

GENERAL CERTIFICATE OF SECONDARY EDUCATION

**MATHEMATICS A**

Higher Paper 4

**SPECIMEN**

Candidates answer on the question paper.

Time: 2 hours

Additional Materials:

- Calculator
- Geometrical instruments
- Tracing paper (optional)

**H** **J512/4**



Candidate Name

Centre Number

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Candidate Number

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**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the boxes above.
- Answer **all** the questions.
- Write your answers on the dotted lines unless the question says otherwise.
- Use blue or black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- There is a space after most questions. Use it to do your working. In many questions marks will be given for a correct method even if the answer is incorrect.
- Do **not** write in the bar code.
- Do **not** write outside the box bordering each page.
- **WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. ANSWERS WRITTEN ELSEWHERE WILL NOT BE MARKED.**

**INFORMATION FOR CANDIDATES**

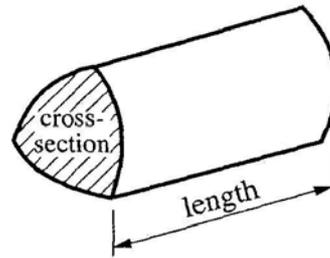
- The number of marks is given in brackets [ ] at the end of each question or part question.
- Unless otherwise instructed take  $\pi$  to be 3.142 or use the  $\pi$  button on your calculator.
- The total number of marks for this paper is 100.

For Examiner's Use	
Total	

This document consists of **25** printed pages.

## FORMULAE SHEET

**Volume of prism** = (area of cross-section) x length

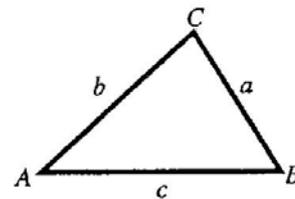


**In any triangle ABC**

**Sine rule**  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

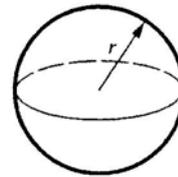
**Cosine rule**  $a^2 = b^2 + c^2 - 2bc \cos A$

**Area of triangle** =  $\frac{1}{2} ab \sin C$



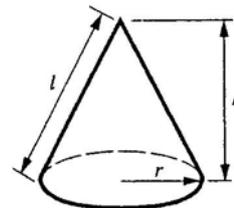
**Volume of sphere** =  $\frac{4}{3} \pi r^3$

**Surface area of sphere** =  $4\pi r^2$



**Volume of cone** =  $\frac{1}{3} \pi r^2 h$

**Curved surface area of cone** =  $\pi r l$



**The Quadratic Equation**

The solutions of  $ax^2 + bx + c = 0$ , where  $a \neq 0$ , are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1 (a) Calculate.

(i)  $7 + 4.1^2$

(a)(i) ..... [1]

(ii)  $\frac{9.21 - 3.79}{1.87 + 0.54}$

Give your answer correct to 2 decimal places.

(ii) ..... [2]

(b) Calculate £72 out of £225 as a percentage.

(b) ..... % [2]

2 (a) An orange costs 20 pence and a pear costs 12 pence.

Write down an expression in pence for the total cost of  $x$  oranges and  $y$  pears.

(a) ..... [2]

(b) Multiply out and simplify.

$x(x + 6) + 3x$

(b) ..... [2]

(c) Factorise.

$3y + 12$

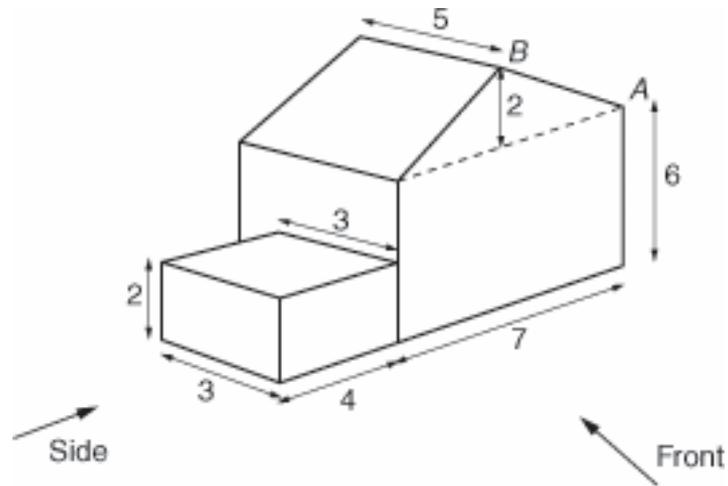
(c) ..... [1]

3 The diagram below represents a house and a garage.

The house has a symmetrical sloping roof.

The diagram has a horizontal roof.

All measurements are in metres.



On the grids draw

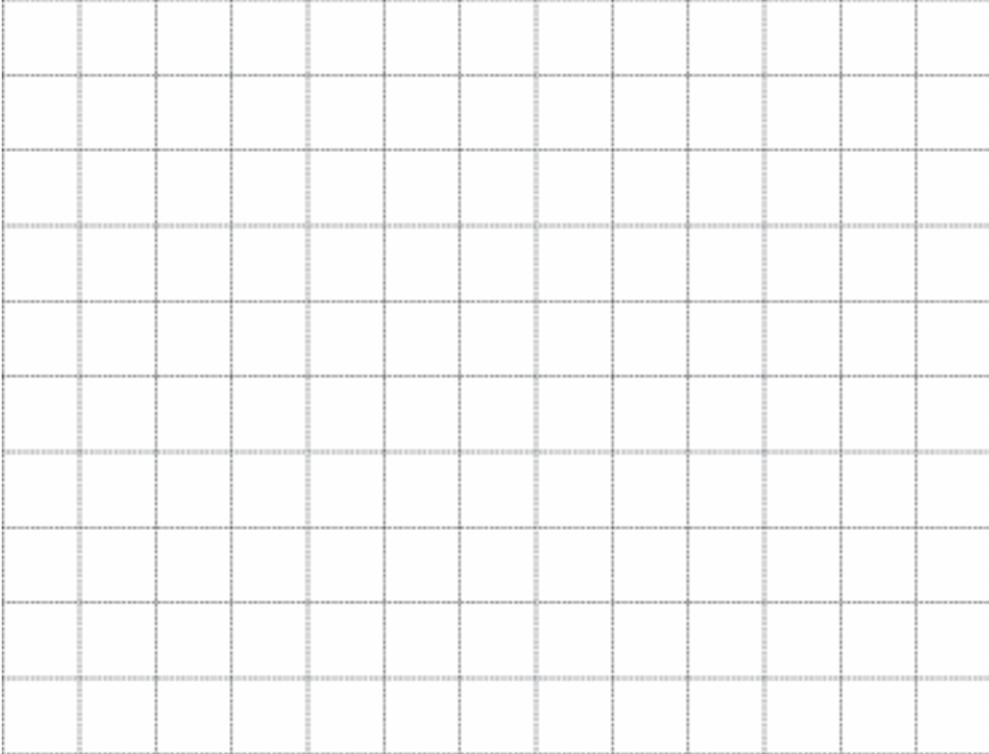
- (a) the front elevation and
- (b) the side elevation.

Use a scale of 1 cm to 1 m.

**(a) Front elevation**



[2]

**(b) Side elevation**

[3]

- 4 (a) Mark cycled 54 miles in  $4\frac{1}{2}$  hours.  
Calculate his average speed in miles per hour.

(a) ..... mph [2]

- (b) Kay drove 60 miles at an average speed of 40 miles per hour.  
How long did her journey take?  
Give your answers in hours and minutes.

(b) .....hours .....minutes [2]

5 A bag contains one yellow and three blue counters.

How many yellow counters must be added to the bag to double the probability of choosing a yellow counter?

..... [2]

6 At the beginning of 2006 the number of animals in a zoo was 800.

It is expected that each year the number of animals will increase by 5% of the number at the beginning of that year.

Work out how many animals the zoo will expect to have at the beginning of 2008.

..... [3]

- 7 The distribution of the times that each of 97 students take to travel to school is given in this table.

Time ( $t$ minutes)	Number of Students
$0 < t \leq 10$	47
$10 < t \leq 20$	14
$20 < t \leq 30$	23
$30 < t \leq 40$	13

- (a) Which class contains the median?

Explain how you found your answer.

Class..... because .....

.....

.....

..... [2]

- (b) (i) Calculate an estimate of the mean time taken to travel to school by these 97 students.

(b)(i) .....minutes [4]

- (ii) Explain why your answer to part (i) is only an **estimate** of the mean.

.....

..... [1]

8 How many 5-digit cube numbers have 8 or 9 as their units digit?

Explain, using mathematics, how you found your answer.

..... [4]

9 The letters  $W$ ,  $X$ ,  $Y$  and  $Z$  represent numbers.

$X$  is three times bigger than  $W$ ;  $Y$  is three times bigger than  $X$ ;  $Z$  is three times bigger than  $Y$ .

If  $W + X + Y = n$ , find, in terms of  $n$ , the value of  $X + Y + Z$ .

..... [3]

10 (a) Solve.

(i)  $3y + 2 = y + 7$

(a)(i) ..... [3]

(ii)  $5y - 3 < 4$

(ii) ..... [2]

(b) Simplify.

(i)  $t^3 \times t^5$

(b)(i) ..... [1]

(ii)  $\frac{t^6}{t^2}$

(ii) ..... [1]

(c) Factorise.

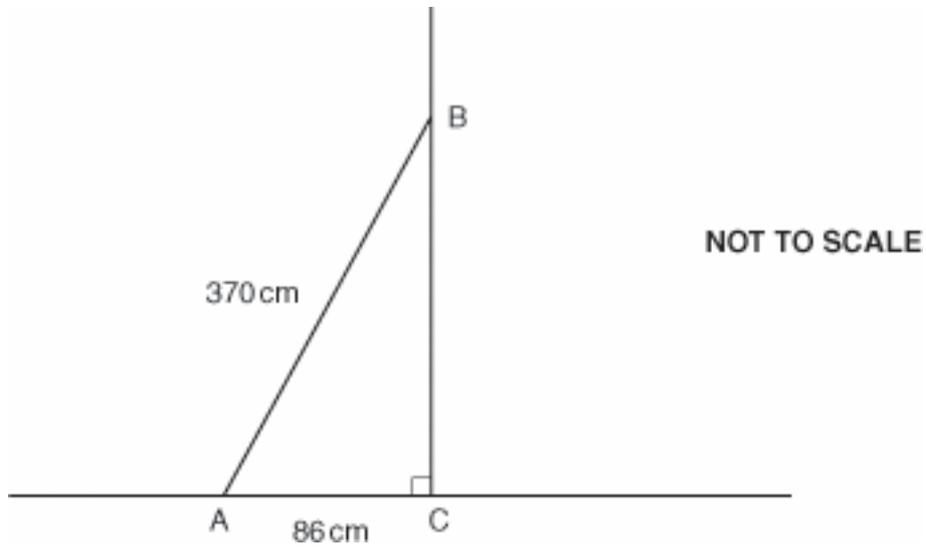
$x^2 - 2x$

(c) ..... [1]

11 The diagram represents a ladder AB leaning against a vertical wall.

The ladder is 370 cm long.

The ground AC is horizontal and the bottom of the ladder is 86 cm from the foot of the wall.



Calculate the length BC.

Give your answer to a sensible degree of accuracy.

.....cm [4]

**12** David collected 1p and 2p coins in a jar.

There are 374 coins in a jar.

The coins have a total value of £5.02.

Use algebra to find how many 1p and 2p coins there are in the jar.

.....1p coins and ..... 2p coins [4]

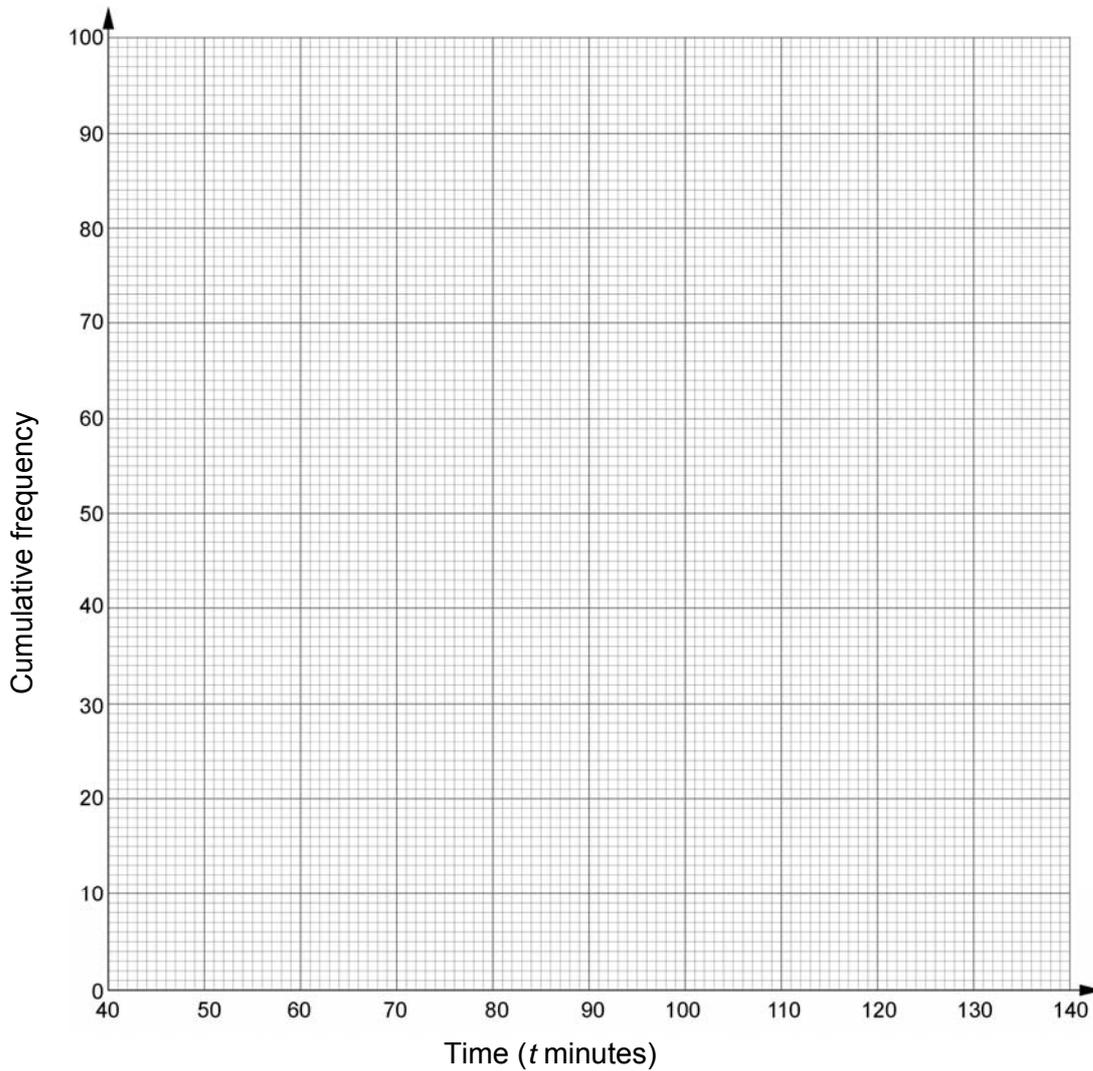
13 The table summarises the time taken by 100 runners to complete a 15 km run.

<b>Time (<math>t</math> minutes)</b>	$40 < t \leq 60$	$60 < t \leq 80$	$80 < t \leq 100$	$100 < t \leq 120$	$120 < t \leq 140$
<b>Frequency</b>	7	23	36	24	10

(a) Complete the cumulative frequency diagram for these data. [1]

<b>Time (<math>t</math> minutes)</b>	$t \leq 60$	$t \leq 80$	$t \leq 100$	$t \leq 120$	$t \leq 140$
<b>Cumulative frequency</b>					

(b) Draw a cumulative frequency diagram for these data. [3]



(c) What was the median time taken to complete the run?  
 (c).....minutes [1]

(d) A news report quoted:

'The range of the times taken to complete the run was 74 minutes and the last runner to complete the run took 137 minutes.'

Could this quote be accurate?

Explain your answer.

.....  
 ..... [2]

14 (a) Two graphs are sketched below.

Underneath the graph, write down its equation.

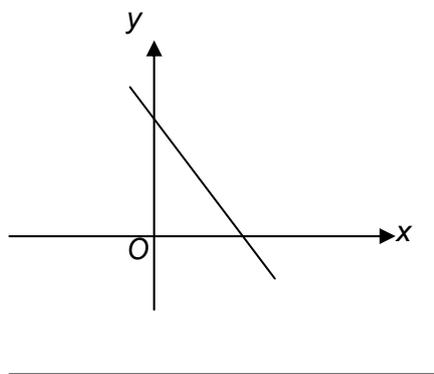
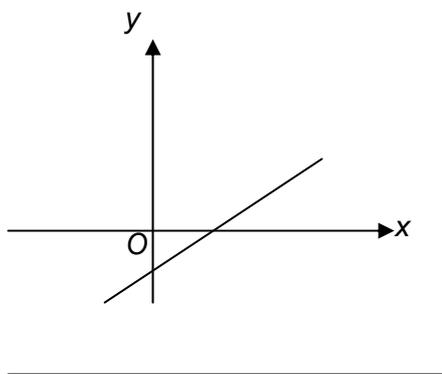
Choose the equation from the following list.

$y = 3x + 5$

$y = 5 - 3x$

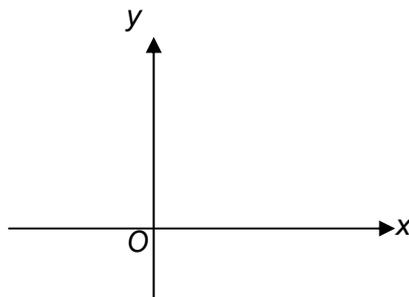
$y = 3x - 5$

$5y = x + 3$



[2]

(b) Sketch the graph of  $y = -3x - 5$ .



[2]

(c) The graphs of the following equations are all straight lines.

$y = 5x + 3$

$3y = x - 5$

$3x + y = 5$

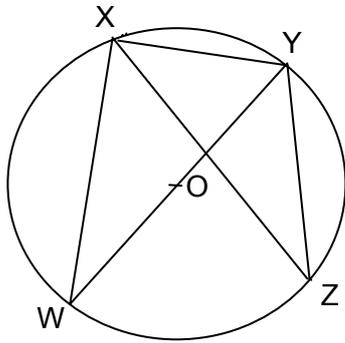
Which of these three lines is parallel to  $y = -3x - 5$ ?

(c)..... [1]

15 W, X, Y and Z are points on the circumference of a circle.

The line WY passes through O, the centre of the circle.

Angle YWX = 49°.



NOT TO SCALE

Find the following angles, giving a reason for each answer.

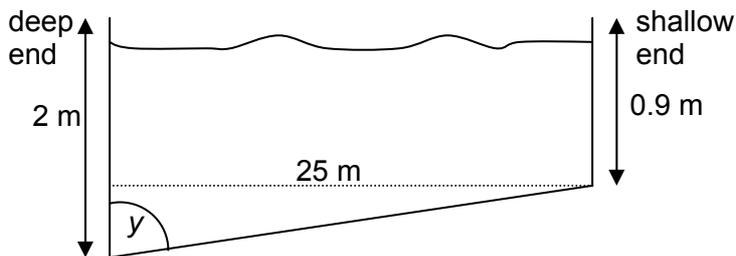
(a) Angle WXY = .....°. Reason ..... [2]

(b) Angle YZX = .....°. Reason ..... [2]

16 The diagram shows the cross-section of a swimming pool.

The walls of the pool are vertical.

The bottom of the pool slopes as shown.



NOT TO SCALE

The depth of the pool is 0.9 m at the shallow end and 2 m at the deep end.

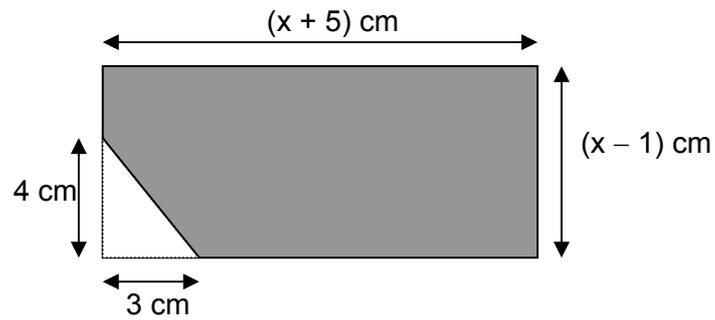
The pool is 25 m long.

Calculate angle y.

.....° [4]

17 A rectangle has length  $(x + 5)$  cm and width  $(x - 1)$  cm.

A corner is removed from the rectangle as shown.



(a) Show that the shaded area is given by  $x^2 + 4x - 11$ . [3]

(b) The shaded area is  $59 \text{ cm}^2$ .

(i) Show that  $x^2 + 4x - 70 = 0$ . [1]

(ii) Calculate the value of  $x$ .

(b)(ii) ..... [3]

**18** The hands on a clock are 8 cm and 11 cm long as shown.

How far apart are the tips of the hands at 4 o'clock?



.....cm [4]

**19** For values of  $x$  greater than 0, write the following in ascending order of size.

You will find that there is more than one answer, depending on the size of  $x$ .

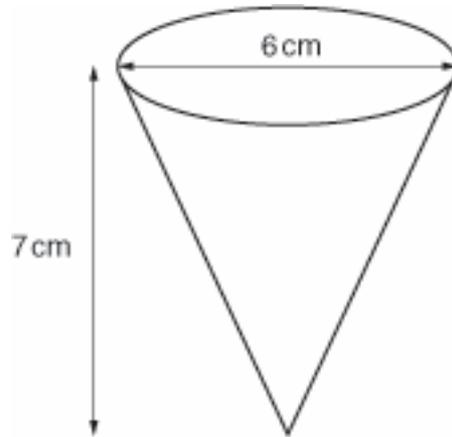
$$x^0 \quad \sqrt{x} \quad \frac{1}{x} \quad x^2$$

Clearly show the values of  $x$  that you considered.

[4]



- 21 A solid circular cone is made of clay.  
Its dimensions are shown in the diagram.



The clay is reshaped into a sphere.  
Find the radius of the sphere.

.....cm [5]



**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**  
**GENERAL CERTIFICATE OF SECONDARY EDUCATION**

**MATHEMATICS A**

Higher Paper 4

**Specimen Mark Scheme**

**J512/4**

The maximum mark for this paper is 100.

<b>1 (a) (i)</b>	23·81	<b>B1</b>		
<b>(ii)</b>	5·42 and 2·41 or 2·24896... or 2·25	<b>M1</b> <b>A1</b>		
<b>(b)</b>	$\frac{72}{225}(100)$ 32	<b>M1</b> <b>A1</b>	<b>5</b>	
<b>2 (a)</b>	$20x + 12y$ condone $20px + 12py$	<b>B2</b>		<b>B1</b> for either term correct or if + missing or further work after correct answer. Condone if given as equality e.g. $T=.$ but not if equal to a number
<b>(b)</b>	$x^2 + 6x + 3x$ $x^2 + 9x$	<b>M1</b> <b>A1</b>		
<b>(c)</b>	$3(y + 4)$	<b>B1</b>	<b>5</b>	Condone missing final bracket
<b>3 (a)</b>	House correct Garage correct	<b>B1</b> <b>B1</b>		
<b>(b)</b>	House side correct House roof correct Garage correct	<b>B1</b> <b>B1</b> <b>B1</b>	<b>5</b>	
<b>4 (a)</b>	$54 \div 4.5$ 12	<b>M1</b> <b>A1</b>		
<b>(b)</b>	$60 \div 40$ 1 hour 30 minutes	<b>M1</b> <b>A1</b>	<b>4</b>	
<b>5</b>	$P(\text{Yellow}) = \frac{1}{4}$ & evidence finding $P = \frac{1}{2}$ 2	<b>M1</b> <b>A1</b>	<b>2</b>	
<b>6</b>	$(800) \times 1.05$ $(800) \times 1.05^2$ 882	<b>M1</b> <b>M1</b> <b>A1</b>	<b>3</b>	
<b>7 (a)</b>	$10 < t \leq 20$ $\frac{1}{2}$ of $(97 + 1) = 49$ ; (49 <sup>th</sup> in $47 + 14$ ) or 47 is just below half	<b>B1</b> <b>B1</b> or <b>B1dep</b>		accept equivalent notation condone 48·5 or 48  <i>need numeric reason</i>
<b>(b) (i)</b>	use of midpoints $\times f$ $\Sigma$ any value in class $\times f$ (1475) $\div 97$ 15·2..... or 15	<b>M1</b> <b>M1</b> <b>M1dep</b> <b>A1</b>		at least one product 990 to 1960 incl 10.2 to 20.2 incl SC2 ans 10.2 or 20.3 & no working
<b>(ii)</b>	Data is grouped	<b>B1</b>	<b>7</b>	exact times unknown

<b>8</b>	Evidence finding cube numbers Isolating cubes of 2 and 9 Evidence cube nos with 2 & 9 as unit digit 5	<b>M1</b> <b>A1</b> <b>M1dep</b> <b>A1</b>	4	Dep 1 <sup>st</sup> <b>M1</b>
<b>9</b>	Evidence sequence x 3x 9x 27x $W + X + Y = 13^*$ $X + Y + Z = 39^*$ $3n$	<b>M1</b> <b>M1</b> <b>A1</b>	3	
<b>10 a (i)</b>  <b>(ii)</b>  <b>b (i)</b>  <b>(ii)</b>  <b>c</b>	$2y + 2 = 7$ or $3y = y + 5$ $2y = 5$ 2.5 $5y < 7$ $t^8$ $t^4$ $x(x - 2)$	<b>M1</b> <b>M1</b> <b>A1</b> <b>M1</b> <b>B1</b> <b>B1</b> <b>B1</b>	8	
<b>11</b>	$370^2 - 86^2$ $\sqrt{\text{their } 370^2 - 86^2}$ $359 - 86 \dots$ 360	<b>M1</b> <b>M1dep</b> <b>A1</b> <b>B1</b>	4	129504  rounding their answer provided < 370
<b>12</b>	$x + y = 374$ $x + 2y = 502$ $x = 246; y = 128$	<b>M1</b> <b>M1</b> <b>A1 A1</b>	4	
<b>13 a</b>  <b>b</b>	7 30 66 90 100 Plotted at correct height within correct interval at upper bound all points joined polygon or with smooth curve	<b>B1</b>  <b>B1</b> <b>B1</b> <b>B1</b>	1	ft (a)  strict ft their cf graph

<p><b>13 c</b></p> <p><b>d</b></p>	<p>(91)</p> <p><math>137 - 74 &gt; 60</math> or <math>60 + 74 &lt; 137</math></p> <p>lowest value outside first class interval</p>	<p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>7</p>	<p>63 or 134 or 77 cited in explanation</p> <p>But there are <math>7 \leq 60</math></p> <p><b>SC1</b> for clear indication, with no contradiction, that if values correct there would be no runners <math>\leq 60</math></p>
<p><b>14 a</b></p> <p><b>b</b></p> <p><b>c</b></p>	<p><math>y = 3x - 5</math>      <math>y = 5 - 3x</math></p> <p>Straight line with negative gradient        passing through negative part of y-axis</p> <p><math>3x + y = 5</math></p>	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p>	<p>5</p>	
<p><b>15 a</b></p> <p><b>b</b></p>	<p>90 angle in a semi-circle</p> <p>49 angles on same chord/arc/segment</p>	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p>	<p>4</p>	
<p><b>16</b></p>	<p>1.1 seen or used</p> <p><math>\tan \theta = 25/1.1</math></p> <p><math>\theta = \tan^{-1} 25/1.1</math></p> <p>87.4806... or 87.5 or 87 from correct method</p>	<p><b>B1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>4</p>	<p>Accept 87.5 or 87 if no method shown</p>

<p><b>17 a</b></p> <p><b>b (i)</b></p> <p><b>(ii)</b></p>	$(x+5)(x-1) - \frac{1}{2} \times 4 \times 3$ $x^2 - x + 5x - 5 - 6$ $x^2 + 4x - 11$ <p><math>x^2 + 4x - 11 = 59</math> rearranged to</p> $x^2 + 4x - 70 = 0$ <p><math>x = \frac{-4 \pm \sqrt{4^2 - 4 \times -70}}{2}</math> or</p> $(x+2)^2 - 74 = 0$ <p>6.6</p>	<p><b>M1</b></p> <p><b>M1dep</b></p> <p><b>A1 ag</b></p> <p><b>B1 ag</b></p> <p><b>M1</b></p> <p><b>A2</b></p>	<p>4</p>	<p><b>A1</b> if two answers given</p>
<p><b>18</b></p>	<p>120° seen or used</p> $8^2 + 11^2 - 2 \times 8 \times 11 \times \cos 120$ $\sqrt{273}$ <p>16.5</p>	<p><b>B1</b></p> <p><b>M1</b></p> <p><b>M1dep</b></p> <p><b>A1</b></p>	<p>4</p>	<p><b>SC1</b> 12 and/or 4 used allow for 273</p> <p>Allow 17 with working seen</p>
<p><b>19</b></p>	<p><math>x = 1,</math>    <math>\sqrt{x} = x^0 = x^2 = \frac{1}{x}</math></p> <p><math>x &gt; 1,</math>    <math>\frac{1}{x} \quad x^0 \quad \sqrt{x} \quad x^2</math></p> <p><math>0 &lt; x &lt; 1</math>    <math>x^2 \quad \sqrt{x} \quad x^0 \quad \frac{1}{x}</math></p>	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>M1 A1</b></p>	<p>4</p>	

<p><b>20 a</b></p> <p><b>b</b></p>	$\frac{2}{9} \quad \frac{7}{9}$ $\frac{3}{10} \quad \frac{7}{10}$ $\frac{2}{9} \quad \frac{7}{9}$ $\frac{3}{10} \times \frac{7}{10} \quad \text{or} \quad \frac{3}{10} \times \frac{7}{10}$ <p>Other product seen and added</p> $\frac{42}{90}$	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p><b>6</b></p>	
<p><b>21</b></p>	$\frac{1}{3} \pi 3^2 \times 7$ $= \frac{4}{3} \pi r^3$ <p><math>r^3 = 15.75</math></p> <p>cube root</p> <p>2.51</p>	<p><b>M1</b></p> <p><b>M1dep</b></p> <p><b>M1dep</b></p> <p><b>M1dep</b></p> <p><b>A1</b></p>	<p><b>5</b></p>	<p>Accept 2.5 – 2.1 with working shown</p>

**Assessment Objectives Grid**

<b>Question</b>	<b>AO2</b>	<b>AO3</b>	<b>AO4</b>	<b>Total</b>
1	5			5
2	5			5
3		5		5
4	4			4
5			2	2
6	3			3
7			7	7
8	4			4
9	3			3
10	8			8
11		4		4
12	4			4
13			7	7
14	5			5
15		4		4
16		4		4
17	7			7
18		4		4
19	4			4
20			6	6
21		5		5
<b>Totals</b>	<b>52</b>	<b>26</b>	<b>22</b>	<b>100</b>