Stage 1 Intermediate Revision Sheet

This document attempts to sum up the contents of the Intermediate Tier Stage 1 Module.

There are two exams, each 25 minutes long. One allows use of a calculator and the other doesn't. Together they represent 15% of the GCSE.

Before you go into the exam make sure you are fully equipped with two pens, two pencils, a calculator, a ruler, a protractor and a pair of compasses. Also be sure not to panic; it is not uncommon to get stuck on a question (I've been there!). Just continue with what you can do and return at the end to the question(s) you have found hard. If you have time check all your work, especially the first question you attempted... always an area prone to error.

I am always available on `jonathan.m.stone@gmail.com` to answer any questions you may have. Please do not hesitate.

JMS

Integers & Powers

- The number 1405.67 should be thought of as

...	1000s	100s	10s	units	.	$\frac{1}{10}$s	$\frac{1}{100}$s	$\frac{1}{1000}$s	...
...	1	4	0	5	.	6	7	0	...

- You need to be able to round to the nearest 10, 100, 1000 etc. For example rounding 14769 to the nearest thousand is 15000.
- When adding or subtracting two numbers think about the first number as a point on the number line. We will then move a distance along the number line given by the second number. It's just a question of which way. The following gives all possibilities (notice they have all moved 2 from 4, either to 2 or 6):

$$4 + (-2) = 4 - 2 = 2$$
$$4 - (+2) = 4 - 2 = 2$$
$$4 + (+2) = 4 + 2 = 6$$
$$4 - (-2) = 4 + 2 = 6$$

- Know the rules governing multiplication and division of positive and negative number. They are

$$\begin{array}{ccccc} \text{Pos} & (\times \text{ or } \div) & \text{Pos} & = & \text{Pos} \\ \text{Pos} & (\times \text{ or } \div) & \text{Neg} & = & \text{Neg} \\ \text{Neg} & (\times \text{ or } \div) & \text{Pos} & = & \text{Neg} \\ \text{Neg} & (\times \text{ or } \div) & \text{Neg} & = & \text{Pos} \end{array}$$

- The order of operations is dictated by BoDMAS. For example

$$\begin{aligned} & 4 + 6 \times 2 \div (3 - 2) & \\ =& 4 + 6 \times 2 \div 1 & \text{Bracket} \\ =& 4 + 6 \times 2 & \text{Division} \\ =& 4 + 12 & \text{Multiplication} \\ =& 16 & \text{Addition} \end{aligned}$$

- When multiplying or dividing a number by a power of 10 (i.e. 10, 100, 1000, etc.) know that the decimal place is moved by the number of zeroes in the power of 10. For example, $1000 \times 3.45 = 3450$ (decimal place moved 3 right) and $0.35 \div 100 = 0.0035$ (decimal place moved 2 left).

- The *factors* of a number are all the integers which divide exactly into it. For example the factors of 16 are 1, 2, 4, 8, 16. A number is *prime* if it has only two factors; itself and one.

- We can write any whole number in *prime factor form.* If the number is prime then it just equals itself ($17 = 17$). But when the number is not prime I need to divide the number by prime numbers *until it reaches one.* For example write 500 as a product of its prime factors;

$$\begin{array}{r|l} 2 & 500 \\ \hline 2 & 250 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

  So we can write $500 = 2^2 \times 5^3$.

- The Highest Common Factor (HCF) of two numbers is the largest number which divides them both. For example the HCF of 18 and 30 is 6.

- The Lowest Common Multiple (LCM) of two numbers is the smallest number they both divide into. For example the LCM of 15 and 20 is 60. To find this number it is often quickest to write out the first few terms of the 'times table' of each number and spot the lowest common number.

15	30	45	**60**	75	90	105	...
20	40	**60**	80	100	120	140	...

Fractions & Decimals

- Fractions should *always* be canceled down. So $\frac{16}{40} = \frac{8}{20} = \frac{4}{10} = \frac{2}{5}$.

- When adding, subtracting, multiplying and dividing mixed fractions (e.g. $5\frac{2}{3}$) it is always easiest to convert them to improper (top heavy) fractions (but don't forget to convert back at the end). Also when dealing with whole numbers, remember that they can always be put over one to make them into a fraction. So $8 = \frac{8}{1}$.

- When adding fractions we must find a common denominator. This is always the LCM of the two denominators. So with $\frac{3}{8}$ and $\frac{2}{5}$ this would be 40 so $\frac{3}{8} + \frac{2}{5} = \frac{15}{40} + \frac{16}{40} = \frac{31}{40}$.

- Ditto for subtracting; $\frac{2}{5} - \frac{3}{8} = \frac{16}{40} - \frac{15}{40} = \frac{1}{40}$.

- Multiplying fractions we just need to multiply the tops and multiply the bottoms and cancel down. $\frac{2}{5} \times \frac{3}{4} = \frac{6}{20} = \frac{3}{10}$.

- To divide fractions then change the divide to a times and flip the final fraction over. So $\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$.

- To order fractions they must all be put over a common denominator. For example write $\frac{3}{8}, \frac{1}{4}$ and $\frac{2}{5}$ in descending[1] order. We can put them all over 40, so they become $\frac{15}{40}, \frac{10}{40}$ and $\frac{16}{40}$ and their relative sizes become apparent.

[1] [descending = largest to smallest], [ascending = smallest to largest]

- All (terminating) decimals can be expressed as fractions. For example $0.125 = \frac{125}{1000} = \frac{25}{200} = \frac{5}{40} = \frac{1}{8}$ and $0.07 = \frac{7}{100}$.
- To convert fractions to decimals we just do the division. For example $\frac{3}{8} = 3 \div 8 = 0.375$.
- You must be able to multiply and divide decimals. See page 19 Mid-sized book.

Percentages

- Percentages are amounts out of 100. So 36% is $\frac{36}{100} = 0.36$. So to convert a percentage to a decimal just move the decimal place two places to the left. So $35\% \Rightarrow 0.35$ and $0.67 \Rightarrow 67\%$.
- Percentages can also be expressed as fractions. For example $36\% = \frac{36}{100} = \frac{18}{50} = \frac{9}{25}$. If required to convert a fraction to a percentage then convert to decimal and move decimal place twice to the right. For example $\frac{3}{8} = 0.375 = 37.5\%$.
- You must be able to convert between the three different forms[2].
- Given a test score of 36 out of 60, the percentage is given by $\frac{36}{60} \times 100 = 60\%$.
- To work out (say) 65% of 350, then it is $0.65 \times 350 = 227.5$.
- For % increases/decreases, work out the amount of increase/decrease and then add/subtract it away from the original. For example a coat worth £125 is reduced by 15% then the amount it is reduced by is $0.15 \times 125 = £18.75$, so the new price is $125 - 18.75 = £106.25$.
- For percentage changes use the formula

$$\text{percentage change} = \frac{\text{actual change}}{\text{original quantity}} \times 100.$$

For example in the coat example above, percentage change $= \frac{18.75}{125} \times 100 = 15\%$.

Coordinates & Elements of Algebra

- Know that points in the xy-plane are written (x, y). So the point $(2, -4)$ represents the point 2 right and 4 down.
- Know that vertical lines (|) are of the form $x =$ constant. Horizontal lines (—) are $y =$ constant. Also know the lines $y = x$ (╱) and $y = -x$ (╲).
- Expressions can be simplified by bringing together *like terms*. For example $3a+2b+2a-b = 5a + b$. We can only bring together terms when the 'jumble' of letters is the same. So $2a + 3ab + 3a + 4b = 5a + 4b + 3ab$ and $3a^2 + 4a - 2a^2 = a^2 + 4a$. Also note that ab is the same as ba, so $2ab - 3ba = -ab$.
- We can expand (or multiply out) brackets. The item outside the bracket multiplies *everything* in the brackets. So $5(a + b) = 5a + 5b$. It is essential to be careful with the signs so take care with the following;

$$\begin{aligned} 4x(2y - 5x) - 2y(x - 3y) &= 8xy - 20x^2 - 2xy + 6y^2 \\ &= 6xy - 20x^2 + 6y^2. \end{aligned}$$

- When factorising, we must bring *everything* we can out of the bracket. First we look at the numbers, then we look at the letters. So $4ab + 2a = 2a(2b + 1)$.

[2]A good question is Mid-sized book, Page 26, Question 3.

- When we have large powers of a variable it is often useful to write it out in full and underline common variables. So

$$\begin{aligned} 12a^3b^2 - 15a^2b &= 12\underline{a}\underline{a}a\underline{b}b - 15\underline{a}\underline{a}\underline{b} \\ &= 3a^2b(4ab-5). \end{aligned}$$

Equations

- Recognize the difference between equations, formulae, identities and expressions. An equation has only one variable and is correct for only one value of that variable. An identity is true for all values that the variable can take. A formula is an equation with more than one variable. An expression is a collection of terms with no '=' sign. Below is a table of examples:

Equations	Identities	Formulae	Expressions
$2x+3=10$	$2(x+3) = 2x+6$	$E=mc^2$	$mx+c$
$2x+4=3x-2$	$x+y=y+x$	$F=ma$	$\frac{1}{2}mv^2$
$2(x+4)=3x-10$	$(x+2)^2 = x^2+4x+4$	$y=mx+c$	$2a+4b$

- You must be able to calculate values from a formula given values. For example, given $E = \frac{1}{2}mv^2$, calculate E when $m=2$ and $v=-5$. So $E = \frac{1}{2} \times m \times v \times v = \frac{1}{2} \times 2 \times (-5) \times (-5) = 1 \times 25 = 25$.

- With simple equations we must get the x's to one side and the numbers to the other. We must do this by doing the *same thing to both sides.* For example:

$$\begin{aligned} 3(x+2) &= 4(2x-3) && \\ 3x+6 &= 8x-12 && \text{Multiply out brackets} \\ 6 &= 5x-12 && \text{Subtract } 3x \\ 18 &= 5x && \text{Add 12} \\ x &= \frac{18}{5} = 3\tfrac{3}{5} && \text{Divide by 5} \end{aligned}$$

- When equations have denominators (e.g. $\frac{q+3}{5} = \frac{q}{4} + 1$) we can get rid of them by multiplying by the entire equation by the LCM of the denominators. So in this example there are 3 terms in the equation so I must make sure I multiply *all* 3 of them by 20.

$$\begin{aligned} \frac{q+3}{5} &= \frac{q}{4} + 1 && \\ 20 \times \frac{q+3}{5} &= 20 \times \frac{q}{4} + 20 \times 1 && \text{Multiply by 20} \\ 4(q+3) &= 5q+20 && \text{Cancel and tidy up} \\ 4q+12 &= 5q+20 && \text{Expand brackets} \\ 12 &= q+20 && \text{Subtract } 4q \\ q &= -8 && \text{Subtract 20 (and flip round)} \end{aligned}$$

- When equations have the unknown in the denominator we must multiply by the denominator. For example

$$\begin{aligned}
\frac{20}{x} &= 4 \\
x \times \frac{20}{x} &= x \times 4 && \text{Multiply by } x \\
20 &= 4x && \text{Cancel } x \text{ and tidy} \\
4x &= 20 && \text{Flip round} \\
x &= 5 && \text{Solve}
\end{aligned}$$

- With quadratic[3] equations we use the same technique as with other simpler equations; i.e. get the x^2 to one side and the numbers to the other and then remember to take the positive *and* negative square roots. For example

$$\begin{aligned}
4x^2 + 5 &= 69 \\
4x^2 &= 64 \\
x^2 &= 16 \\
x &= \pm 4
\end{aligned}$$

If you end up being left with $x^2 = \frac{49}{16}$ then take the square root of the top and the bottom to get $x = \pm\frac{\sqrt{49}}{\sqrt{16}} = \pm\frac{7}{4} = \pm 1\frac{3}{4}$.

Sequences

- When given a sequence of numbers it is often useful to look at the difference between terms to find a pattern. For example given the triangular numbers we can see the difference has a readily identifiable pattern.

△ Numbers	1		3		6		10		15		21		28
Difference		+2		+3		+4		+5		+6		+7	

We can use this pattern to work out the next few terms.

- When there is a constant difference between the terms then the n^{th} term is given by

$$T = (\text{diff. between terms})n + (\text{zeroth term}).$$

So for example

$$\begin{aligned}
5, 8, 11, 14, 17 \ldots &\Rightarrow T = 3n + 2 \\
5, 1, -3, -7, -11 \ldots &\Rightarrow T = -4n + 9 \\
-3, -1, 1, 3, 5 \ldots &\Rightarrow T = 2n - 5
\end{aligned}$$

Properties of Shapes

- Polygons are many (straight) sided shapes. For example triangles, quadrilaterals, pentagons, hexagons etc. It is *regular* if all its sides and angles are equal. For example a regular quadrilateral is a square.
- A vertex is the point where 2 sides meet.

[3] Equations with x^2

- **Triangles**. There are many types of triangles you must know. Internal angles in a triangle *always* sum to $180°$. You need to know the following types:
  - Scalene. Nothing special (no sides or angles equal).
  - Isosceles. Two sides and two angles equal.
  - Equilateral. Three equal length sides and three $60°$ angles.
  - Right angled triangle. One right ($90°$) angle.
  - Obtuse triangle. One angle greater than $90°$.
  - Acute triangle. No angle greater than $90°$.
- **Quadrilaterals**. Four sided shapes. The internal angles of a quadrilateral *always* sum to $360°$. You need to know the following types:[4]
  - Trapezium.
  - Parallelogram.
  - Rhombus. (Squashed square.)
  - Rectangle.
  - Square. (A non-squashed square :-)
  - Kite.
  - Arrowhead.
- **Circles**. Page 67/68. Learn it also.

Symmetry & Transformations

- A *line* of symmetry divides a 2D shape into two halves and one half is the mirror image of the other. 3D shapes can have *planes* of symmetry which divide it into two halves and one half is the mirror image of the other.
- The order of rotational symmetry of a shape is the number of times it fits onto itself in one turn. So every shape has at least order 1 rotational symmetry. For example a square has order 4 rotational symmetry.
- **Transformations** can be of three types. We will deal with each in turn:
  1. **Translations**. These are just movements in the xy-plane without rotation or reflection. They are described by column vectors where the top number is the "left-right" movement and the bottom number representing the "up-down" movement. As always right and up are regarded as positive.

     So $\binom{3}{4}$ represents a translation 3 to the right and 4 up, whereas $\binom{3}{-1}$ represents 3 right and 1 down.
  2. **Rotations**. Three things define a rotation;
     - The *centre of rotation*. That is the point about which your shape is rotated.
     - The *direction* of rotation. Either clockwise ↻ or anti-clockwise ↺.
     - The *size* of the angle through which the shape is rotated. For example $45°$, $90°$, $180°$.
  3. **Reflections**. To reflect a shape in a line we must make sure that for *each point* on the object we go *to* the line at right angles and then go the *same* distance again to find the equivalent point on the image.

[4] An excellent table is given in Mid-sized book, Pages 65/66... learn it(!)

Angles, Constructions & Bearings

- A 'net' of a 3D shape is what you would get if you cut it up and lay it down. See Mid-sized book, page 83.
- Know the angle laws involving parallel lines (corresponding and alternate angles equal). Know vertically opposite angles equal and angles on a line sum to $180°$. See Mid-sized book, page 85.
- Be able to construct, accurately, any triangle or polygon given to you with the use of ruler, protractor and compasses. Never rub out construction lines as examiners look for these to give marks.
- Internal angles of *any* triangle sum to $180°$. Internal angles of *any* quadrilateral sum to $360°$. In an n sided polygon, the internal angles sum to $(n-2)\times 180$. So a nonagon's internal angles sum to $(9-2)\times 180 = 1260°$.
- The external angles of *all* polygons add to $360°$. So in a *regular* 10 sided polygon, each individual exterior angle is $\frac{360}{10} = 36°$. (We can then use this to find the internal angle; internal angle $= 180° - 36° = 144°$.)
- Shapes 'tessellate' if they can tile a floor without leaving any gaps and without overlapping. So rectangles, kites and parallelograms can. Circles and arrowheads can't.
- Bearings are *always* written with 3 digits. They represent the angle from North measured clockwise $\circlearrowright$. So $\nearrow$ would be represented by $045°$ and $\nwarrow$ by $315°$. Also *always* draw in the North $\uparrow$ lines and remember that these are all parallel so we can use our angle facts for parallel lines to help us.

Handling Data

- Know the difference between *primary* and *secondary* data. Primary data is collected through experiments, surveys, questionnaires etc. Secondary data is data extracted from published sources.
- When designing and giving a questionnaire every effort should be made to eliminate bias in both the design of the questionnaire and in the way it is given. (i.e. when surveying people don't just pick every good looking female you see as their views may not be representative of the population as a whole.)
- When collecting data from a lot of people have a pre-designed questionnaire with sections for all different possible answers. Have a tally column and next to it a frequency column. For example if surveying people's favourite food this would do nicely:

Type of Food	Tally	Frequency
Italian		
Fish 'n Chips		
Chinese		
Indian		
French		
Other		

- Data can be [qualitative], [quantitative discrete] or [quantitative continuous]. Examples below:

Qualitative	Quantitative Discrete	Quantitative Continuous
Hair colour	Score on a dice	Weight of a person
Make of jeans	Number of children in a class	Length of a field
Favourite football team	Buses per hour	Temperature

- Know how to create and/or complete a *two way table.* Never, *ever* miss the totals' sections. For example if wanting to create a table of boys and girls who pass or fail a test then the following is the two way table of the data;

	Boys	Girls	Total
Pass			
Fail			
Total			

Probability

- Probabilities are *always* given as a number between[5] 0 and 1. So they can be written as proper fractions or decimals (but not percentages or ratios).
- The sum of the probabilities of *all* possible outcomes is 1.
- We write probabilities with the following notation; for example, when flicking a coin we write $\mathrm{P}(\text{head}) = \frac{1}{2}$.
- Be able to list *joint outcomes* of two successive events. If I flick a coin and then role a dice the joint outcomes are;

$$\begin{array}{ll} (\mathrm{H}, 1) & (\mathrm{T}, 1) \\ (\mathrm{H}, 2) & (\mathrm{T}, 2) \\ (\mathrm{H}, 3) & (\mathrm{T}, 3) \\ (\mathrm{H}, 4) & (\mathrm{T}, 4) \\ (\mathrm{H}, 5) & (\mathrm{T}, 5) \\ (\mathrm{H}, 6) & (\mathrm{T}, 6) \end{array}$$

[5] and including