INTERNATIONAL BACCALAUREATE

MATHEMATICS

Higher Level

Monday 13 November 1995 (afternoon)

Paper 1   2 hours

This examination paper consists of 20 questions.

The maximum mark for each question is 4.

The maximum mark for this paper is 80.

This examination paper consists of 14 pages.

INSTRUCTIONS TO CANDIDATES

Write your candidate reference number in the box:

DO NOT open this examination paper until instructed to do so.

Answer ALL questions in the spaces provided.

Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.

EXAMINATION MATERIALS

Required/Essential:

      IB Statistical Tables
      Electronic calculator
      Ruler and compasses

Allowed/Optional:

      A simple translating dictionary for candidates not working in their own language
      Millimetre square graph paper

FORMULAE

Trigonometrical identities:

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha - \cos \beta = 2 \sin \frac{\alpha + \beta}{2} \sin \frac{\beta - \alpha}{2}$$

$$\cos 2\theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta = \cos^2 \theta - \sin^2 \theta$$

$$\text{If } \tan \frac{\theta}{2} = t \text{ then } \sin \theta = \frac{2t}{1 + t^2} \text{ and } \cos \theta = \frac{1 - t^2}{1 + t^2}$$

Integration by parts:

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Standard integrals:

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a} + c$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + c \quad (|x| < a)$$

Statistics: If $(x_1, x_2, \ldots, x_n)$ occur with frequencies $(f_1, f_2, \ldots, f_n)$ then the mean m and standard deviation s are given by

$$m = \frac{\Sigma f_i x_i}{\Sigma f_i} \qquad s = \sqrt{\frac{\Sigma f_i (x_i - m)^2}{\Sigma f_i}}, \qquad i = 1, 2, \ldots, n$$

Binomial distribution:

$$p_x = \binom{n}{x} p^x (1 - p)^{n-x}, \quad x = 0, 1, 2, \ldots, n$$

Maximum marks will be given for correct answers. When an answer is wrong, some marks will be given for a correct method provided this is shown by written working.

1. Given $f(x) = e^x \ln x$, $x > 0$, find $f''(x)$.

Working:

Answer:

2. Given the matrices

$$A = \begin{pmatrix} 4 & 2 & k \\ 5 & 0 & -1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & 3 \\ 1 & 0 \\ -2 & 6 \end{pmatrix},$$

find the products AB and BA.

Working:

Answers:

Turn over

3. Find $\int x^2 e^{-x}\, dx$.

Working:

Answer:

4. Find the term that is independent of x in the expansion of $\left(2x - \dfrac{3}{x^2}\right)^9$.

Working:
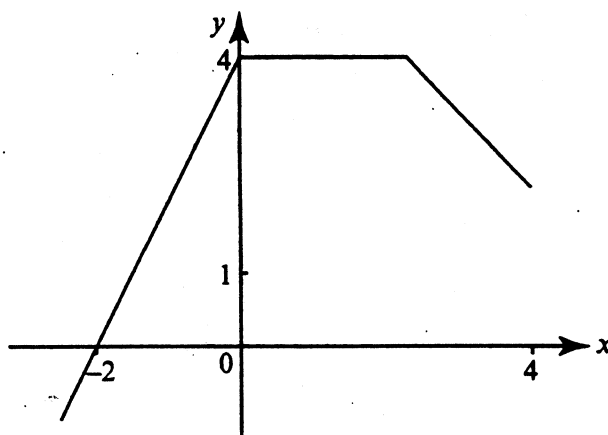
Answer:

5. Points A and B are the representations in the complex plane of the numbers $z = 2 - 2i$ and $w = -1 - \sqrt{3}i$ respectively.

(a) Given that the origin is O , find the angle AOB in radians, expressing your answer in terms of π.

(b) Calculate the argument of zw in radians, again expressing your answer in terms of π.

Working:

Answers:

_____

_____

6. The graph of $y = f(x)$ is given below.

(a) State the y-intercept of the graph of $y = \dfrac{1}{f(x)}$.

(b) Sketch on the same axes the graph of $y = \dfrac{1}{f(x)}$.

Working:

Answer:

7. Given $f(x) = \dfrac{x(x-3)(x-8)}{7}$, find the greatest and least values of $f(x)$ in the domain $2 \leqslant x \leqslant 9$, giving your answers as reduced fractions.

Working:

Answers:

8. Given $A(2, -1, 4)$, $B(6, 0, 5)$, $C(1, 3, -3)$, find $\cos A\widehat{B}C$.

Working:

Answer:

9. A discrete random variable takes the values $X = 0$, 1, 5 with probabilities $\frac{1}{3}$, $\frac{2}{5}$, and k respectively.

(a) Find k.

(b) Find the mean of X.

(c) Find the variance of X.

Working:

Answers:

_____

_____

_____

10. Given $p(x) = x^3 - x^2 + 4x - 4$,

(a) find $p(1)$;

(b) factorise $p(x)$ completely;

(c) find the partial fraction decomposition of $\dfrac{8x^2 - 7x + 14}{p(x)}$.

Working:

Answers:

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_____

_____

11. Find the equation of the tangent to the curve

$$2x^2 - 3xy + y^2 = 4$$

at the point $(3, 2)$.

Working:

Answer:

_____

Turn over

12. An advanced mathematics class consists of 6 girls and 4 boys.

    (a) How many different committees of 5 students can be chosen from this class?

    (b) How many such committees can be chosen if class members Jack and Jill cannot both be on the committee?

    (c) How many such committees can be chosen if there must be more girls than boys on the committee?

    Working:

    Answers:
    _____
    _____
    _____

13. Given two vectors $\vec{u}$ and $\vec{v}$, with $|\vec{u}| = 4$, find the value of $|\vec{u} + 2\vec{v}|$ in the following cases:

    (a) $\vec{v} = -3\vec{u}$;

    (b) $\vec{u}$ and $\vec{v}$ are perpendicular and $|\vec{v}| = 3|\vec{u}|$.

    Working:

    Answers:
    _____
    _____

14. Find $\int x \sqrt[3]{7 - 8x^2} \, dx$.

Working:

Answer:

15. Find $\dfrac{dy}{dx}$ and $\dfrac{d^2 y}{dx^2}$, given that:

$$y = 5 \sin t \,; \; x = 4t^2.$$

Working:

Answers:

Turn over

16. Find the values of k so that the matrix

$$\begin{pmatrix} 3 & -2 & 1 \\ 5 & k & 7 \\ 0 & 8 & k \end{pmatrix}$$

is singular, expressing your answer in surd (radical) form.

Working:

Answers:

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17. A box contains two cubic dice whose faces are numbered 1 to 6. One of the dice is fair, and the other is weighted so that the probability of a six appearing is 0.9.

(a) If a die is selected at random and tossed, what is the probability that a six appears?

(b) If a six appears, what is the probability that the chosen die was the weighted one?

Working:

Answers:

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18. A rectangular metal plate measuring 60 cm by 40 cm is inclined so that a **longer** edge remains on a horizontal floor, while the **shorter** edges make an angle of 50° with the horizontal.

Calculate

(a) the height above the floor of the top edge of the metal plate;

(b) the angle that a diagonal of the metal plate makes with the floor.

Working:

Answers:
_____
_____

19. Find the real and imaginary parts of $(i + \sqrt{3})^5$.

Working:

Answers:
_____
_____

Turn over

20. Find the cartesian equation of the circle described by the point z in the complex plane, given that

$$|z - 3| = 2|z + i|.$$

State the radius and the coordinates of the centre of the circle.

Working:

Answers: