

DO NOT WRITE ON THIS PAPER	TIME – 2 hours	Paper 1 of 5 from ZigZag Education
Sample GCSE Examination Paper Intermediate tier non-calculator paper	Standard Equipment: pen, pencil, ruler, protractor. Compasses (Q15).	

1. Below are the results from a child's spelling tests over a term.
 3 3 5 5 5 5 6 7 9 10
 Calculate the mean mark over the whole term. [3]

$$\text{Mean} = \frac{\text{Sum of all results}}{\text{Number of results}}$$

$$\text{Mean} = \frac{3 + 3 + 5 + 5 + 5 + 5 + 6 + 7 + 9 + 10}{10} = \frac{58}{10} = 5.8$$

2. (a) Simplify the expression: $7x + 5x + 3x$. [1]

$7x$ means $7 \times x$. Remember, the same letter in \pm equations can be grouped.
 $15x$

- (b) Solve the following equations:

- (i) $12x = 48$ [1]

$12x$ means $12 \times x$. We can solve this equation by dividing both sides by 12.
 $12x \div 12 = 48 \div 12$
 $x = 4$

- (ii) $8 + x + 6 + 2x = 17$ [2]

$2x$ means $2 \times x$. Additions can be done in any order, so we can group together the numbers and the terms involving x .

$$2x + x + 8 + 6 = 17$$

$$3x + 14 = 17$$

Now we just rearrange the equation.

$$3x = 3$$

$$x = 1$$

Take 14 from both sides.
Divide both sides by 3.

- (c) For the formula $f = 3s - 4$ find the value of f , when $s = 7$. [1]

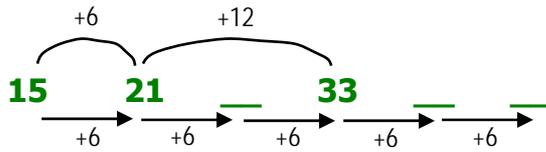
$3s$ means $3 \times s$. This is a simple substitution.
 $f = 3 \times 7 - 4$
 $f = 21 - 4 = 17$

3. (a) Write the three missing terms of the sequence

15 21 ___ 33 ___ ___

[2]

The key here is to look at the differences between the terms.



There is a gap of six between the first two terms, and a gap of twelve between the second and fourth terms. This suggests the sequence is increasing by six each time.

15 21 27 33 39 45

- (b) Write down the values of the following, in the simplest form.

(i) $\sqrt{64}$

[1]

We need to find the number which times itself is 64 ($? \times ? = 64$). This is called the square root of 64.

8

(ii) 10^2 **100**

$10^2 = 10 \times 10$

[1]

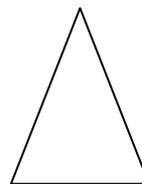
(iii) 2^3 **8**

$2^3 = 2 \times 2 \times 2 = 4 \times 2$

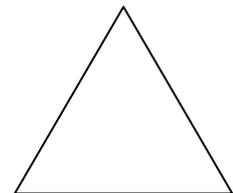
[1]

4. Here are two triangles.

Triangle A has two sides the same length.
Triangle B has all its sides the same length.



A



B

- (a) (i) Write down the special name for triangle A.

Isosceles

[1]

- (ii) Write down the special name for triangle B.

Equilateral

[1]

- (b) (i) Write down the order of rotational symmetry for triangle A

1

[1]

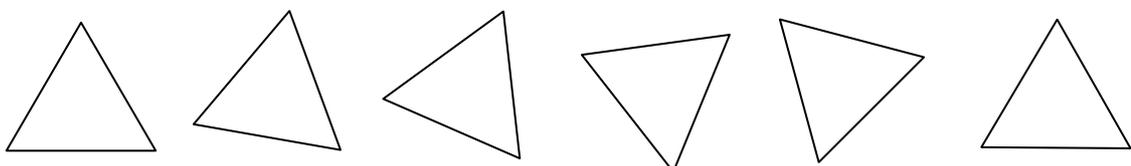
- (ii) Write down the order of rotational symmetry for triangle B.

3

[1]

The order of rotational symmetry of a shape is the number of times that shape looks the same when rotated through 360° . An isosceles triangle will only look the same after a full rotation of 360° .

The diagram below shows that when triangle B is rotated through 120° , it looks the same as it did before the rotation.



5. A car mechanic buys engine oil in 1.5 litre bottles. He buys 7 bottles.

(a) How many millilitres of oil does the mechanic have?

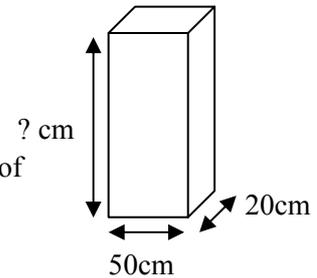
cm² [2]

Remember, 1 litre = 1000 millilitres.

$$\begin{aligned} & 1.5 \times 7 \times 1000\text{ml} \\ & = 10.5 \times 1000\text{ml} \\ & = 10,500\text{ml} \end{aligned}$$

(b) The mechanic pours 3000cm³ of the oil into a cuboid tank, the base of the tank measures 50cm × 20cm.

What height would the tank have to be, for it to be full of oil?



cm [5]

Remember, the Volume of a Cuboid = width × height × length

If the tank is to be completely full after 3000cm³ of oil has been poured in, it must have a volume of 3000cm³.

$$3000\text{cm}^3 = 50\text{cm} \times ?\text{cm} \times 20\text{cm}$$

$$3000\text{cm}^3 = 1000\text{cm}^2 \times ?\text{cm}$$

$$\frac{3000\text{cm}^3}{1000\text{cm}^2} = ?\text{cm}$$

Don't forget the units!

$$\frac{3000\text{cm}^3}{1000\text{cm}^2} = 3\text{cm}$$

6. (a) Simplify

$$6r + 5s - 3s + r$$

[1]

Remember, additions and subtractions can be written in any order, and the same letter in ± equations can be grouped.

$$= 6r + r + 5s - 3s$$

$$= 7r + 2s$$

(b) Factorise

$$x^2 + 7x$$

[1]

Factorise means 'write with brackets'. Look for what is the same in each term, and write this on the outside of the brackets. Remember, $x^2 = x \times x$, and $7x = 7 \times x$. The common term is x .

$$x^2 + 7x = x(? + ?)$$

Now think, what times x is x^2 ? Answer x .

What times x is $+7x$? Answer $+7$.

$$= x(x + 7)$$

Notice that the signs are the same.

(c) Solve the equations–

(i) $4(3x + 5) = 38$

$x = [3]$

$4(3x + 5)$ means $4 \times 3x + 4 \times 5$. Multiply out the brackets and rearrange the result to find x .

$$\begin{aligned}4(3x + 5) &= 38 \\12x + 20 &= 38 \\12x &= 18 \\x &= 1.5\end{aligned}$$

**Multiply out the brackets.
Take 20 from each side.
Divide both sides by 12.**

(ii) $27 + 3x - 9 = 9x$

[3]

In \pm equations the terms can be written in any order. Group the numbers and terms with letters together and then rearrange the result to find x .

$$\begin{aligned}27 + 3x - 9 &= 9x \\3x + 27 - 9 &= 9x \\3x + 18 &= 9x \\18 &= 6x \\3 &= x\end{aligned}$$

**(Group letters and numbers)
(Simplify)
(Take $3x$ from each side)
(Divide each side by 6)**

$$x = 3$$

7. Estimate the answer to the following: $\frac{10.33 + 889}{101 - 1.01}$

[2]

If the a question asks you to 'estimate', round the numbers to make the sum easier. In this case round 10.33 and 1.01 to its nearest whole number and 899 to the nearest 10.

Remember to round up for decimals or numbers ending in 5 or more and down for 4 or less.

$$\frac{10.33 + 890}{101 - 1.01} \approx \frac{10 + 890}{101 - 1} \approx \frac{900}{100} \approx 9$$

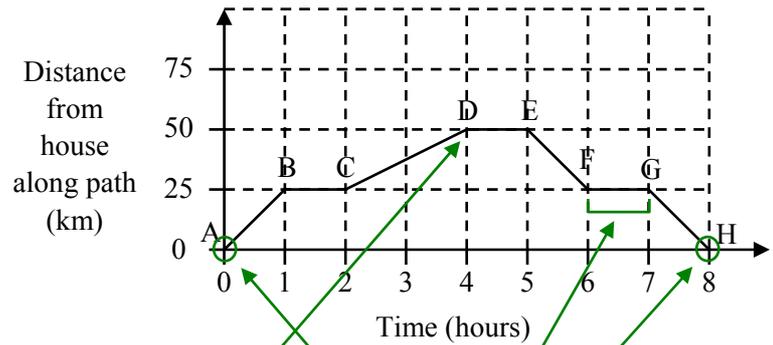
This symbol means approximately equal to

Extra Hint: For single fractions like this (one fraction line) always work out the top (numerator) and bottom (denominator) of the fraction before estimating the division. **Remember** the fraction line acts like a simple division. So: $\frac{900}{100}$ is the same as $900 \div 100 = 9$.

8. Rose goes for a bike ride down a long path, from her house to a church. She then returns back down the path, from the church to her house.



Her ride is represented by this graph.



- (a) How far is the church from the house along the path? [1]

The church is at the end of the path. Looking at the graph we can see that the furthest distance Rose was from her house is 50km, this is when she is at the church.

50km

- (b) During her cycle Rose takes rests and sits down.

- (i) How many hours in total during the cycle is Rose stopped for? [1]

The flat part of the graph show the times when Rose has stopped moving – that is B to C and D to E and F to G. They are each 1 hour stops.

3 hours

- (ii) How many hours is she away from the house? [1]

Rose is at her house when the graph shows her as being 0km away. That is A and H. the time between these 2 points is:

8 hours

- (iii) What is her average speed during her first hour's cycle? [1]

Average Speed = $\frac{\text{Distance travelled}}{\text{Time}}$

We can see from the graph that Rose travelled 25km in the first hour.

25 km/h

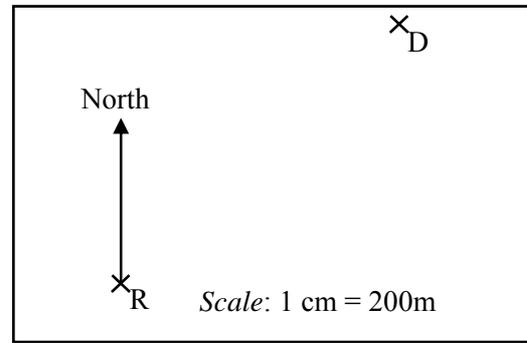
- (iv) On which section did she cycle slowest? [1]

Speed is represented by the steepness of the graph; the less steep the graph, the slower Rose was cycling. C to D is the least steep.

Section CD

9. **Copy the diagram.** The diagram shows the position of, Rose's house (R) and Damian's house (D).

(a) Measure and use the scale to work out the true distance of R from D.



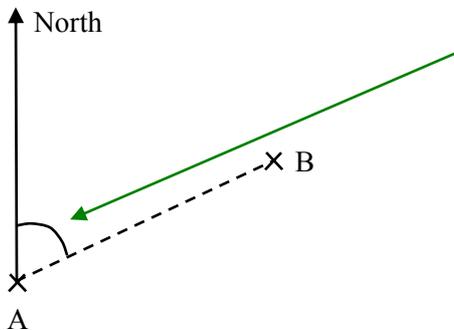
[1]

The scale shows that every cm on the diagram represents 200m in the real world. You should measure the distance on the diagram as 5cm.

$5 \times 200\text{m} = 1000\text{m}$

(b) Measure and write down the bearing (in degrees) of D from R.

[1]



The bearing between two points is the angle between the straight line that connects them and North, starting at 0° travelling clockwise around the circle.

To measure the bearings between the 2 points place a protractor on A with 0° facing north and measure clockwise the angle to B.

10. Below is a recipe for making a cake. To make one cake you will need:

- 150 g Self raising flour
- 150 g Sugar
- 3 eggs
- $\frac{1}{2}$ pint of milk

(a) Complete the list of ingredients to make 8 cakes.

To make eight cakes, we will need eight times the amount of ingredients required for one cake.

(i) Self raising flour

$8 \times 150\text{g} = 1200\text{g}$

g [1]

(ii) Sugar

$8 \times 150\text{g} = 1200\text{g}$

g [1]

(iii) Eggs

$3 \times 8 \text{ eggs} = 24 \text{ eggs}$

[1]

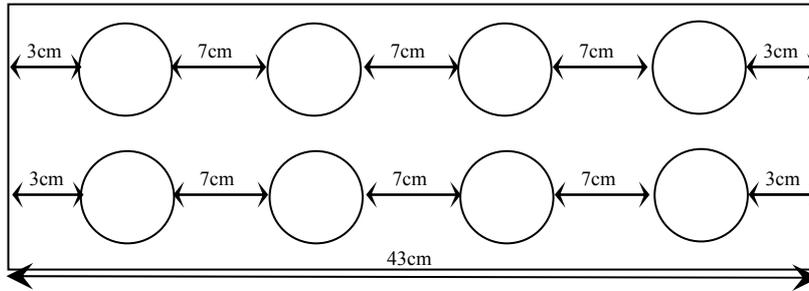
(iv) Milk

$8 \times \frac{1}{2} \text{ pints} = 4 \text{ pints}$

pints [1]

Don't forget the units!

- (b) The cakes are baked in the following baking tray. [not drawn to scale]

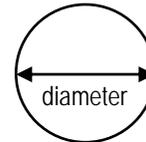


The cake mixture is placed in the circular spaces, making cylindrical cakes.

This diagram represents one of the cakes.



- (i) Calculate the diameter of each of the cakes.



cm [2]

The diameter of a circle means its width.

Each cake has the same diameter. Subtract the measurements between the spaces from the total length of the tray. This gives the combined length of all four cakes. Divide this result by four to find the diameter of one cake.

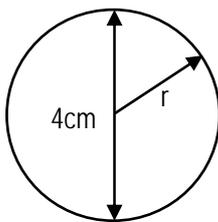
$$43\text{cm} - (3 + 7 + 7 + 7 + 3)\text{cm} = 43\text{cm} - 27\text{cm} = 16\text{cm}$$

$$16\text{cm} \div 4 = 4\text{cm}$$

- (ii) Calculate the shaded area of the cake shown in the diagram.
Take the value of π to be 3.14.

cm² [3]

The shaded area is a circle, with diameter 4cm.



Area of a circle = πr^2

πr^2 means $\pi \times r \times r$, where r is the radius of the circle.

The radius is half of the diameter, so $r = 2\text{cm}$.

$$\pi \times 2\text{cm} \times 2\text{cm} = \pi \times 4\text{cm}^2$$

$$= 3.14 \times 4\text{cm}^2$$

$$= 12.56\text{cm}^2$$

**Don't forget the units!
cm \times cm = cm²**

11. a) Solve the inequality $3x + 2 \leq 5$

\leq means 'less than or equal to'. Don't be scared by the sign – inequalities can be solved in a similar way as linear equations.

$$\begin{aligned}3x + 2 &\leq 5 \\3x &\leq 3 \\x &\leq 1\end{aligned}$$

**Take 2 from both sides.
Divide both sides by 3.**

Solve the following equations:

b) $x^2 = 9$

x^2 means $x \times x$, so you need to think 'what number times itself is nine?'. This is called the square root of nine. The answer is 3 because $3 \times 3 = 9$.

DON'T FORGET THE NEGATIVE NUMBERS! Remember, a negative times a negative equals a positive, so any number with a positive square root also has a negative square root. So the answer is also, -3 because $-3 \times -3 = 9$.

3 or -3 .

c) $\frac{x}{2} + \frac{x}{3} = 2$

Remove the fractions by multiplying both sides by 6 (2×3), and then solve as a normal linear equation.

$$\frac{x}{2} + \frac{x}{3} = 2$$

$$3x + 2x = 12$$

$$5x = 12$$

$$x = \frac{12}{5} = 2.4$$

Multiply both sides by 6.

Simplify.

Divide both sides by 5.

d) $\frac{x+1}{2} + \frac{x}{3} = 1$

**Note that $3(x + 1)$
means $3 \times x + 3 \times 1$.**

7 marks

Similarly to part c)

$$\frac{x+1}{2} + \frac{x}{3} = 1$$

$$3(x+1) + 2x = 6$$

$$3x + 3 + 2x = 6$$

$$5x + 3 = 6$$

$$5x = 3$$

$$x = \frac{3}{5} = 0.6$$

Multiply both sides by 6.

Multiply out the brackets.

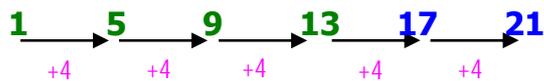
Simplify.

Take 3 from both sides.

Divide both sides by 5.

12. a) Write down the next 2 numbers in the sequences
 i) 1, 5, 9, 13,...

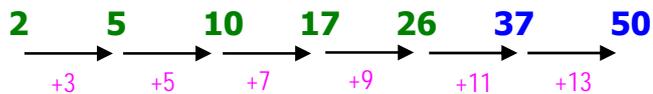
Look at the difference between each term in the sequence.



The sequence is increasing by 4 each time. So the next to terms in the sequence will be $13+4=17$ and $17+4=21$

- ii) 2, 5, 10, 17, 26,...

Look at the difference between each term in the sequence.



The difference is growing by two each time, so the next two differences will be 11 and 13, add these differences on. So the next 2 differences will be $26+11=37$ and $37+13=50$.

- b) Determine a formula for the nth term of each of the above sequences?

The n^{th} term is the rule, usually given as a formula, that gives the value of all the terms of the sequence.

To find the n^{th} term, look at the differences between the terms. If the difference is constant (say K), then the n^{th} term will be $nK + ?$, and we can substitute back into the series to find ?.



**Here, the difference is 4, so $K = 4$. Therefore the n^{th} term is given by $4n + ?$
 Substituting in for the first term ($n = 1$):**

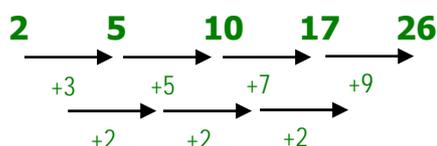
$$4 \times 1 + ? = 1$$

$$4 + ? = 1$$

$$? = -3$$

n^{th} term for the first sequence: $4n - 3$

When the sequence does not have a common difference, look at the difference between the differences.



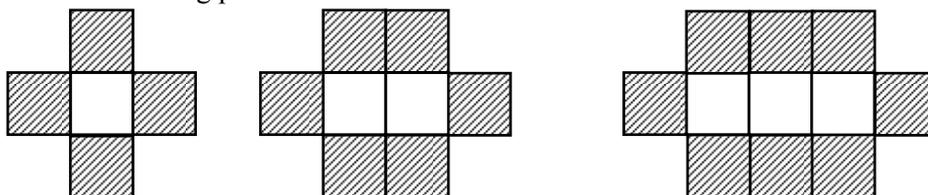
When this second row shows a constant difference, it means the n^{th} term formula involves the term n^2 . Compare this sequence to the sequence of square numbers.

1	4	9	16	25
2	5	10	17	26

To change the sequence of square numbers into our original sequence, we must add one.

n^{th} term for the second sequence: $n^2 + 1$

Consider the following pattern:



- c) How many dark squares will there be when there are 100 white squares?

The pictures show that there are two dark squares for every white square, plus two more that are on the left and right edges of the pattern.

$$2 \times 100 + 2 = 202$$

- d) How many dark squares will there be when there are n white squares?

6 marks

n means any number, if n is substituted in for 100 in the above question we will have a rule that will apply to any number.

$$2 \times n + 2$$

$$2n + 2$$

13. X and Y are lengths.

2 marks

$$J = X^2 + Y^2$$

$$K = 2X + Y$$

- a) State whether J represents i) a length ii) an area iii) a volume iv) none of the previous
 b) State whether K represents i) a length ii) an area iii) a volume iv) none of the previous

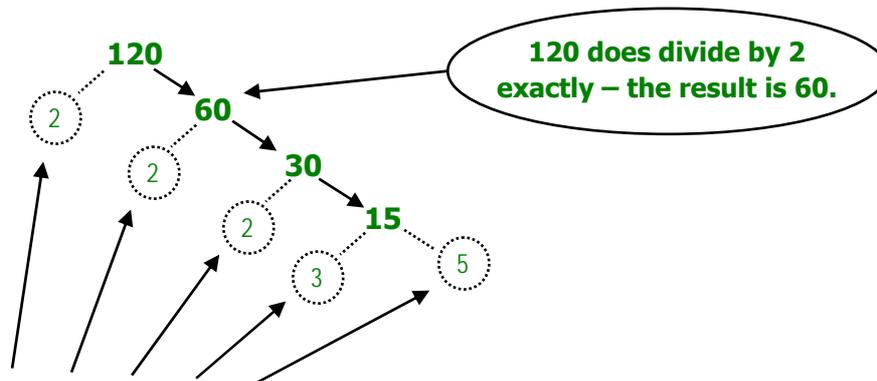
Formulas that are lengths include lengths or added lengths, formulas that are areas include the product of 2 lengths and formulas that are volumes include the product of 3 lengths.

Strategy: firstly write out the formulas without constants. Then replace the letters representing lengths with 'cm':

$X^2 + Y^2 \rightarrow \text{cm}^2 + \text{cm}^2 \rightarrow \text{Area}$
 $2X + Y \rightarrow \text{cm} + \text{cm} \rightarrow \text{length}$

14. a) Write 120 as the product of primes.

Draw a factor tree. Begin by dividing by the first prime number, 2. If the number does not divide by 2, try dividing by the next prime number, and so on.



$$120 = 2 \times 2 \times 2 \times 3 \times 5$$

- b) Write 1.234×10^{-5} as an ordinary number.

$$10^{-5} = \frac{1}{100,000}$$

$$1.234 \times 10^{-5} = 1.234 \times \frac{1}{100,000} = \frac{1.234}{100,000} = 0.00001234$$

An alternative strategy: move the decimal point to the left by the index number above the 10 (5). So it has to move 5 places.

So 1.234×10^{-5} can be written as 0.00001.234

0.00001234

c) Estimate: $\frac{13.8 \times 0.022}{133}$

4 marks

If the a question asks you to 'estimate', round the numbers to make the sum easier. In this case round all the numbers to 1 significant figure (s.f.). 1 s.f. is the first digit which isn't a zero.

Strategy: To round the numbers; look at the number after the 1st significant number, round the 1st digit up if this number is 5 or more and round down if it is 4 or less.

$$\frac{13.8 \times 0.022}{133} \approx \frac{10 \times 0.02}{100} \approx \frac{0.2}{100} \approx 0.002$$

This symbol means approximately equal to

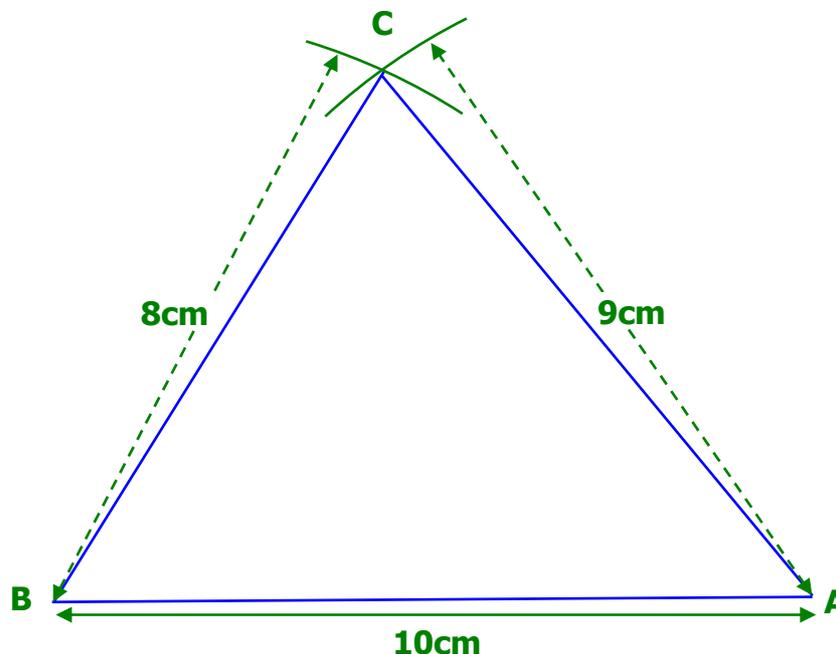
Extra Hint: For single fractions like this (one fraction line) always work out the top (numerator) and bottom (denominator) of the fraction before estimating the division.

Remember the fraction line acts like a simple division. So: $\frac{0.2}{100}$ is the same as $0.2 \div 100 = 0.002$.

15. a) Construct a triangle ABC such that AB = 10cm BC = 9cm and AC = 8cm.

4 marks

Draw the longest side of the triangle as the base, 10cm long. With compasses centred at one end, draw an arc with radius 9cm. With compasses centred at the other end, draw an arc with radius 8cm. Join the ends of the base line to the point where the arcs cross.

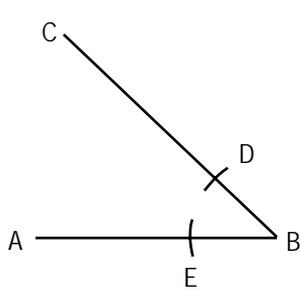


For further information, see section iSS8 of the Intermediate GCSE Mathematics Revision Programme from ZigZag Education – see www.MathsAngel.co.uk

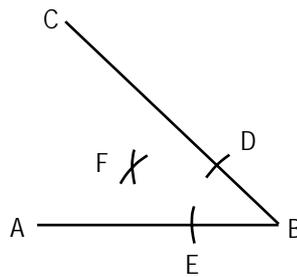
- b) Shade all the points inside the triangle that are within 3cm of AB and are nearer to AB than BC.

Firstly, find all points inside the triangle that are within 3cm of AB. Draw a circle of radius 3cm centred in point A, and another centred on point B. With compasses near the centre of the line, draw an arc above and below the line. Now draw a line that touches the top of the two circles and the top arc, and draw another that touches the bottom two circles and the bottom arc. This region contains all points, both in and outside the triangle, that are within 3cm of AB.

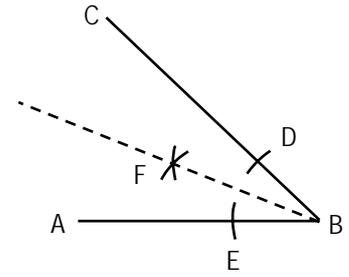
To find all points nearer AB than BC, we must bisect angle ABC.



With compasses centred on B, draw arcs to cut the lines at D and E.

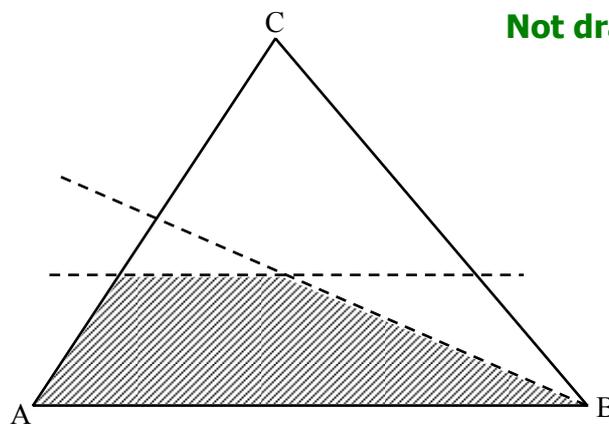


With the same radius, draw arcs centred on D and E to cross at F.



Draw a line through points B and F. This is the bisector of angle ABC.

Your diagram should look something like this:



Not drawn to scale.

For further information, see section SS12 of the Intermediate GCSE Mathematics Revision Programme from ZigZag Education – see www.MathsAngel.co.uk

16. Bag A, and bag B both have green and yellow balls in.
 The ratio of green to yellow balls in bag A is 1:3.
 The ratio of green to yellow balls in bag B is 1:4.
 The number of balls in each bag is the same.
 a) Calculate the smallest number of balls that can be in bag A.

The number of balls in bag a must be a whole number.

**LCM of 4
and 5 is 20**

Bag A has a minimum of 4 balls in it (1+3)

Bag A could also have a multiple of 4 balls in it: 8 (2:6) or 12 (3:9)

Bag B has a minimum of 5 balls in it (1+5)

Bag B could also have a multiple of 5 balls in it: 10 (2:10) or 15 (3:15)

Bag A has a minimum or Multiple of 4: 4, 8, 12, 16, 20

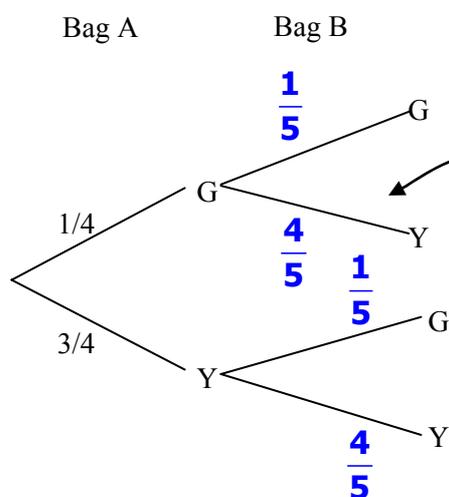
Bag B has a minimum or Multiple of 5: 5, 10, 15, 20, 25

What is the smallest, common number? 20

The least number of balls in Bag A is 20

Notice this is the same as working out the lowest common multiple of 4 and 5.

- (b) A ball is selected at random from each bag.
 Copy and complete the tree diagram.



**Each branch represents an event,
with the probability of the event
showing on the branch.**

**This branch is for first selecting a
green ball from bag A, and then a
yellow ball from bag B.**

- c) Calculate the probability that both balls are of the same colour.

6 marks

This could happen in two ways: both balls could be green or both balls could be yellow. Using the tree diagram above and multiplying along the branches we can see that:

$$P(\text{Both balls green}) = \frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$$

$$P(\text{Both balls yellow}) = \frac{3}{4} \times \frac{4}{5} = \frac{12}{20}$$

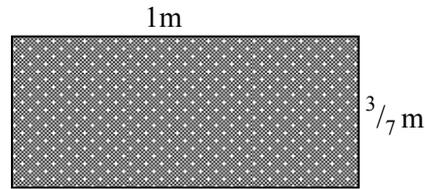
**When multiplying fractions,
multiply the tops together then
multiply the bottoms together.
i.e. $3 \times 4 = 12$ & $4 \times 5 = 20$**

The probability that either of these two events occur is the sum of their individual probabilities.

$$\frac{1}{20} + \frac{12}{20} = \frac{13}{20}$$

When adding fractions, add the tops and keep the bottoms the same.

17. I have enough paint for 100m^2 .
I am painting the front of these panels.
How many complete panels do I have enough paint for?



2 marks

Area of a rectangle = base \times height

Each panel has an area of $1\text{m} \times \frac{3}{7}\text{m} = \frac{3}{7}\text{m}^2$

To find out how many panels can be painted, we need to divide 100 by $\frac{3}{7}$. To divide fractions, turn them upside-down and change the sign to a multiply.

$$100 \div \frac{3}{7} = 100 \times \frac{7}{3} = \frac{700}{3}$$

To solve this we can use long division.

1 3 divides into 7, twice, with a remainder of **1**.
($3 \times 2 = 6$ and $7 - 6 = 1$) Write the remainder above the next zero on the bottom line, and the 2 on the top line.

2 3 divides into 10, 3 times, with a remainder of **1**.
($3 \times 3 = 9$ and $10 - 9 = 1$) Write the remainder above the next zero on the bottom line, and the 3 on the top line.

$$\begin{array}{r} 233.33 \\ 3 \overline{)700.00} \end{array}$$

3 3 divides into 10, 3 times, with a remainder of **1**.
($3 \times 3 = 9$ and $10 - 9 = 1$) Write the remainder above the next zero on the bottom line, and the 3 on the top line.

Remember, we want to know how many COMPLETE panels can be painted. Answer is 233.

We can now see that the pattern is repeating itself so 3 will be a recurring decimal.

233 Panels.

18. The share price of a company was recorded every quarter for two years. The results are shown in the table below.
- a) Find a four point moving average for the data.

	1998	1999
1 st quarter	£1.80	£2.00
2 nd quarter	£2.00	£2.20
3 rd quarter	£2.10	£2.30
4 th quarter	£2.10	£2.30

The average of the data is the mean. $\text{Mean} = \frac{\text{Sum of all results}}{\text{Number of results}}$

Place the data in date order. The first four point moving average is the mean of prices 1-4, the second is the mean of prices 2-5, the third is the mean of prices 3-6 and so on.

1.80, 2.00, 2.10, 2.10, 2.00, 2.20, 2.30, 2.30

$$\begin{aligned} \text{MA}_1 &= \frac{1.80 + 2.00 + 2.10 + 2.10}{4} = \frac{8.00}{4} = \text{£}2.00 \\ \text{MA}_2 &= \frac{2.00 + 2.10 + 2.10 + 2.00}{4} = \frac{8.20}{4} = \text{£}2.05 \\ \text{MA}_3 &= \frac{2.10 + 2.10 + 2.00 + 2.20}{4} = \frac{8.40}{4} = \text{£}2.10 \\ \text{MA}_4 &= \frac{2.10 + 2.00 + 2.20 + 2.30}{4} = \frac{8.60}{4} = \text{£}2.15 \\ \text{MA}_5 &= \frac{2.00 + 2.20 + 2.30 + 2.30}{4} = \frac{8.80}{4} = \text{£}2.20 \end{aligned}$$

Don't forget the units!

- b) Comment on the trend of the moving average.

4 marks

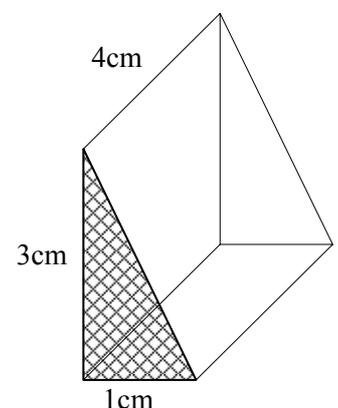
The trend of the moving average is how it changes over time, i.e. whether it tends to go up or down, and by how much if it is a constant amount.

The moving average steadily increases by £0.05 a quarter.

19. a) Calculate the area shaded.

Area of a triangle = $\frac{1}{2} \times \text{base} \times \text{perpendicular height}$

$$\frac{1}{2} \times 1\text{cm} \times 3\text{cm} = 1.5\text{cm}^2$$



- b) Calculate the volume of the smaller prism.

Remember, the volume of a prism = Area of cross-section \times length

The Prism can be cut along its length (like bread) to give the same shape and size faces. In this case the Prism can be cut along the 4cm length to give equally sized triangle bread. This means that the triangular face can be used as the objects uniform cross section.

We have already calculated the area of this cross-section in part a).

$$1.5\text{cm}^2 \times 4\text{cm} = 6\text{cm}^3$$

The two prisms are similar.

c) Calculate the missing length x .

If two shapes are similar, it means their angles are all the same, and their sides are corresponding.

The 4cm length side on the small prism corresponds to the 12cm length on the large prism. The ratio of the corresponding sides is 4:12. $4 \times 3 = 12$ so 3 is the scale factor.

Apply this scale factor to the 3cm length to find the corresponding side x .

$$x = 3 \times 3 = 9\text{cm}$$

Check these results are sensible in relation to the original diagram.

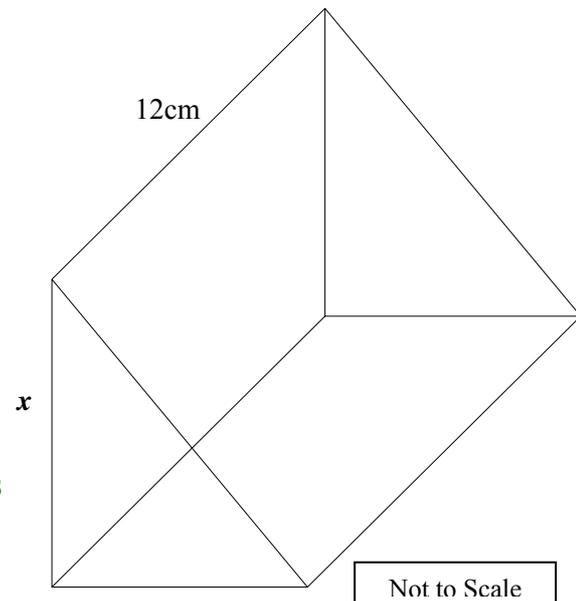
Alternatively: we can solve this by using algebra, by using the fact that we know the ratios will be the same for all the sides.

$$12 \div 4 = x \div 3$$

$$3 = x \div 3$$

$$9 = x$$

$$x = 9\text{cm}$$



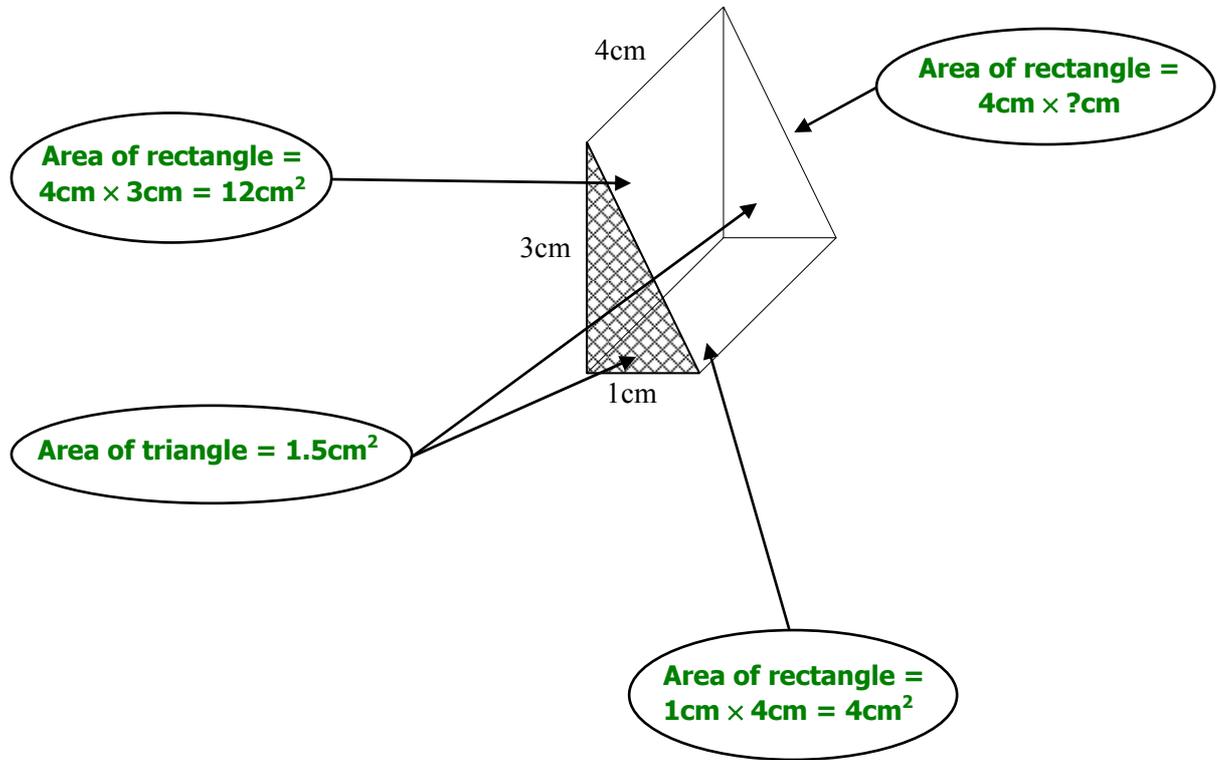
The surface area of the smaller prism is given by: $a + b\sqrt{c}$ where a , b and c are integers

d) Find a , b and c .

7 marks

The surface area of the prism is the sum of the areas of all the different faces of the prism. From the diagram we can see that the prism is made up of two identical triangles and three rectangles. We have already calculated the area of the triangles in part a), so we must now find the area of the rectangles.

Area of a rectangle = base \times height

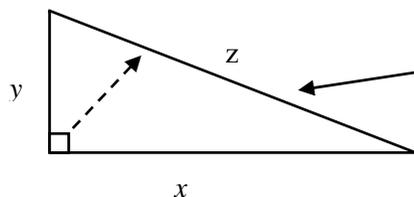


To find the area of the slanted face we must use Pythagoras' Theorem, as the height of the rectangle is the same as the length of the hypotenuse of the triangle.

Pythagoras' Theorem:

On a right-angled triangle with sides of length x , y and z , where z is the hypotenuse:

$$x^2 + y^2 = z^2$$



Note: The hypotenuse is the longest side of a right-angled triangle. It is always opposite the right angle.

Substituting the lengths from the diagram into the formula we have:

$$x^2 + y^2 = z^2$$

$$3^2 + 1^2 = z^2$$

$$9 + 1 = z^2$$

$$10 = z^2$$

$$z = \sqrt{10}$$

So the area of the slanted face is $4 \times \sqrt{10}$

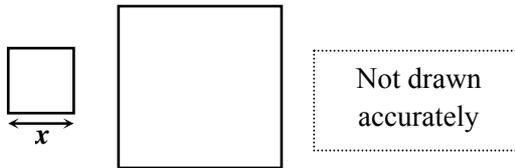
$$2 \times 1.5 + 12 + 4 + 4\sqrt{10} = 19 + 4\sqrt{10}$$

$$a = 19$$

$$b = 4$$

$$c = 10$$

20. Here are two squares.



The perimeter of the bigger square is 4cm more than the smaller one.

- a) Work out an expression in terms of x for the difference in the areas of the 2 squares, and simplify your answer.

The perimeter of a shape is the sum of the lengths of all its edges.

The perimeter of the Small Square = $4x$.

The perimeter of the Large Square = $4x + 4$

Each side of the Large Square = $(4x + 4) \div 4 = x + 1$.

Area of a Rectangle = Base \times Height.

Area of the small square = $x \times x = x^2$

Area of the large square = $(x + 1) \times (x + 1) = (x + 1)^2$

Find the difference by subtracting the smaller area from the larger one:

$$D = (x + 1)^2 - x^2$$

Firstly multiply out the brackets.

$$(x + 1)^2 = (x + 1)(x + 1) = x^2 + 2x + 1$$

$$D = x^2 + 2x + 1 - x^2$$

\pm equations can be written in any order, so group together the powers of x .

$$D = x^2 - x^2 + 2x + 1$$

Simplify.

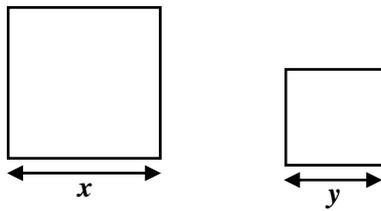
$$D = 2x + 1$$

The difference between the area of the small square above and an even smaller square is given by the expression: $6x - 9$.

b) Find an expression for the perimeter of the smallest square in terms of x .

8 marks

Assign the length of one side of the smallest square a letter, say y , and then construct an expression in x and y to find y in terms of x .



Area of small square = x^2

Area of smallest square = y^2

Find the difference by subtracting the smaller area from the larger one:

$$x^2 - y^2 = 6x - 9$$

Rearrange the equation to put all the x 's on one side and the y 's on the other.

$$x^2 - 6x + 9 = y^2$$

Square-root both sides.

$$\sqrt{(x^2 - 6x + 9)} = y$$

To find the square root of a quadratic, we must factorise the quadratic into the form $(x + a)(x + a)$. Remember, $a^2 = 9$ and $2a = -6$; so $a = -3$

$$y = x - 3$$

The perimeter of the smallest square is $4 \times y$.

$$4 \times y = 4 \times (x - 3) = 4x - 12$$