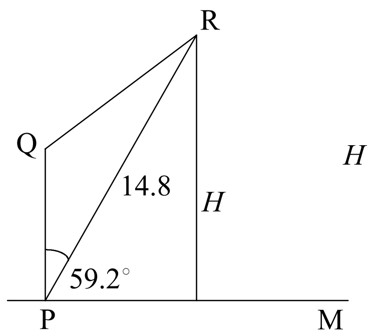
(b) $\dfrac{11.1}{\sin \hat{R}} = \dfrac{14.8}{\sin 102^\circ}$ (Follow through with candidate's answer to part (a))

$\Rightarrow \sin \hat{R} = \dfrac{11.1 \sin 102^\circ}{14.8} = 0.7336$ ***(M1)***

$\Rightarrow \hat{R} = 47.2^\circ \left(\text{or } 47.0^\circ \text{ from } \sqrt{220.05}\right)$ ***(A1)***

[2 marks]

(c)



$H = h + 6.5$

Angle $QPR = 180^\circ - (102^\circ + 47.2^\circ)$

$= 30.8^\circ$ (or 31.0°) ***(M1)***

$\Rightarrow R\hat{P}M = 90^\circ - 30.8^\circ = 59.2^\circ$ (or 59.0°)

$\sin 59.2^\circ = \dfrac{H}{14.8}$ ***(M1)***

$\Rightarrow H = 14.8 \sin 59.2^\circ = 12.7 \text{ m}$

OR

$\cos 30.8^\circ = \dfrac{H}{14.8}$ ***(M1)***

$\Rightarrow H = 14.8 \cos 30.8^\circ = 12.7 \text{ m}$

Therefore, $h = 12.7 - 6.5$

$= 6.2 \text{ m}$ ***(A1)***

[3 marks]

Total [14 marks]

6. (a) (i)

$$\begin{array}{ccc} \text{Ranch} & \text{Cape Cod} & \text{Colonial} \\ (5 & 7 & 12) \end{array}$$ *(A1)*

$$\begin{array}{l} \\ \text{Ranch} \\ \text{Cape Cod} \\ \text{Colonial} \end{array} \begin{array}{c} \begin{array}{cccc} \text{Steel} & \text{Wood} & \text{Glass} & \text{Paint} \end{array} \\ \begin{pmatrix} 5 & 20 & 16 & 7 \\ 7 & 18 & 12 & 9 \\ 6 & 25 & 8 & 5 \end{pmatrix} \end{array}$$ *(A2)*

Note:	Accept the transpose of these matrices, or different, correct ordering of rows/columns.

(ii) Amount of glass $= 5 \times 16 + 7 \times 12 + 12 \times 8$ *(M1)*

$= 260$ units *(A1)*

[5 marks]

(iii)

$$\begin{array}{l} \\ \text{Steel} \\ \text{Wood} \\ \text{Glass} \\ \text{Paint} \end{array} \begin{array}{c} \text{Cost} \\ \begin{pmatrix} 150 \\ 80 \\ 50 \\ 10 \end{pmatrix} \end{array}$$ *(A1)*

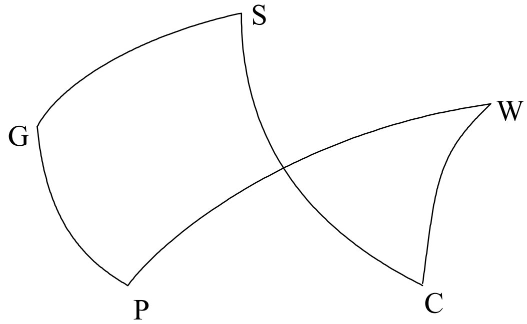
(iv) Total cost of raw materials for a colonial style house

$= 6 \times 150 + 25 \times 80 + 8 \times 50 + 5 \times 10$ *(M1)*

$= \$\,3350$ *(A1)*

[3 marks]

(b) (i)



(M2)(A2)

$C \to S \to G \to P \to W \to C$ or vice versa.

Note:	Award ***(M1)(A1)*** if a subgraph is drawn and is partially correct. Award ***(M1)(A0)*** if a subgraph is drawn but is totally wrong. Award ***(A2)*** for the correct path with no graph shown.

(ii) Distance $= 15 + 9 + 10 + 14 + 13 = 61$ km. *(A1)*

[5 marks]

Question 6 continued

(c) (i)

$$\begin{array}{c} \\ C \\ P \\ G \\ S \\ W \end{array}\begin{array}{c} \begin{array}{ccccc} C & P & G & S & W \end{array} \\ \begin{pmatrix} 2 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 0 & 2 & 0 \end{pmatrix} \end{array}$$

(A5)

Note: Award ***(A1)*** for each correct row.

(ii) $\boldsymbol{M}^2$ tells us how many ways the contractor can travel in 2 stages from one town to another (or return to the same town). ***(R1)***

[6 marks]

(d) **Note:** In part (d) penalize for additional answers which are incorrect by deducting ***[1 mark]*** for each incorrect pair.

(i) **Connected**: Fig 1, Fig 2, Fig 3 ***(A2)***

Note: Award ***(A2)*** for all 3 correct, ***(A1)*** for 1 or 2 correct.

(ii) **Complete**: Fig 3 ***(A1)***

(iii) **Tree**: Fig 1, Fig 2 ***(A1)(A1)***

[5 marks]

6. (e) (i) C2 F2 Contractor loses 2 ***(A1)***
Friend wins 2 ***(A1)***

(ii) C1 ***(A1)***

(iii) The friend should play F2 to minimise losses. ***(M1)***

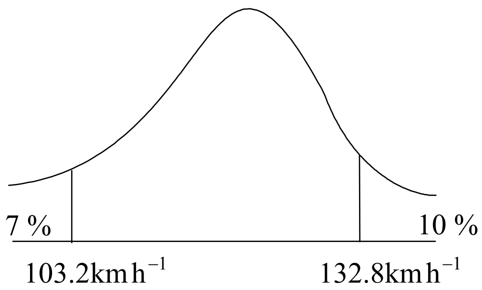
Thus required strategy is C1 F2 ***(M1)***

so the contractor wins 1 and his friend loses 1. ***(A1)***

[6 marks]

Total [30 marks]

7. (i) (a)

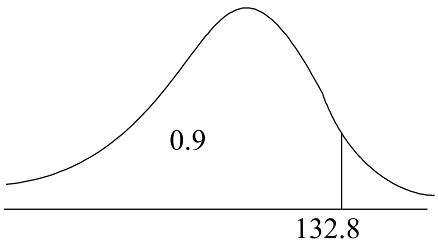


(A1)(A1)

Notes:	Award ***(A1)*** for correctly indicating the areas represented by 7 % and 10 %. Award ***(A1)*** for correctly indicating both the ordinates 103.2 and 132.8.

[2 marks]

(b)



$\Phi(z) = 0.9 \Rightarrow z = 1.28$ ***(M1)***

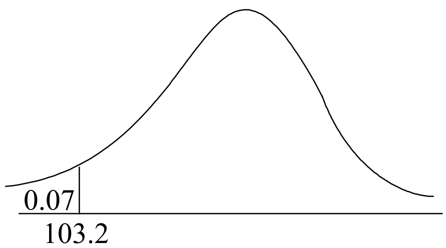
$z = \dfrac{x-\mu}{s} \Rightarrow 1.28 = \dfrac{132.8-\mu}{10.7}$ ***(M1)***

$\Rightarrow \mu = 132.8 - 1.28 \times 10.7 = 119.104$

$= 119$ (3 s.f.) ***(A1)***

OR

Candidates may use the 7 % giving:



$\Phi(-z) = 0.93 \Rightarrow -z = 1.48 \Rightarrow \quad z = -1.48$ ***(M1)***

$-1.48 = \dfrac{103.2-\mu}{10.7}$

$\Rightarrow \quad \mu = 103.2 - 1.48 \times 10.7 = 119.036$ ***(M1)***

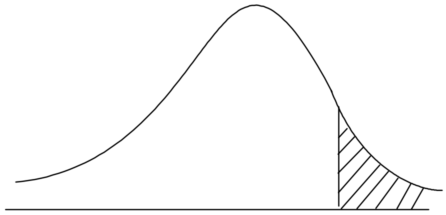
$= 119$ (3 s.f.) ***(A1)***

[3 marks]

continued…

Question 7 (i) continued

(c)



$z = \frac{130-119}{10.7} = 1.03$

$A(1.03) = 0.8485$ ***(M1)***

Required area $= 1 - 0.8485$

$= 0.1515$

$= 15.2\,\%$ (3 s.f.) ***(A1)***

Notes:	Award full marks for same answer obtained using GDC. Follow through with candidate's answer to part (b).

[2 marks]

(ii) (a) $p = 25.2 \quad q = 16.8 \quad r = 12.4$ ***(A1)(A1(A1)***

[3 marks]

(b) (i) H_0 : There is no connection between gender and subject taken. ***(C1)***

(ii) Degrees of freedom $= (3-1)(2-1) = 2 \times 1$ ***(M1)***

$= 2$ ***(AG)***

(iii) $\chi^2(2) = 5.99$ ***(A1)***

[3 marks]

(c) Accept H_0 ***(C1)***

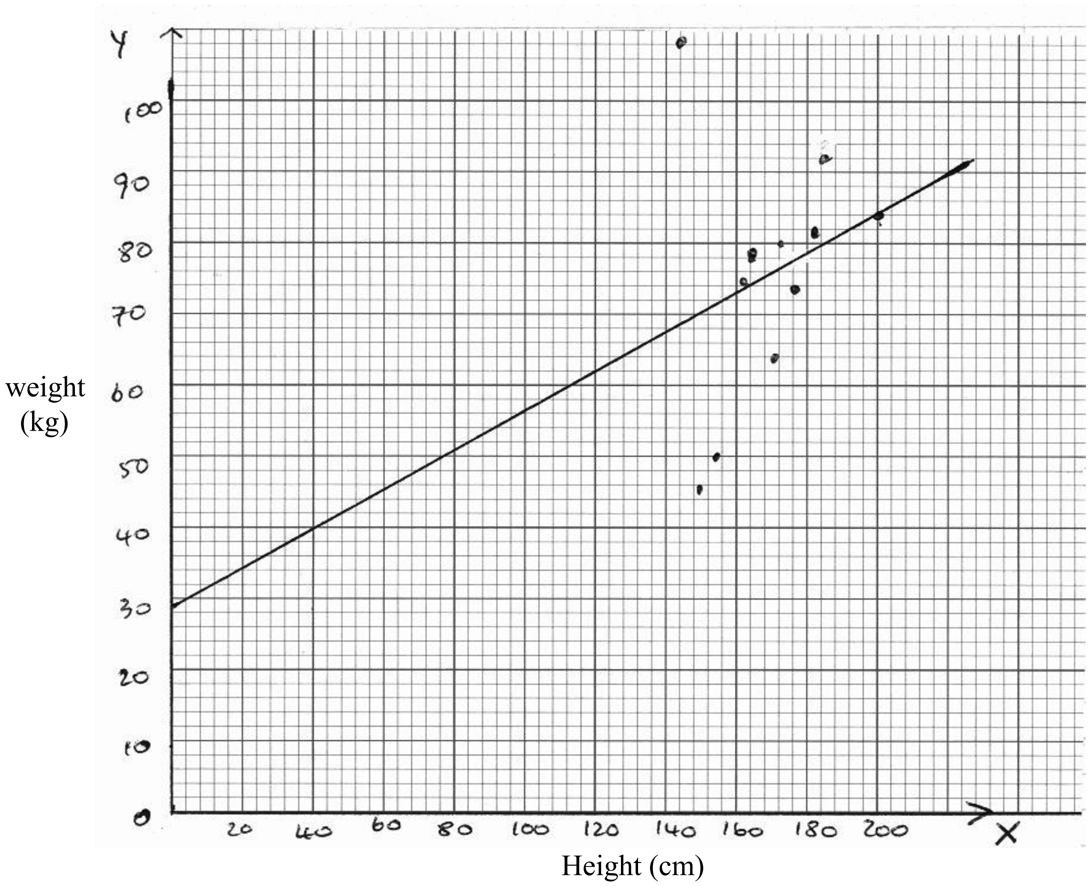
Since $1.78 < 5.99$ ***(R1)***

[2 marks]

Question 7 (ii) continued

(iii) (a)

x	155	161	173	150	182	165	170	185	175	145
y	50	75	80	46	81	79	64	92	74	108



(A2)
(A2)

Notes: Award ***(A1)*** for axes correctly labelled, and ***(A1)*** for correct scales. Award ***(A1)*** for 4, 5 6, or 7 correctly plotted points, ***(A2)*** for 8 or more.

[4 marks]

(b) Mean height $= 166.1 = 166$ (3 s.f.) ***(A1)***

[1 mark]

(c) Mean weight = 74.9 (3 s.f.) ***(A1)***

[1 mark]

continued…

Question 7 (iii) continued

(d) (i) $Sx = 12.68$ ***(A1)***

Gradient $= \dfrac{Sxy}{Sx^2} = \dfrac{44.31}{(12.68)^2} = 0.276$ ***(M1)(AG)***

(ii) $y - 74.9 = 0.276(x - 166)$ ***(M1)***

$y = 0.276x + 29.1$ ***(A1)***

OR ***(G2)***

$y = 0.276x + 29.1$

(iii) Line on graph. ***(A2)***

Note:	Award ***(A1)*** for the y-intercept at 29.1, and ***(A1)*** for a straight line through (166, 74.9).

[6 marks]

(e) (i) $y = 0.276 \times 190 + 29.1$

$= 81.5$ kg. ***(A1)***

(ii) $72 = 0.276x + 29.1$

$x = \dfrac{72 - 29.1}{0.276}$

$= 155$ cm. ***(A1)***

OR

From the graph ***(A1)***

(i) $y = 81\ (\pm 1)$ ***(A1)***

(ii) $x = 155\ (\pm 1)$ ***(A1)***

Note:	Follow through with candidate's line.

[2 marks]

(f) The 'line of best fit' becomes closer to the remaining points. ***(R1)***

OR

Gradient becomes steeper and the line is more accurate 'best fit'. ***(R1)***

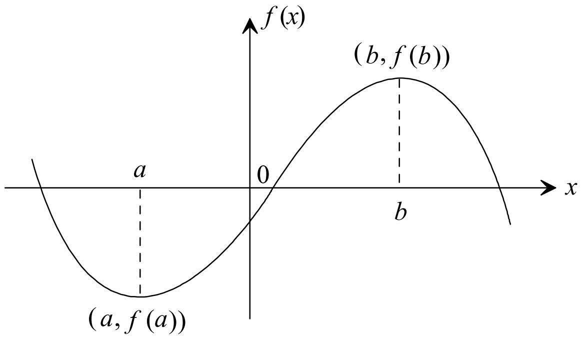
OR

Any reasonable explanation. (Line becomes $y = 1.10x - 113$) ***(R1)***

[1 mark]

Total [30 marks]

8. (i) (a)



(A2)

Note:	The curve need not be exactly like this one. The candidate's sketch must have $(a, f(a))$ as a minimum with $a < 0$, and $(b, f(b))$ as a maximum with $b > 0$. The turning points do not need to be on opposite sides of the x-axis.

[2 marks]

(b) (i) False ***(A1)***

(ii) True ***(A1)***

(iii) False ***(A1)***

(iv) True ***(A1)***

(v) False ***(A1)***

(vi) False ***(A1)***

[6 marks]

(ii) (a) $g'(x) = 2px + q$ ***(A1)***

[1 mark]

(b) $2px + q = 2x + 6$

$\Rightarrow p = 1$ and $q = 6$ ***(A1)(A1)***

[2 marks]

(c) (i) $g'(x) = 0$

$\Rightarrow 2x + 6 = 0$ ***(M1)***

$\Rightarrow \quad x = -3$ ***(A1)***

(ii) $-12 = (-3)^2 + 6(-3) + c$

$-12 = 9 - 18 + c$ ***(M1)***

$\Rightarrow c = -3$ ***(A1)***

[4 marks]

(iii) (a) $s = \int v\,\mathrm{d}t = \int 9t^2\,\mathrm{d}t = \dfrac{9t^3}{3} + d$ (candidates do not have to use the integral sign) ***(M1)***

$s = 3t^3 + d$

$s = 0$ when $t = 0 \Rightarrow d = 0$

Therefore, $s = 3t^3$ ***(A1)***

[2 marks]

continued...

Question 8 (iii) continued

(b) Reaches the building when $s = 192$

$\Rightarrow 192 = 3t^3$ ***(M1)***

$\Rightarrow \quad t^3 = 64$

$\Rightarrow \quad t = \sqrt[3]{64} = 4$ seconds ***(A1)***

[2 marks]

(c) $\frac{\mathrm{d}v}{\mathrm{d}t}$ represents acceleration ***(A1)***

[1 mark]

(d) $\frac{\mathrm{d}v}{\mathrm{d}t} = 18t$ ***(M1)***

when $t = 2$, acceleration $= 18 \times 2$

$= 36\ \mathrm{ms}^{-2}$ ***(A1)***

[2 marks]

(iv) (a) (i) $l = 24 - 2x$ ***(A1)***

(ii) $w = 9 - 2x$ ***(A1)***

[2 marks]

(b) $B = x(24 - 2x)(9 - 2x)$ ***(M1)***

$= 4x^3 - 66x^2 + 216x$ ***(AG)***

[1 mark]

(c) $\frac{\mathrm{d}B}{\mathrm{d}x} = 12x^2 - 132x + 216$ ***(A1)***

[1 mark]

(d) (i) $\frac{\mathrm{d}B}{\mathrm{d}x} = 0 \Rightarrow x^2 - 11x + 18 = 0$

$(x-2)(x-9) = 0$ ***(M1)***

$\Rightarrow x = 2$ or $x = 9$ (not possible)

Therefore, $x = 2$ cm. ***(A1)***

(ii) $B = 4(2)^3 - 66(2)^2 + 216(2)$ (or $2 \times 20 \times 5$) ***(M1)***

$= 200\ \mathrm{cm}^3$ ***(A1)***

[4 marks]

Total [30 marks]