MATHEMATICS

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Duration: Three Hours ***Maxim ... 50***

Notations and Definitions used in the paper

R: The set of real numbers.

$R^n = \{(x_1, x_2, ..., x_n) : x_i \in R,\ i = 1, 2, ..., n\}$

C: The set of complex numbers.

ϕ : The empty set.

For any subset E of X (or a topological space X).

$\overline{E}$: The closure of E in X.

E°: The interior of E in X.

E^c : The complement of E in X.

$Z_n = \{0, 1, 2, ..., n-1\}$

A^t : The transpose of a matrix A.

ONE MARKS QUESTIONS (1-20)

1. Consider R2 with the usual topology. Let $S = \{(x, y) \in R^2 : x \text{ is an integer }\}$. Then S is
   (a.) Open but Not Closed
   (b.) Both open and closed
   (c.) Neither open nor closed
   (d.) Closed but Not open

2. Suppose X=$\{\alpha, \beta, \delta\}$. Let $\Im_1 = \{\phi, X, \{\alpha\}, \{\alpha, \beta\}\}$ and $\Im_2 = \{\phi, X, \{\alpha\}, \{\beta, \delta\}\}$.
   Then
   (a.) Both $\Im_1 \cap \Im_2$ and $\Im_1 \cup \Im_2$ are topologies
   (b.) Neither $\Im_1 \cap \Im_2$ nor $\Im_1 \cup \Im_2$ is a topology
   (c.) $\Im_1 \cup \Im_2$ is a topology but $\Im_1 \cap \Im_2$ is Not a topology
   (d.) $\Im_1 \cap \Im_2$ is a topology but $\Im_1 \cup \Im_2$ is not a topology

3. For a positive integer n, let $f_n : R \to R$ be defined by

$$f_n(x) = \begin{cases} \dfrac{1}{4n+5}, & \text{If } 0 \le x \le n \end{cases}$$

   (b.) Uniformly and also in L^1 n...
   (c.) Point wise but Not uniformly
   (d.) In L^1 norm but Not point wise

4. Let P_1 and P_2 be two projection opera... on a vector space. Then
   (a.) P_1+P_2 is a projection if $P_1P_2=P_2P_1 = 0$
   (b.) P_1-P_2 is a projection if $P_1P_2 = P_2P_1 = 0$
   (c.) P_1+P_2 is a projection
   (d.) P_1-P_2 is a projection

5. Consider the system of linear equations
   $x + y + z = 3,\ x - y - z = 4,\ x - 5y + kz = 6$
   Then the value of k which this system has an infinite number of solutions is
   (a.) k = –5
   (b.) k = 0
   (c.) k = 1
   (d.) k = 3

6. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ x & y & z \end{bmatrix}$ and let $V = \{(x, y, z) \in R^3 : \det(A) = 0\}$. Then the dimension of V equals
   (a.) 0
   (b.) 1
   (c.) 2
   (d.) 3

7. Let $S = \{0\} \cup \left\{\dfrac{1}{4n+7} : n = 1, 2, ...\right\}$. Then the number of analytic functions which banish only on S is
   (a.) Infinite
   (b.) 0
   (c.) 1
   (d.) 2

8. It is given that $\sum_{n-0}^{\infty} a_n z^n$ converges at z = 3+i4. Then the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n z^n$ is

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9. The value of α for which $G=\{\alpha,1,3,9,19,27\}$ is a cyclic group under multiplication modulo 56 is
(a.) 5
(b.) 15
(c.) 25
(d.) 35

10. Consider Z_{24} as the additive group modulo 24. Then the number of elements of order 8 in the group Z_{24} is
(a.) 2
(b.) 2
(c.) 3
(d.) 4

11. Define $f:R^2 \to R$ by
$$f(x,y)=\begin{cases}1, & if \ \ xy=0,\\ 2, & otherwise\end{cases}$$
If $S=\{(x,y): f$ is continuous at the point $(x,y)\}$, then
(a.) S is open
(b.) S is closed
(c.) $S=\phi$
(d.) S is closed

12. Consider the linear programming problem,
max. $z=c_1x_1+c_2x_2, c_1,c_2>0$
Subject to. $x_1+x_2\le 3$
$2x_1+3x_2\le 4$
$x_1,x_2\ge 0$.
Then,
(a.) The primal has an optimal solution but the dual does Not have an optimal solution
(b.) Both the primal and the dual have optimal solutions
(c.) The dual has an optimal solution but the primal does not have an optimal solution
(d.) Neither the primal nor the dual have optimal solutions

13. Let $f(x)=x^{10}+x-1, x\in R$ and let $x_k=k,\ k=0,1,2,....,10$. Then the value of the divided difference $f[x_0,x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9,x_{10}]$ is

14. Let X and ... distributed random variables ha... joint probability density function
$$f(x,y)=\begin{cases}\frac{1}{\pi}, & if \ \ldots\\ 0 & otherw\ldots\end{cases}$$
Then $P(Y>\max(X,-X))=$
(a.) $\frac{1}{2}$
(b.) $\frac{1}{3}$
(c.) $\frac{1}{4}$
(d.) $\frac{1}{6}$

15. Let $X_1,X_2,.....$ be a sequence of independent and identically distributed chi-square random variables, each having 4 degree of freedom. Define
$$S_n=\sum_{i=1}^{n}X_1^2,\ n=1,2,....,$$
If $\frac{S_n}{n}\xrightarrow{p}\mu$, as $n\to\infty$, then $\mu=$
(a.) 8
(b.) 16
(c.) 24
(d.) 32

16. Let $\{E_n:n=1,2,....\}$ be a decreasing sequence of Lebesgue measurable sets on R and let F be a Lebesgue measurable set on R such that $E_1\cap F=\phi$. Suppose that F has Lebesgue measure 2 and the Lebesgue measure of E_n equals $\frac{2n+2}{3n+1}$, $n=1,2,....$
Then the Lebesgue measure of the set $\left(\bigcap_{n=1}^{\infty}E_n\right)\cup F$ equals
(a.) $\frac{5}{3}$
(b.) 2
(c.) $\frac{7}{3}$
(d.) $\frac{8}{\ }$

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$$\int_0^{\frac{\pi}{8}} \left((y')^2 + 2yy' - 16y^2 \right) dx, \; y(0) = 0, \; y\left(\frac{\pi}{8}\right) = 1$$

occurs for the curve

(a.) $y = \sin(4x)$

(b.) $y = \sqrt{2}\sin(2x)$

(c.) $y = 1 - \cos(4x)$

(d.) $y = \dfrac{1 - \cos(8x)}{2}$

18. Suppose $y_p(x) = x\cos(2x)$ is a particular solution of $y'' + \alpha y = -4\sin(2x)$.

Then the constant α equals

(a.) –4

(b.) –2

(c.) 2

(d.) 4

19. If $F(s) = \tan^{-1}(s) + k$ is the Laplace transform of some function $f(t), t \geq 0$, then k =

(a.) $-\pi$

(b.) $-\dfrac{\pi}{2}$

(c.) 0

(d.) $\dfrac{\pi}{2}$

20. Let $S = \{0,1,1)(1,0,1),(-1,2,1)\} \subseteq R^3$. Suppose R^3 is endowed with the standard inner product $\langle , \rangle$. Define $M = \{x \in R^3 : (x, y) = 0 \text{ for all } y \in S\}$.

Then the dimension of M equals

(a.) 0

(b.) 1

(c.) 2

(d.) 3

TWO MARKS QUESTIONS (21-75)

21. Let X be an uncountable set and let $\Im = \{U \subseteq X : U = \phi \text{ or } U^c \text{ if finite } \}$

Then the topological space $(X, \Im)$

(a.) Is separable

(b.) Is Hausdorff

22. Suppose ([illegible] pological space. Let $\{S_n\}_{n\geq 1}$ be a se[illegible]bsets of X.

Then

(a.) $(S_1 \cup S_2)^\circ = S_1^\circ \cup S_2^\circ$

(b.) $\left(\bigcup_n S_n \right)^\circ = \bigcup_n S_n^\circ$

(c.) $\overline{\bigcup_n S_n} = \bigcup_n \overline{S}_n$

(d.) $\overline{S_1 \cup S_2} = \overline{S}_1 \cup \overline{S}_2$

23. Let (X, d) be a metric space. Consider the metric ρ on X defined by

$\rho(x, y) = \min\{\frac{1}{2}, d(x, y)]\, x, y \in X$.

Suppose $\Im_1$ and $\Im_2$ are topologies on X defined by d and ρ, respectively. Then

(a.) $\Im_1$ is a proper subset of $\Im_2$

(b.) $\Im_2$ is a proper subset of $\Im_1$

(c.) Neither $\Im_1 \subseteq \Im_2$ nor $\Im_2 \subseteq \Im_1$

(d.) $\Im_1 = \Im_2$

24. A basis of $V = \{(x, y, z, w) \in R^4 : x + y - z = 0, \; y + z + w = 0, 2x + y - 3z - w = 0\}$

(a.) $\{(1,1,-1,0),(0,1,1,1),\{(2,1,-3,1)\}$

(b.) $\{(1,-1,0,1)\}$

(c.) $\{(1,0,1,-1)\}$

(d.) $\{(1,-1,0,1),(1,0,1-1)\}$

25. Consider R^3 with the standard inner product. Let $S = \{(1,1,1),(2,-1,2),(1,-2,1)\}$.

For a subset W of R^3, let L(W) denote the linear span of W in R^3. Then an orthonormal set T with $L(S) = L(T)$ is

(a.) $\left\{ \frac{1}{\sqrt{3}}(1,1,1), \frac{1}{\sqrt{6}}(1,-2,1) \right\}$

(b.) $\{(1,0,0),(0,1,0),(0,0,1)\}$

(c.) $\left\{ \frac{1}{\sqrt{3}}(1,1,1), \frac{1}{\sqrt{2}}(1,-1,0) \right\}$

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respective eigen vectors $(1,-1,0)^t$, $(1,1-2)^t$ and $(1,1,1)^t$. Then 6A equals

(a.) $\begin{bmatrix} -1 & 5 & 2 \\ 5 & -1 & 2 \\ 2 & 2 & 2 \end{bmatrix}$

(b.) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(c.) $\begin{bmatrix} 1 & 5 & 3 \\ 5 & 1 & 3 \\ 3 & 3 & 3 \end{bmatrix}$

(d.) $\begin{bmatrix} -3 & 9 & 0 \\ 9 & -3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

27. Let $T: R^3 \to R^3$ be a linear transformation defined by $T((x,y,z)) = (x+y-z, x+y+z, y-z)$.
Then the matrix of the linear transformation T with respect to the ordered basis $B = \{(0,1,0),(0,0,1),(1,0,0)\}$ of R^3 is 0

(a.) $\begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$

(b.) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$

(c.) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$

(d.) $\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix}$

28. Let $Y(x) = (y_1(x), y_2(x))$ and let $A = \begin{bmatrix} -3 & 1 \\ k & -1 \end{bmatrix}$.
Further, let S be the set of values of k for

(a.) $\{k : k$

(b.) $\{k : k \le 3\}$

(c.) $\{k : k < -1\}$

(d.) $\{k : k < 3\}$

29. Let $u(x,y) = f(xe^y) + g($
Where f and g are differentiable functions. Then the differential equation of minimum satisfied by u is

(a.) $u_{xy} + xu_{xx} = u_x$

(b.) $u_{xy} + xu_{xx} = xu_x$

(c.) $u_{xy} - xu_{xx} = u_x$

(d.) $u_{xy} - xu_{xx} = xu_x$

30. Let C be the boundary of the triangle formed by the points $(1,0,0),(0,1,0),(0,0,1)$.
Then the value of the line integral $\oint_C -2y\,dx + (3x - 4y^2)\,dy + (z^2 + 3y)\,dz$ is

(a.) 0
(b.) 1
(c.) 2
(d.) 4

31. Let X be a complete metric space and let $E \subseteq X$. Consider the following statements:

(S_1) E is compact
(S_2) E is closed and bounded
(S_3) E is closed and totally bounded
(S_4) Every sequence in E has a subsequence converging in E

(a.) S_1
(b.) S_2
(c.) S_3
(d.) S_4

32. Consider the series $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \sin(nx)$.
Then the series

(a.) Converges uniformly on R
(b.) Converges point wise but not uniformly on R
(c.) Converges in L1 norm to an integrable function on $[0, 2\pi]$ but does not

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33. Let f(z) be an analytic function. Then the value of $\int_0^{2\pi} f\left(e^{it}\right)\cos(t)\,dt$ equals

(a.) 0

(b.) $2\pi f(0)$

(c.) $2\pi f'(0)$

(d.) $\pi f'(0)$

34. Let G_1 and G_2 be the images of the disc $\{z \in C : |z+1| < 1\}$ under the transformations $w = \frac{(1-i)z+2}{(1-i)z+2}$ and $w = \frac{(1+i)z+2}{(1+i)z+2}$ respectively. Then

(a.) $G_1 = \{w \in C : Im(w) < 0\}$ and $G_2 = \{w \in C : Im(w) ? 0\}$

(b.) $G_1 = \{w \in C : Im(w) > 0\}$ and $G_2 = \{w \in C : Im(w) < 0\}$

(c.) $G_1 = \{w \in C : Im(w) > 2\}$ and $G_2 = \{w \in C : Im(w) < 2\}$

(d.) $G_1 = \{w \in C : Im(w) < 2\}$ and $G_2 = \{w \in C : Im(w) > 2\}$

35. Let $f(z) = 2z^2 - 1$. Then the maximum value of $|f(z)|$ on the unit disc $D = \{z \in C : |z| \le 1\}$ equals

(a.) 1

(b.) 2

(c.) 3

(d.) 4

36. Let $f(z) = \frac{1}{z^2 - 3z + 2}$

Then the coefficient of $\frac{1}{z^3}$ in the Laurent series expansion of $f(z)$ and is

(a.) 0

(b.) 1

(c.) 3

(d.) 5

37. Let $f : C \to C$ be an arbitrary analytic function satisfying $f(0) = 0$ and $f(1) = 2$. Then

(a.) there exists a sequence $\{z_n\}$ such that $|z_n|$ and $|f(z_n)|>n$

(d.) there ... ence $\{z_n\}$ such that $z_n \to 0$ a...

38. Define $f : C \to$ ...

$$f(z) = \begin{cases} 0, & if\ Re(\ldots (z = 0,) \\ z, & ot\ldots \end{cases}$$

Then the set of points wher... is

(a.) $\{z : Re(z) \neq 0$ and $Im(z) \neq$ ...

(b.) $\{z : Re(z) \neq 0\}$

(c.) $\{z : Re(z) \neq 0$ or $Im(z) \neq 0\}$

(d.) $\{z : Im(z) \neq 0\}$

39. Let U(n) be the set of all positive integers less than *n* and relatively prime to n. Then U(n) is a ground under multiplication modulo n. For n = 248, the number of elements in U(n) is

(a.) 60

(b.) 120

(c.) 180

(d.) 240

40. Let R(x) by the polynomial ring in x with real coefficients and let I = $(x^2 + 1)$ be the ideal generated by the polynomial $x^2 + 1$ in R[x]. Then

(a.) I is a maximal ideal

(b.) I is a prime ideal but NOT a maximal ideal

(c.) I is NOT a prime ideal

(d.) R[x] /I has zero divisors

41. Consider Z_5 and Z_{20} as ring modulo 5 and 20, respectively. Then the number of homomorphism $\varphi : Z5 \to Z_{20}$ is

(a.) 1

(b.) 2

(c.) 4

(d.) 5

42. Let Q be the field of rational number and consider Z_2 as a field modulo 2. Let f(x) = $x_3 - 9x_2 + 9x + 3$.

Then f(x) is

(a.) irreducible over Q but reducible over Z_2

(b.) irreducible over both Q and Z_2

(c.) reducible over Q but irreducible over Z_2

(d.) reducible over both Q and Z_2

43. Consider Z_5 as field modulo 5 and let $f(x) = x^5 + 4x^4 + 4x^3 + 4x^2 + x + 1$

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(c.) 2 and 2

(d.) 1 and 2

44. Consider the Hilbert space $l^2 = \left\{ x = \{x_n\} : x_n \in R, \sum_{n=1}^{\infty} x_n^2 < \infty \right\}$

Let $E = \left\{ \{x_n\} : | x_n | \leq \in \frac{1}{n} \text{ for all } n \right\}$ be a subset of l^2. Then

(a.) $E^0 = \left\{ x : | x_n | < \frac{1}{n} \text{ for all } n \right\}$

(b.) $E^0 = E$

(c.) $E^0 = \left\{ x : | x_n | < \frac{1}{n} \text{ for all but finitely many } n \right\}$

(d.) $E^0 = \phi$

45. Let X and Y be normed liner spaces and the T : X → Y be a linear map. Then T is continuous if

(a.) Y is finite dimensional

(b.) X is finite dimensional

(c.) T is one to one

(d.) T is onto

46. Let X be a normed linear space and let E_1, $E_2 \subseteq X$. Define

$E_1 + E_2 = \{ x + y : x \in E_1, y \in E_2 \}$.

Then $E_1 + E_2$ is

(a.) open if E_1 or E_2 is open

(b.) NOT open unless both E_1 and E_2 are open

(c.) closed if E_1 or E_2 is closed

(d.) closed if both E_1 and E_2 are closed

47. For each $a \in R$, consider the linear programming problem

Max. $z = x_1 + 2x_2 + 3x_3 + 4x_4$

subject to

$$ax_1 + 2x_2 \leq 1$$

$$x_1 + ax_2 + 3x_4 \leq 2$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

Let S {$a \in R$: the given LP problem has a basic feasible solution}. Then

(a.) $S = \phi$

(b.) S= R

(c.) S = (0, ∞)

(d.) S = (-∞, 0)

48. Consider the linear programming problem

$+ 5x_3 \leq 3$

$x_1,$

Then the dual of this L

(a.) has a feasible soluti... NOT have a basic feasible so...

(b.) has a basic feasible solutio...

(c.) has infinite number of ... solutions

(d.) has no feasible solution

49. Consider a transportation problem with two warehouses and two markets. The warehouse capacities are $a_1 = 2$ and $a_2 = 4$ and the market demands are $b_1 = 3$ and $b_2 = 3$. let x_{ij} be the quantity shipped from warehouse i to market j and c_{ij} be the corresponding unit cost. Suppose that $c_{11} = 1$, $c_{21} = 1$ and $c_{22} = 2$. Then $(x_{11}, x_{12}, x_{21}, x_{22}) = (2, 0, 1, 3)$ is optimal for every

(a.) $x_{12} \in [1, 2]$

(b.) $x_{12} \in [0, 3]$

(c.) $x_{12} \in [1, 3]$

(d.) $x_{12} \in [2, 4]$

50. The smallest degree of the polynomial that interpolates the data

x	-2	-1	0	1	2	3
f(x)	-58	-21	-12	-13	-6	27

is

(a.) 3

(b.) 4

(c.) 5

(d.) 6

51. Suppose that x_0 is sufficiently close to 3. Which of the following iterations $x_{n+1} = g(x_n)$ will converge to the fixed point $x = 3$?

(a.) $x_{n+1} = -16 + 6x_n + \frac{3}{x_n}$

(b.) $x_{n+1} = \sqrt{3 + 2x_n}$

(c.) $x_{n+1} = \frac{3}{x_n - 2}$

(d.) $x_{n+1} = \frac{x_n^2 - 2}{2}$

52. Consider the quadrature formula,

$\int_{-1}^{1} | x | f(x) dx \approx \frac{1}{2}\left[f(x_0) + f(x_1) \right]$

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(a.) 1
(b.) 2
(c.) 3
(d.) 4

53. Let A, B and C be three events such that P(A) = 0.4, P(B) = 0.5, P(A∪B) = 0.6, P(C) = 0.+ and P(A∪B∪C^c) = 0.1
Then P(A∪B|C)=

(a.) $\frac{1}{2}$
(b.) $\frac{1}{3}$
(c.) $\frac{1}{4}$
(d.) $\frac{1}{5}$

54. Consider two identical boxes B_1 and B_2, where the box B (i = 1, 2) contains i +2 red and 5-i-1 white balls. A fair die is cast. Let the number of dots shown on the top face of the die be N. If N is even or 5, then two balls are down with replacement from the box B_1, otherwise, two balls are drawn with replacement from the box B_2. The probability that the two drawn balls are of different colours is

(a.) $\frac{7}{25}$
(b.) $\frac{9}{25}$
(c.) $\frac{12}{25}$
(d.) $\frac{16}{25}$

55. Let X_1, X_2,.... be a sequence of independent and identically distributed random variable with

$$P(X_1 = -1) = P(X_1 = 1) = \frac{1}{2}$$

Suppose for the standard normal random variable Z, P(-0.1< Z ≤ 0.1) = 0.08.

If $S_n = \sum_{i=1}^{n^2} X_i$, *then* $\lim_{n\to\infty} P\left(S_n > \frac{n}{10}\right) =$

(a.) 0.42
(b.) 0.46
(c.) 0.50

$$\bar{X} = \frac{1}{5}\sum_{i=1}^{5} X_i \quad \ldots \sum^{5} \left(X_i - \bar{X}\right)^2$$

Then $E\left(T^2 \bar{X}^2\right) =$

(a.) 3
(b.) 3.6
(c.) 4.8
(d.) 5.2

57. Let x_1 = 3.5, x_2 = 7.5 and x_3 ... observed values of random sample ... three from a population having uni... distribution over the interval (θ, θ +5), where θ ∈ (0, ∞) is unknown and is to be estimated. Then which of the following is NOT a maximum likelihood estimate of θ?

(a.) 2.4
(b.) 2.7
(c.) 3.0
(d.) 3.3

58. The value of $\int_0^\infty \int_{1/y}^\infty x^4 e^{-x^3 y} dx dy$ equals

(a.) $\frac{1}{4}$
(b.) $\frac{1}{3}$
(c.) $\frac{1}{2}$
(d.) 1

59. $\lim_{n\to\infty}\left[(n+1)\int_0^1 x^n \ln(1+x)\,dx\right] =$

(a.) 0
(b.) In 2
(c.) In 3
(d.) ∞

60. Consider the function f: R → R defined by

$$f(x) = \begin{cases} x^4, & \text{if } x \text{ is rational}, \\ 2x^2 - 1, & \text{if } x \text{ is irrational} \end{cases}$$

Let S be the set of points where f is continuous. Then

(a.) S = {1}
(b.) S = {-1}
(c.) S = {-1, 1}
(d.) S = ϕ

61. For a positive real number p, let (f_n: n = 1, 2,....) be a sequence of functions defined on [0, 1] by

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Let $f(x)=\lim_{n\to\infty} f_n(x),\ x\in[0,1]$. Then, on [0, 1]

(a.) f is Riemann integrable

(b.) the improper integral $\int_0^1 f(x)\,dx$ converges for p ≥ 1

(c.) the improper integral $\int_0^1 f(x)\,dx$ converges for p < 1

(d.) f_n converges uniformly

62. Which of the following inequality is NOT true for $x\in\left(\frac{1}{4},\frac{3}{4}\right)$

(a.) $e^{-x} > \sum_{j=0}^{2}\frac{(-x)^j}{j!}$

(b.) $e^{-x} < \sum_{j=0}^{3}\frac{(-x)^j}{j!}$

(c.) $e^{-x} > \sum_{j=0}^{4}\frac{(-x)^j}{j!}$

(d.) $e^{-x} > \sum_{j=0}^{5}\frac{(-x)^j}{j!}$

63. Let u(*x*, *y*) be the solution of the Cauchy problem

$xu_x+u_y=1,\ u(x,0)=2\ln(x),\ x>1$

Then $u(e,1)=$

(a.) -1

(b.) 0

(c.) 1

(d.) e

64. Suppose

$y(x)=\lambda\int_0^{2\pi} y(t)\sin(x+t)\,dt,\ x\in[0,2\pi]$

has eigenvalue $\lambda=\frac{1}{\pi}$ and $\lambda=-\frac{1}{\pi}$, with corresponding eigenfunctions $y_1(x)=\sin(x)+\cos(x)$ and $y_2(x)=\sin(x)-\cos(x)$, respectively. Then the integral equation

$y(x)=f(x)+\frac{1}{\pi}\int_0^{2\pi} y(t)\sin(x+t)\,dt,\ x\in[0,2\pi]$

has a solution when f(x)=

(a.) 1

(b.) cos(x)

$u_{xx}u_{yy}=0,$ [illegible] $1<y<1$

$u_x(0,y)=u_x$ [illegible]

$u_y(x,-1)=0,\ u_y($ [illegible] $\sin(x)$

The problem admits so[illegible]

(a.) $\alpha=0,\ \beta=1$

(b.) $\alpha=-1,\ \beta=\frac{\pi}{2}$

(c.) $\alpha=1,\ \beta=\frac{\pi}{2}$

(d.) $\alpha=1,\ \beta=-\pi$

66. The functional

$\int_0^1 (1+x)(y;)^2\,dx,\ y(0)=0,\ y(1)=1,$

possesses

(a.) strong maxima

(b.) strong minima

(c.) weak maxima but NOT a strong maxima

(d.) weak minima but NOT a strong minima

67. The value of α for which the integral equation $u(x)=\alpha\int_0^1 e^{x-t}u(t)dt$, has a non-trivial solution is

(a.) -2

(b.) -1

(c.) 1

(d.) 2

68. Let $P_n(x)$ be the Legendre polynomial of degree *n* and let

$P_{m+1}(0)=-\frac{m}{m+1.}P_m(0),\ m=1,2,.....$

If $P_n(0)=-\frac{5}{16},$ *then* $\int_{-1}^{1}P_n^2(x)\,dx=$

(a.) $\frac{2}{13}$

(b.) $\frac{2}{9}$

(c.) $\frac{5}{16}$

(d.) $\frac{2}{5}$

69. For which of the following pair of functions $y_1(x)$ and $y_2(x)$, continuous function $p(x)$ and $q(x)$ can be

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$y_2(x)$ give two linearly independent solution of

$y''+p(x)y'+q(x)y=0, x\in[-1,1]$

(a.) $\frac{2}{13}$

(b.) $\frac{2}{9}$

(c.) $\frac{5}{16}$

(d.) $\frac{2}{5}$

70. Let $J_0(.)$ and $J_1(.)$ be the Bessel functions of the first kind of orders zero and one, respectively.

If $£(J_0(t))=\frac{1}{\sqrt{s^2+1}}$, then $£(J_1(t))=$

(a.) $\frac{s}{\sqrt{s^2+1}}$

(b.) $\frac{1}{\sqrt{s^2+1}}-1$

(c.) $1-\frac{s}{\sqrt{s^2+1}}$

(d.) $\frac{s}{\sqrt{s^2+1}}-1$

COMMON DATA QUESTIONS

Common Data for Questions 71, 72, 73:

Let P[0, 1] = {p : p is a polynomial function on [0, 1]}. For p∈ P[0, 1] define

||P|| = sup {|p(x)| :$0\le x\le 1$}

Consider the map T:P[0, 1] → P[0, 1] defined by

$$(Tp)(x)=\frac{d}{dx}(p(x)).$$

71. The linear map T is

(a.) one to one and onto

(b.) one to one but NOT onto

(c.) onto but NOT one to one

(d.) neither one to one nor onto

72. The normal linear space P[0, 1] is

(a.) a finite dimensional normed linear space which is NOT a Banach space

73. The map T

(a.) closed an

(b.) neither conti ... osed

(c.) continuous but

(d.) closed but NOT c

Common Data for Questio

Let X and Y be jointly distributed variables such that the conditional distribu Y, given X = x, is uniform on the interval (x x + 1). Suppose E(X) = 1 and Var (X) = $\frac{5}{3}$.

74. The mean of the random variable Y is

(a.) $\frac{1}{2}$

(b.) 1

(c.) $\frac{3}{2}$

(d.) 2

75. The variance of the random variable Y is

(a.) $\frac{1}{2}$

(b.) $\frac{2}{3}$

(c.) 1

(d.) 2

TWO MARKS QUESTIONS (76-85)

Linked Answer Questions: 76-85 carry two marks each

Statement for Lined Answer Questions 76 and 77:

Suppose the equation $x^2y''-xy'+(1+x^2)y=0$ has a solution of the form $y=x^r\sum_{n=0}^{\infty}c_nx^n, c_0\neq 0$

76. The indicial equation for r is

(a.) $r^2 - 1 = 0$

(b.) $(r-1)^2 = 0$

(c.) $(r+1)^2 = 0$

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(b.) $n^2c_n + c_{n-2} = 0$

(c.) $c_n - n^2c_{n-2} = 0$

(d.) $c_n + n^2c_{n-2} = 0$

Statement for Linked Answer Question 78 and 79:

A particle of mass *m* slides down without friction along a curve $z = 1 + \frac{x^2}{2}$ in the *xz*-plane under the action of constant gravity. Suppose the *z*-axis points vertically upwards. Let $\dot{x}$ and $\ddot{x}$ denote $\frac{dx}{dt}$ and $\frac{d^2x}{dt^2}$, respectively.

78. The Lagrangian of the motion is

(a.) $\frac{1}{2}m\dot{x}^2\left(1+x^2\right) - mg\left(1+\frac{x^2}{2}\right)$

(b.) $\frac{1}{2}m\dot{x}^2\left(1+x^2\right) + mg\left(1+\frac{x^2}{2}\right)$

(c.) $\frac{1}{2}mx^2\dot{x}^2 - mg\left(1+\frac{x^2}{2}\right)$

(d.) $\frac{1}{2}m\dot{x}^2\left(1+x^2\right) - mg\left(1+\frac{x^2}{2}\right)$

79. The Lagrangian equation of motion is

(a.) $\ddot{x}\left(1+x^2\right) = -x\left(g+\dot{x}^2\right)$

(b.) $\ddot{x}\left(1+x^2\right) = x\left(g+\dot{x}^2\right)$

(c.) $\ddot{x} = -gx$

(d.) $\ddot{x}\left(1-x^2\right) = x\left(g+\dot{x}^2\right)$

Statements for Linked Answer Questions 80 and 81:

Let u(*x*, t) be the solution of the one dimensional wave equation

$u_{tt} - 4u_{xx} = 0, -\infty < x < \infty, t > 0$

$u(x,0) = \begin{cases} 16 - x^2, & |x| \le 4, \\ 0, & otherwise, \end{cases}$ and

$u_t(x,0) = \begin{cases} 1, & |x| \le 2, \\ 0, & otherwise, \end{cases}$

(b.) $\frac{1}{2}\left[32 - ([illegible] + 2t)^2\right] + t$

(c.) $\frac{1}{2}\left[32 - (2 - 2t [illegible] \right] + 1$

(d.) $\frac{1}{2}\left[16 - (2-2t)^2\right] + \frac{1}{2}\left[[illegible] \right\}\right]$

81. The value of $u_t(2, 2)$

(a.) equals -15

(b.) equals -16

(c.) equals 0

(d.) does NOT exist

Statement for Linked Answer Questions 82 and 83:

Suppose $E = \{(x, y) : xy \ne 0\}$. Let $f : R^2 \to R$ be defined by

$$f(x,y) = \begin{cases} 0, & if\ xy = 0, \\ y\sin\left(\frac{1}{x}\right) + x\sin\left(\frac{1}{y}\right), & otherwise. \end{cases}$$

Let S_1 be the set of points in R^2 where f_x exists and S_2 be the set of the points in R^2 where f_y exists. Also, let E_1 be the set of points where f_x is continuous and E_2 be the set of points where f_y is continuous.

82. S_1 and S_2 are given by

(a.) $S_1 = E \cup \{x, y): y = 0\}$, $S_2 = E \cup \{(x, y) : x = 0\}$

(b.) $S_1 = E \cup \{x, y): y = 0\}$, $S_2 = E \cup \{(x, y) : y = 0\}$

(c.) $S_1 = S_2 = R^2$

(d.) $S_1 = S_2 = E \cup \{(0, 0)\}$

83. E_1 and E_2 are given by

(a.) $E_1 = E_2 = S_1 \cap S_2$

(b.) $E_1 = E_2 = S_1 \cap S_2/\{(0,0)\}$

(c.) $E_1 = S_1$, $E_2 = S_2$

(d.) $E_1 = S_2$, $E_2 = S_1$

Statement for Linked Answer Questions 84 and 85:

Let $A \begin{bmatrix} 3 & 0 & 0 \\ 0 & 6 & 2 \\ 0 & 2 & 6 \end{bmatrix}$

and let $\lambda_1 \ge \lambda_2 \ge \lambda_3$ be the eigenvalue of A.

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(b.) (8, 4, 3)
(c.) (9, 3, 3)
(d.) (7, 5, 3)

85. The matrix P such that $P^t A P = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_2 \end{bmatrix}$ is

(a.) $\begin{bmatrix} \frac{1}{\sqrt{3}} & 0 & \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{bmatrix}$

(b.) $\begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{-2}{\sqrt{6}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$

(c.) $\begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix}$

(d.) $\begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$

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